Ankara Ü. BLM bölümü

BLM433 Sayısal analiz teknikleri

Multidimensional Optimization

Techniques to find minimum and maximum of a function of

several variables are described.

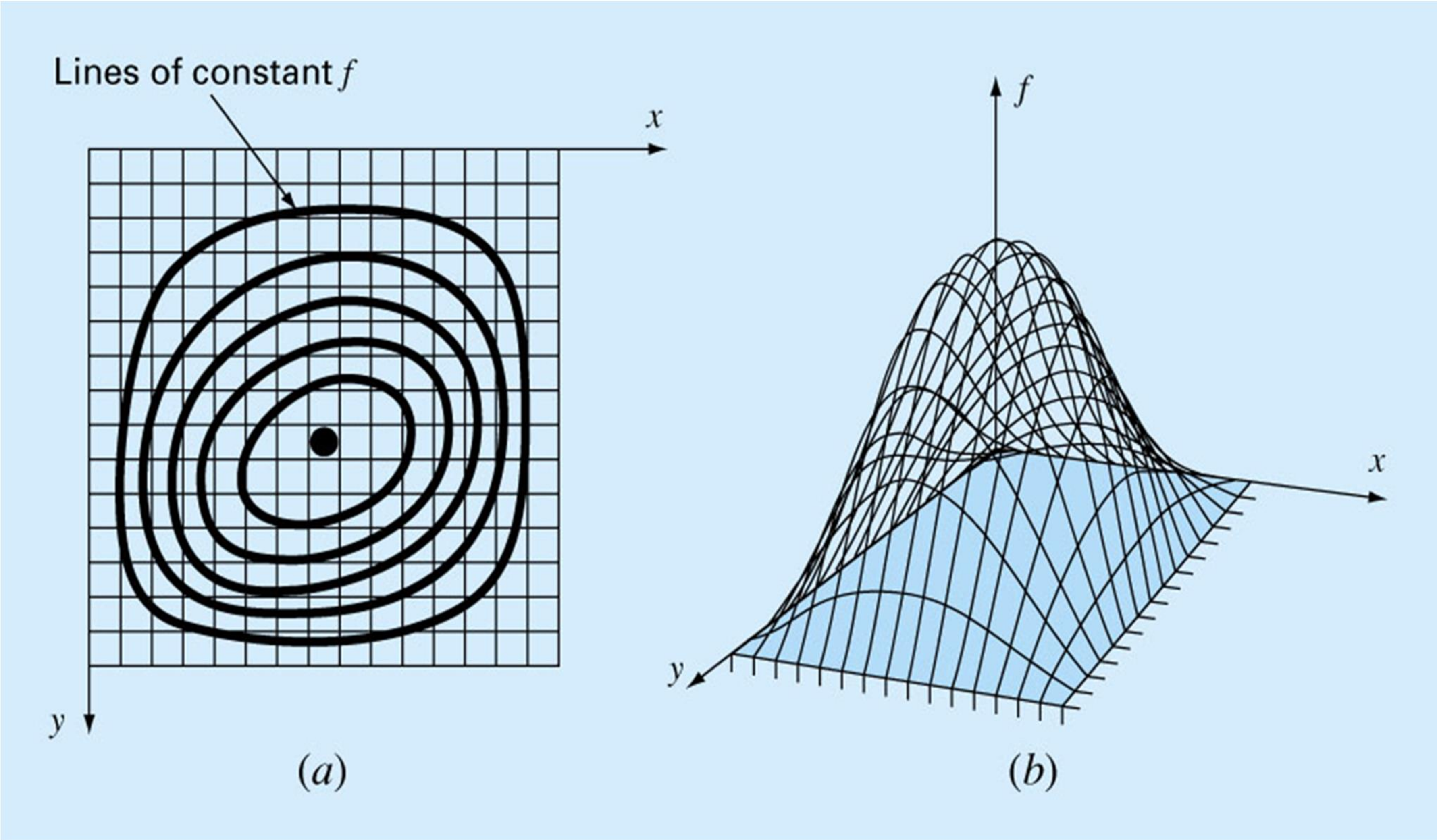
These techniques are classified as:

That require derivative evaluation

Gradient or descent (or ascent) methods

That do not require derivative evaluation

Non-gradient or direct methods.



Lines of constant f
x
y
(a)
f
x
y
(b)

RANDOM SEARCH method

Based on evaluation of the function randomly at selected values of the

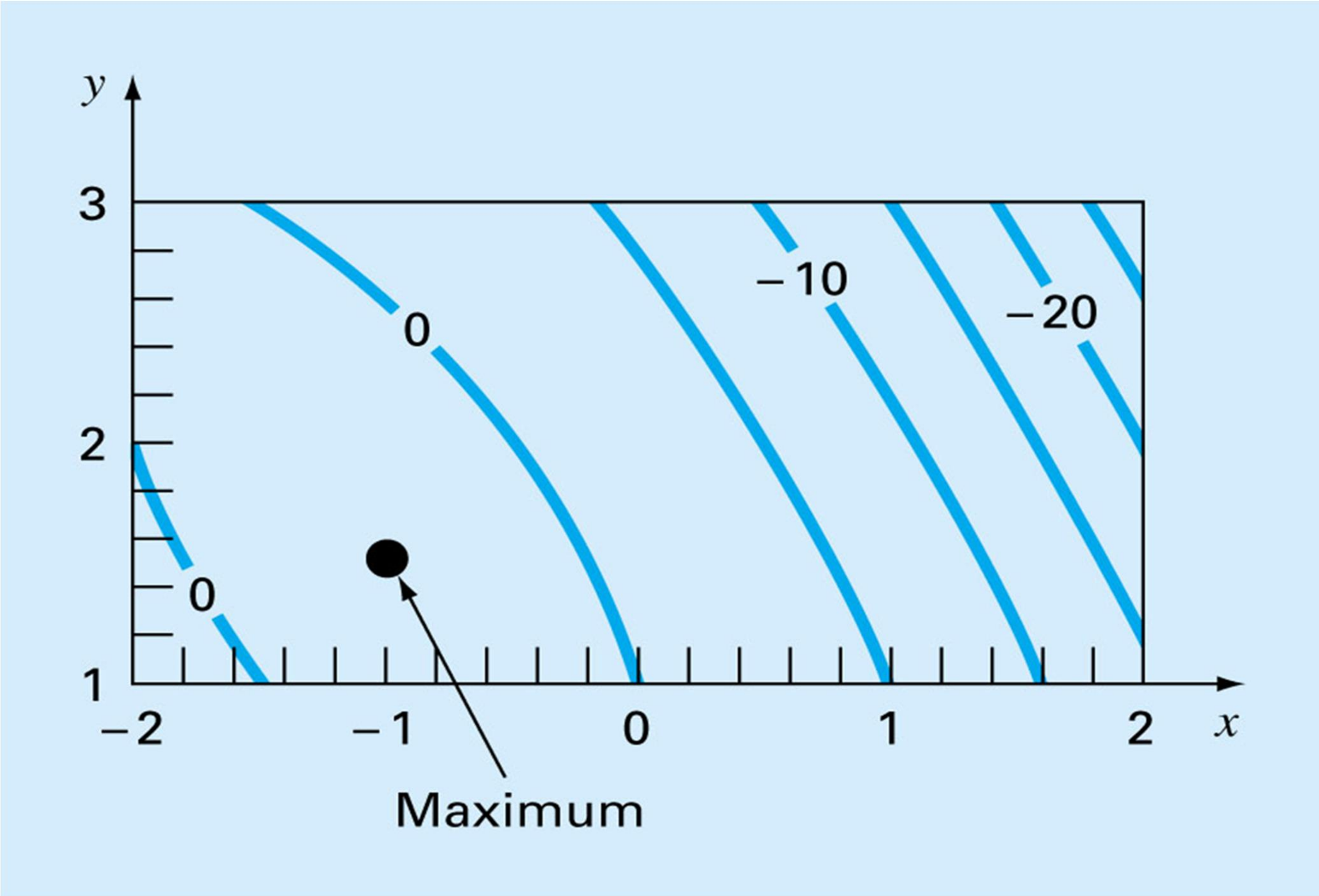
independent variables.

If a sufficient number of samples are conducted, the optimum will be eventually located.

Example: maximum of a function

f (x, y)=y-x-2x2-2xy-y2

can be found using a random number generator.



y
3
2
1
−2
−1
0
1
2
x
0
0
−10
−20
Maximum

Advantages

Works even for discontinuous and nondifferentiable functions.

Always finds the global optimum rather than the global minimum.

Disadvantages

As the number of independent variables grows, the task can become onerous.

Not efficient, it does not account for the behavior of underlying function.

More efficient than random search and still doesn’t

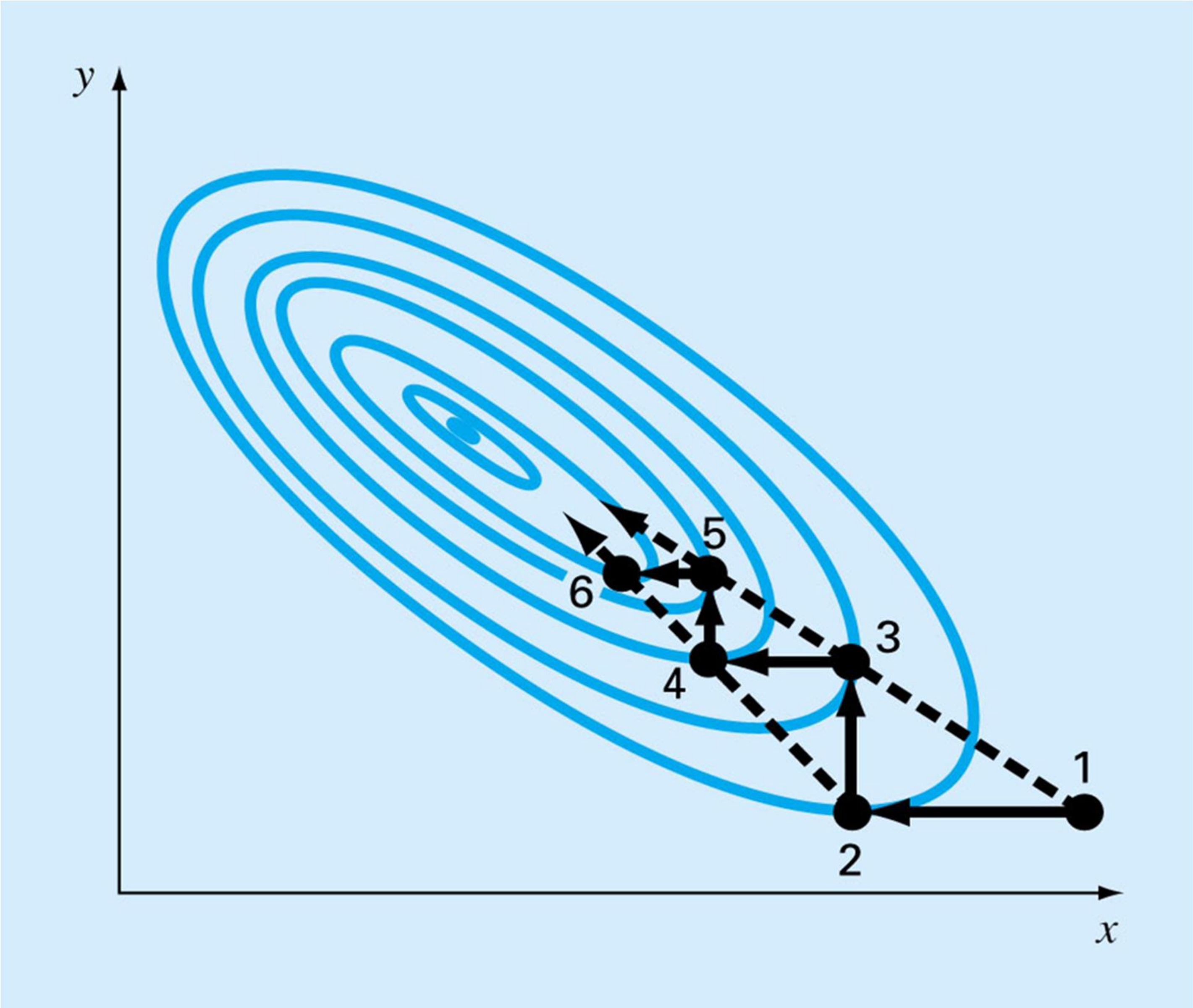
require derivative evaluation.

The basic strategy is:

Change one variable at a time while the other variables are held constant.

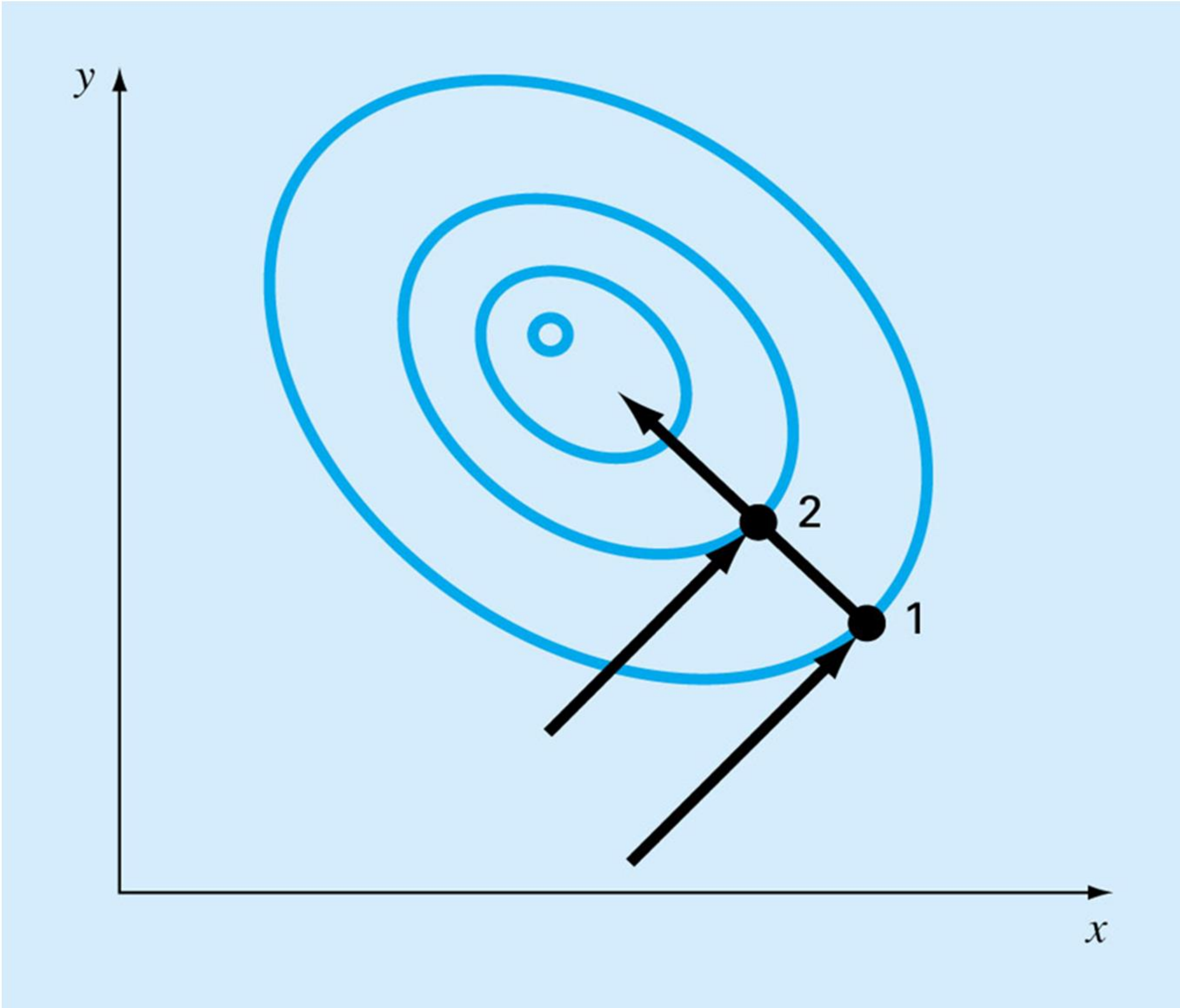
Thus problem is reduced to a sequence of one-dimensional searches that can be solved by variety of methods.

The search becomes less efficient as you approach the maximum.

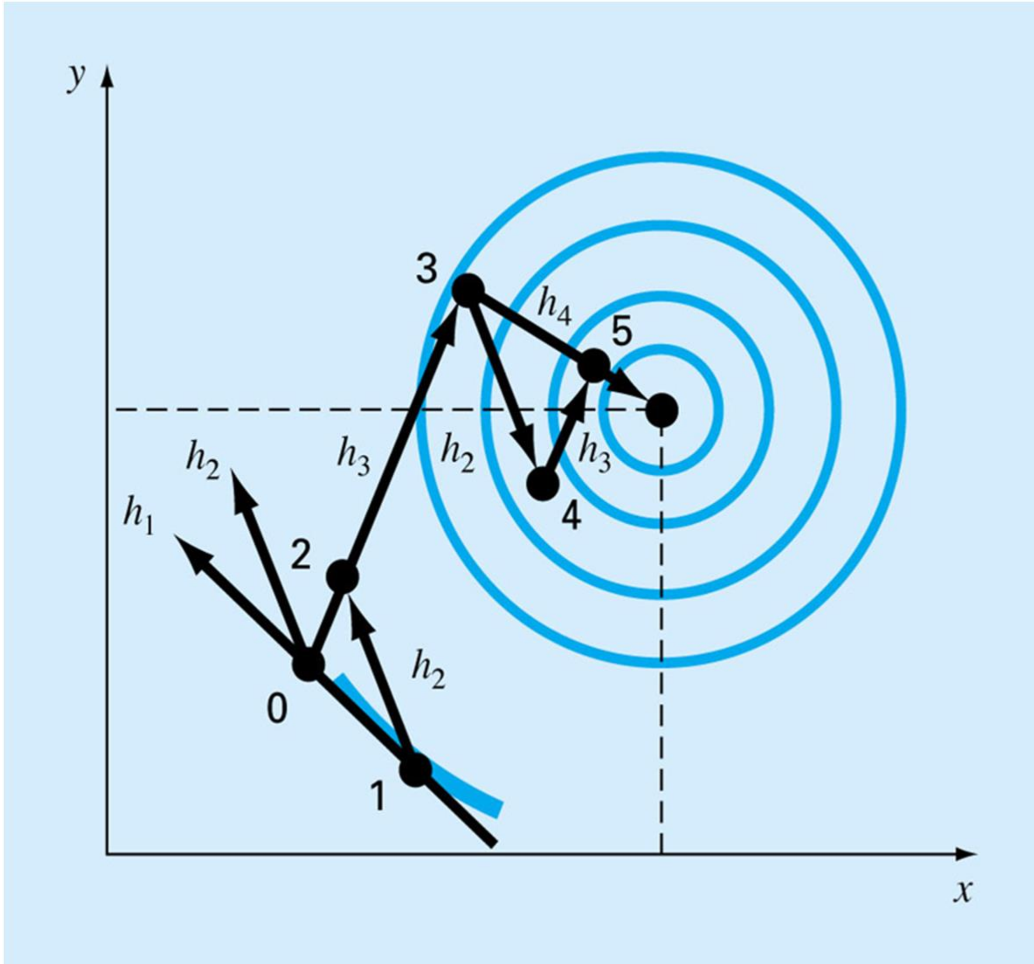


y
x
1
2
3
4
5
6

Pattern direction



Powell method



The Gradient Method

If f(x,y) is a two dimensional function, the

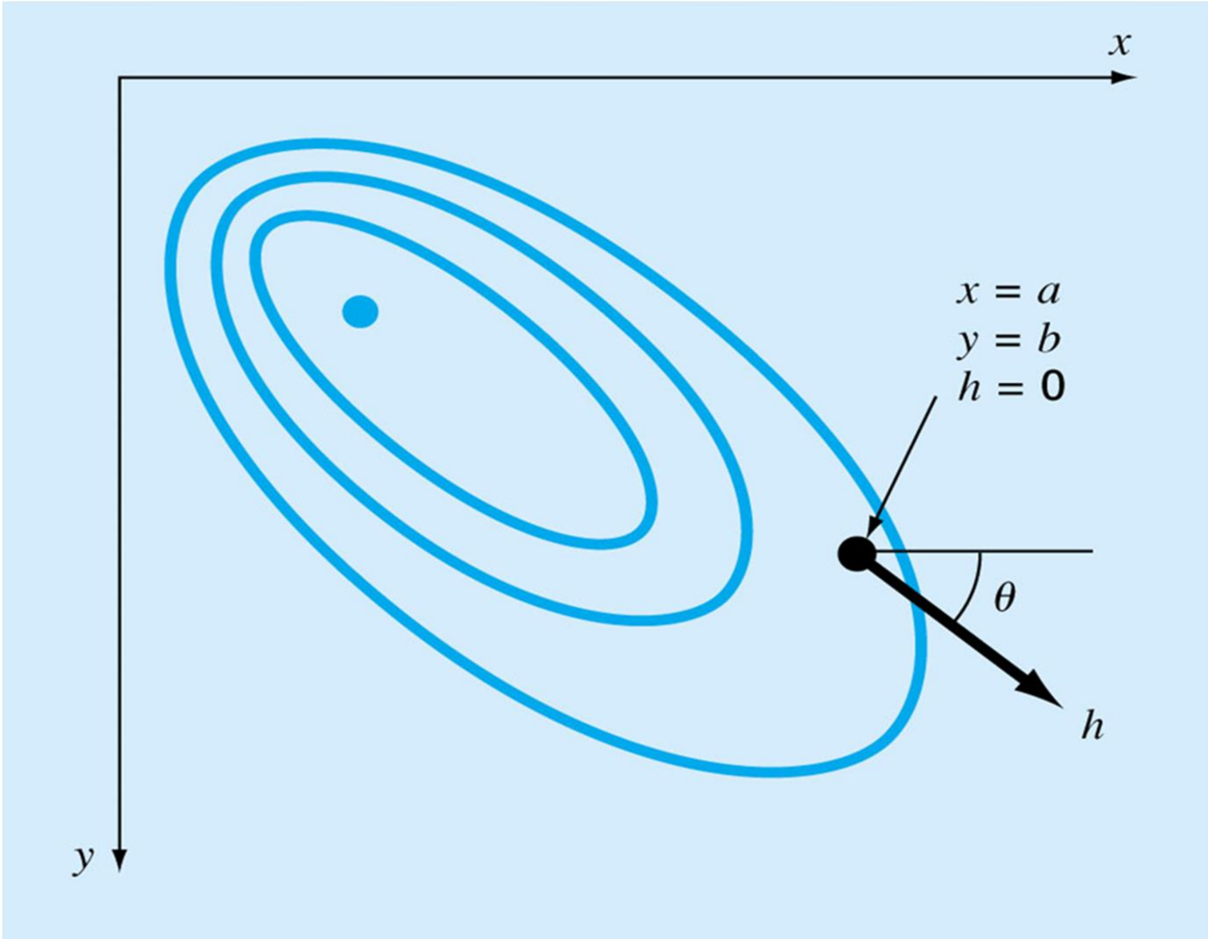
gradient vector tells us

What direction is the steepest ascend?

How much we will gain by taking that step?

$$\nabla f = \frac{\partial f}{\partial x}\mathrm{i} + \frac{\partial f}{\partial y}\mathrm{j} \qquad \text{or } del\ f$$

$$\nabla f(x) = \begin{Bmatrix} \frac{\partial f}{\partial x_1}(\mathrm{x}) \\ \frac{\partial f}{\partial x_2}(\mathrm{x}) \\ \vdots \\ \frac{\partial f}{\partial x_n}(\mathrm{x}) \end{Bmatrix}$$



Hessian

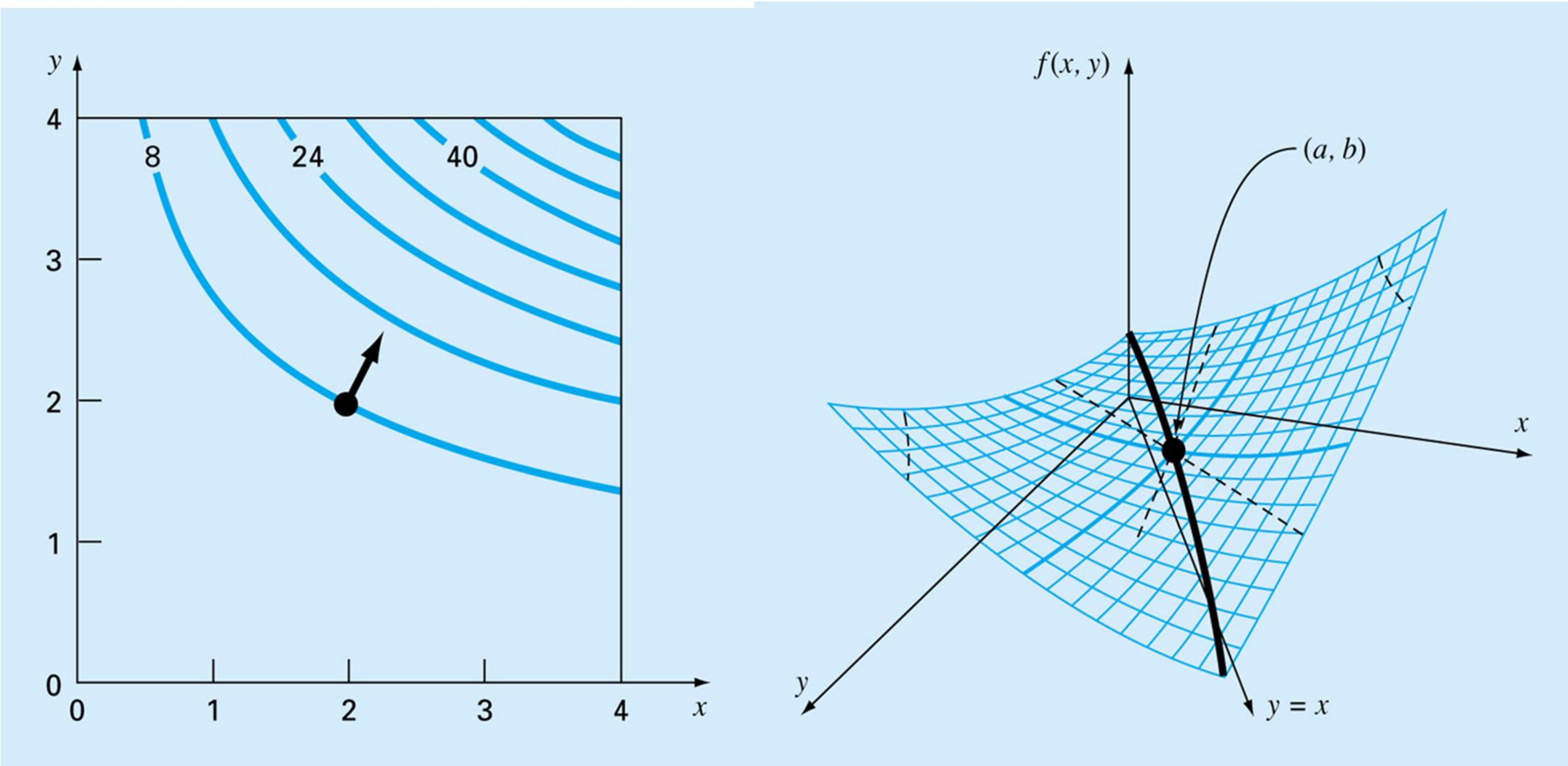
For one dimensional functions both first and second

derivatives valuable information for searching out optima.

First derivative provides (a) the steepest trajectory of the function and (b) tells us that we have reached the maximum.

Second derivative tells us that whether we are a maximum or minimum.

For two dimensional functions whether a maximum or a minimum occurs involves not only the partial derivatives w.r.t. x and y but also the second partials w.r.t. x and y.



y
4
3
2
1
0
0
1
2
3
4
x
8
24
40
f(x, y)
(a, b)
x
y
y = x

$$|H| = \frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$$

If $|\mathrm{H}| \succ 0$ and $\frac{\partial^2 f}{\partial x^2} \succ 0$, then f(x, y) has a local minimum

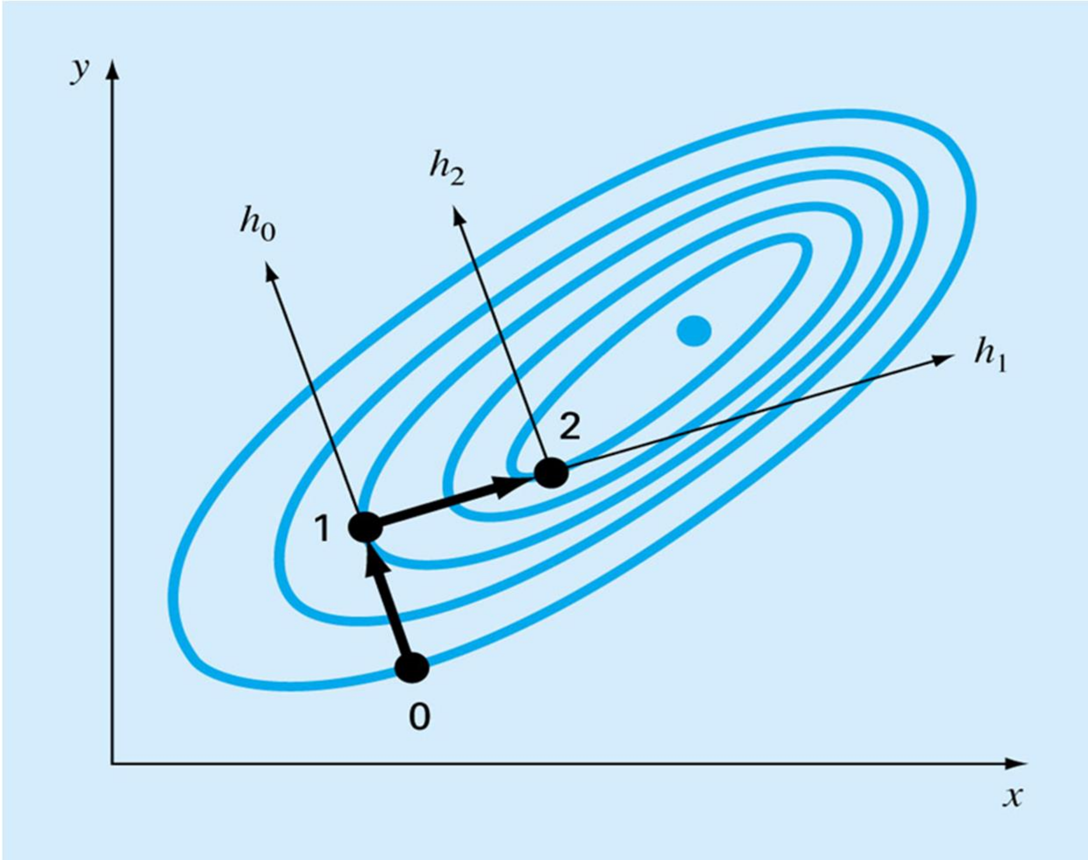
If $|\mathrm{H}| \succ 0$ and $\frac{\partial^2 f}{\partial x^2} \prec 0$, then f(x, y) has a local minimum

If $|\mathrm{H}| \prec 0$, then f(x, y) has a saddle point

The stepest accent method

$$x = x_o + \frac{\partial f}{\partial x} h$$

$$y = y_o + \frac{\partial f}{\partial y} h$$



$$\nabla f = 3\text{i} + 4\text{j}$$

$$x = 1 + 3h$$

$$y = 2 + 4h$$