Ankara Ü. BLM bölümü

BLM433-Sayıdsal Analiz Teknikleri

Notion of numerical differentiation has been introduced early. In this part more accurate formulas that retain more terms will be developed.

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2}h^2 + \cdots$$

$$f'(xi) = \frac{f(x_{i+1}) - f(x_i)}{h} - \frac{f''(x_i)}{2}h + O(h^2)$$

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + O(h)$$

$$f'(xi) = \frac{f(x_{i+1}) - f(x_i)}{h} - \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{2h^2}h + O(h^2)$$

$$f'(xi) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h} + O(h^2)$$

Similar improved versions can be developed for the backward and centered formulas as well as for the approximations of the higher derivatives.

There are two ways to improve derivative estimates when employing finite divided differences:

Decrease the step size, or

Use a higher-order formula that employs more points.

A third approach, based on Richardson extrapolation, uses two derivative estimates t compute a third, more accurate approximation.

$$I \cong I(h_2) + \frac{1}{(h_1/h_2)^2 - 1}[I(h_2) - I(h_1)]$$

$$h_2 = h_1/2$$

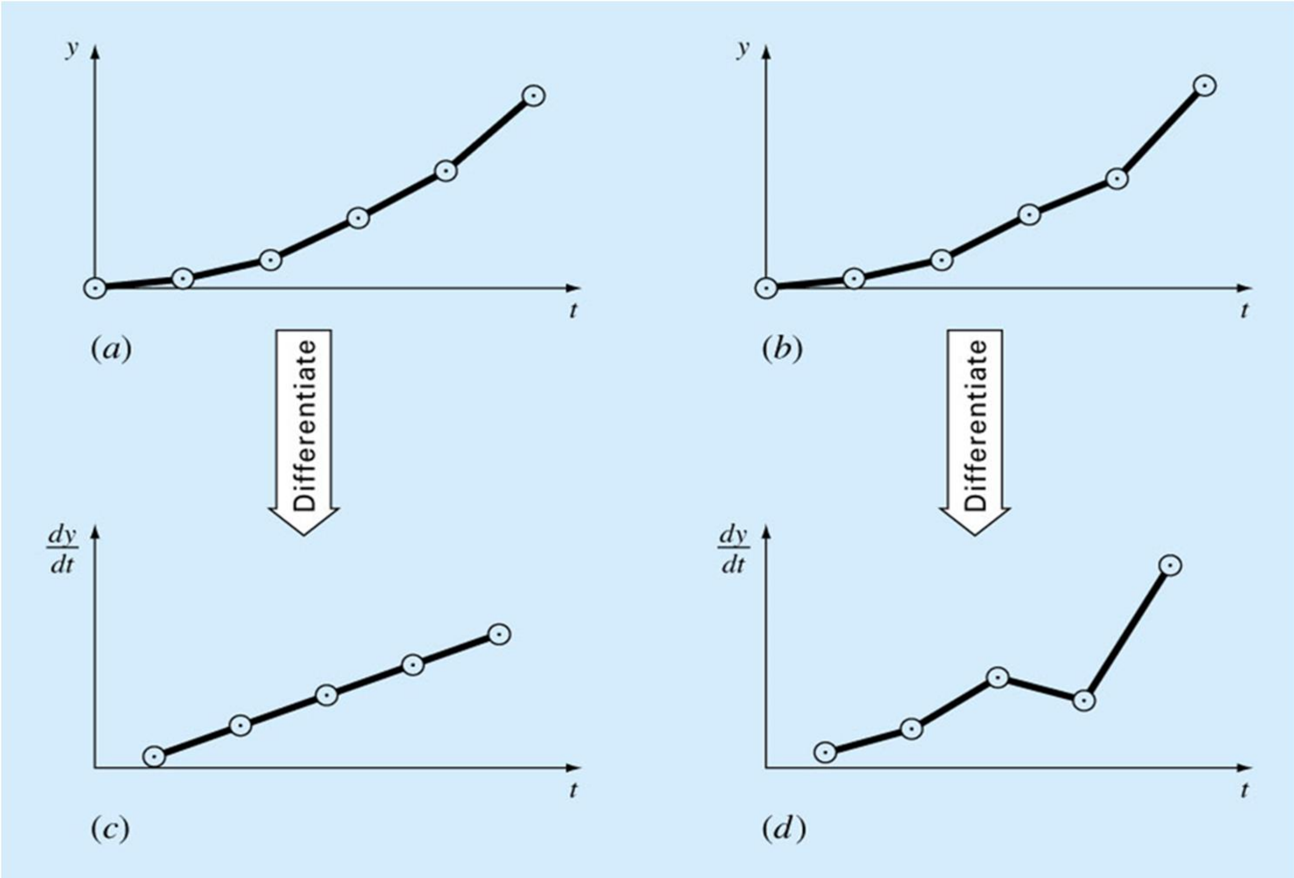
$$I \cong \frac{4}{3} I(h_2) - \frac{1}{3} I(h_1)]$$

$$D \cong \frac{4}{3} D(h_2) - \frac{1}{3} D(h_1)]$$

For centered difference approximations with O(h2). The application of this formula yield a new derivative estimate of O(h4).

Data from experiments or field studies are often collected at unequal intervals. One way to handle such data is to fit a second-order Lagrange interpolating polynomial.

$$f'(x) = f(x_{i-1})\frac{2x - x_i - x_{i+1}}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})} + f(x_i)\frac{2x - x_{i-1} - x_{i+1}}{(x_i - x_{i-1})(x_i - x_{i+1})} +$$
$$f(x_{i+1})\frac{2x - x_{i-1} - x_i}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)}$$



y
t
(a)
Differentiate
y
t
(b)
Differentiate
dy/dt
t
(c)
dy/dt
t
(d)