Physics 122: Electricity &Magnetism
Coulomb's Law

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Electric Force and Coulomb's Law

- Although we can write down a vector form for the force, it is easier to simply use the equation for the magnitude, and just use the "like charges repel, opposites attract" rule to figure out the direction of the force.
- Note that the form for Coulomb's Law is exactly the same as for gravitational force between two masses

$$F = G\frac{m_1 m_2}{r^2} \qquad G \Rightarrow k \qquad m \Rightarrow q$$

Note BIG difference, There is only one "sign" of mass, only attraction.

- Note also that the mass is an intrinsic property of matter. Likewise, charge is also an intrinsic property. We only know it exists, and can learn its properties, because of the force it exerts.
- Because it makes other equations easier to write, Coulomb's constant is actually written $k = \frac{1}{4\pi\varepsilon_0}$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

where $\varepsilon_0 = 8.85\times10^{-12}$ $C^2/N\text{-}m^2$ is called the permittivity of free space.

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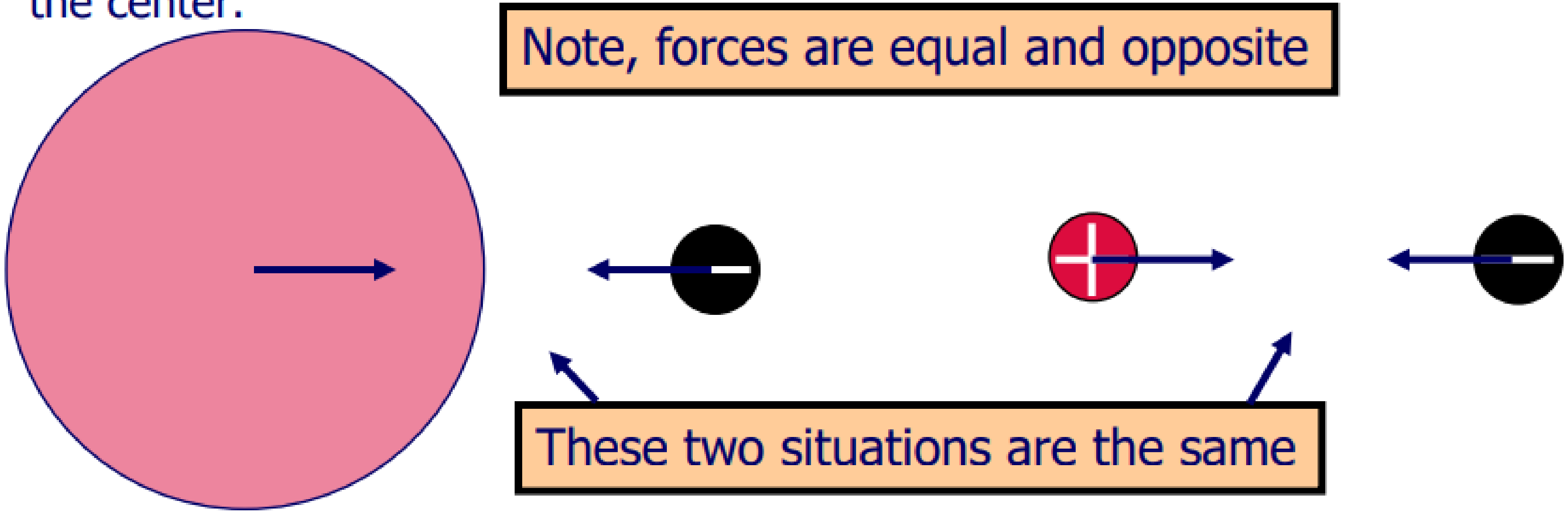
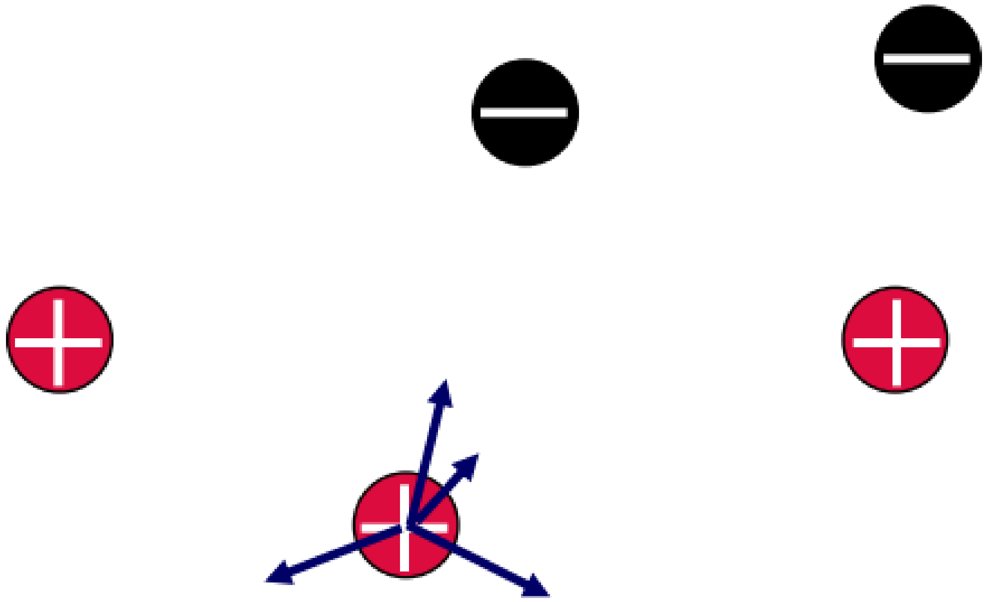
Spherical Conductors

- Because it is conducting, charge on a metal sphere will go everywhere over the surface.
- You can easily see why, because each of the charges pushes on the others so that they all move apart as far as they can go. Because of the symmetry of the situation, they spread themselves out uniformly.
- There is a theorem that applies to this case, called the shell theorem, that states that the sphere will act as if all of the charge were concentrated at the center.



Case of Multiple Charges

- You can determine the force on a particular charge by adding up all of the forces from each charge.



Forces on one charge due to a number of other charges

The smallest unit of charge e known in nature[2] is the charge on an electron $(-e)$ or a proton $(+e)$ and has a magnitude

$$e = 1.602\,19 \times 10^{-19}\ \text{C} \tag{23.5}$$

Therefore, 1 C of charge is approximately equal to the charge of 6.24×10^{18} electrons or protons. This number is very small when compared with the number of free electrons in 1 cm^3 of copper, which is on the order of 10^{23}.

Example 23.1 The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3 x 10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

$$F_e = k_e \frac{|e||-e|}{r^2} = (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19}\ \text{C})^2}{(5.3 \times 10^{-11}\ \text{m})^2} = 8.2 \times 10^{-8}\ \text{N}$$

$$F_g = G \frac{m_e m_p}{r^2}$$

$$= (6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)$$

$$\times \frac{(9.11 \times 10^{-31}\ \text{kg})(1.67 \times 10^{-27}\ \text{kg})}{(5.3 \times 10^{-11}\ \text{m})^2}$$

$$= 3.6 \times 10^{-47}\ \text{N}$$

Let's Calculate the Exact Location

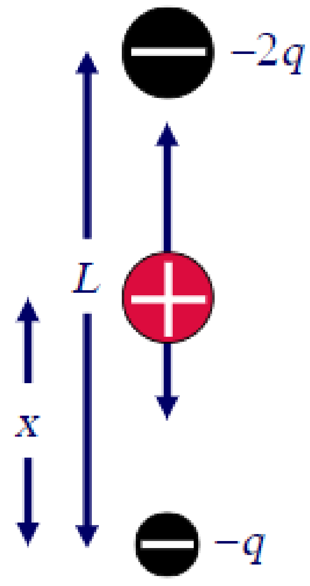
- Force is attractive toward both negative charges, hence could balance.
- Need a coordinate system, so choose total distance as L, and position of + charge from $-q$ charge as x.
- Force is sum of the two force vectors, and has to be zero, so

$$F = F_1 + F_2 = k\frac{2qQ}{(L-x)^2} - k\frac{qQ}{x^2} = 0$$

- A lot of things cancel, including Q, so our answer does not depend on knowing the + charge value. We end up with

$$\frac{2}{(L-x)^2} = \frac{1}{x^2} \qquad \frac{(L-x)^2}{x^2} = 2 \Rightarrow \frac{L-x}{x} = \sqrt{2}$$

- Solving for x, $x = \frac{L}{1+\sqrt{2}} = 0.412L$, so slightly less than half-way between.



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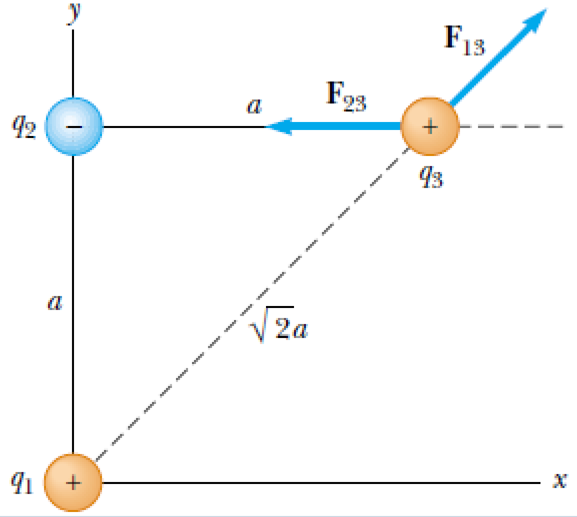
Example 23.2 Find the Resultant Force

Consider three point charges located at the corners of a right triangle as shown in Figure 23.8, where $q_1 = q_3 = 5.0\ \mu C$,



$q_2 = -2.0\ \mu C$, and $a = 0.10$ m. Find the resultant force exerted on q_3.

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Solution First, note the direction of the individual forces exerted by q_1 and q_2 on q_3. The force $\mathbf{F}_{23}$ exerted by q_2 on q_3 is attractive because q_2 and q_3 have opposite signs. The force $\mathbf{F}_{13}$ exerted by q_1 on q_3 is repulsive because both charges are positive.

The magnitude of $\mathbf{F}_{23}$ is

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(2.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{(0.10\ \mathrm{m})^2}$$

$$= 9.0\ \mathrm{N}$$

The magnitude of the force $\mathbf{F}_{13}$ exerted by q_1 on q_3 is

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(5.0 \times 10^{-6}\ \mathrm{C})(5.0 \times 10^{-6}\ \mathrm{C})}{2(0.10\ \mathrm{m})^2}$$

$$= 11\ \mathrm{N}$$

$$F_{13} \cos 45° = 7.9\ \mathrm{N}.$$

$$F_{3x} = F_{13x} + F_{23x} = 7.9\ \mathrm{N} + (-9.0\ \mathrm{N}) = -1.1\ \mathrm{N}$$

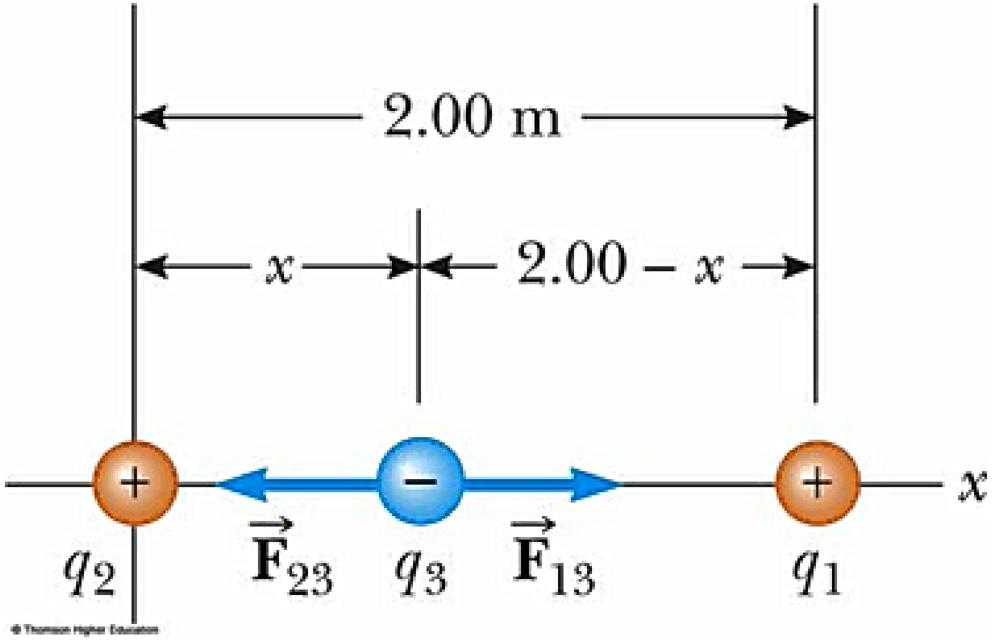
$$F_{3y} = F_{13y} + F_{23y} = 7.9\ \mathrm{N} + 0 = 7.9\ \mathrm{N}$$

We can also express the resultant force acting on q_3 in unit-vector form as

$$\mathbf{F}_3 = (-1.1\hat{\mathbf{i}} + 7.9\hat{\mathbf{j}})\ \mathrm{N}$$

Zero Resultant Force, Example 23.3

- Three point charges lie along the x axis
- The positive charge $q_1 = 15\mu C$ is at x=2m
- The positive charge $q_2 = 6\mu C$ is at the origin
- The net force acting on q_3 is zero
- Where is the resultant force equal to zero?



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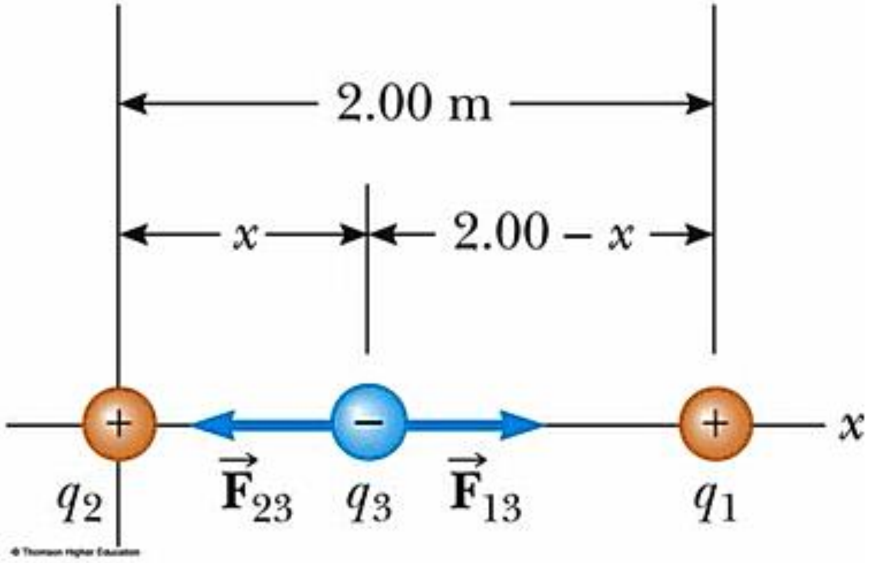
- $0 = \vec{F}_3 = \vec{F}_{23} + \vec{F}_{13}$

$$= -k_e \frac{|q_2||q_3|}{x^2} + k_e \frac{|q_1||q_3|}{(2-x)^2}$$



- Solve

$$k_e \frac{|q_2||q_3|}{x^2} = k_e \frac{|q_1||q_3|}{(2-x)^2}$$

$$(2.00 - x)^2 |q_2| = x^2 |q_1|$$

$$(4.00 - 4.00x + x^2)(6.00 \times 10^{-6}\ \mathrm{C}) = x^2 (15.0 \times 10^{-6}\ \mathrm{C})$$

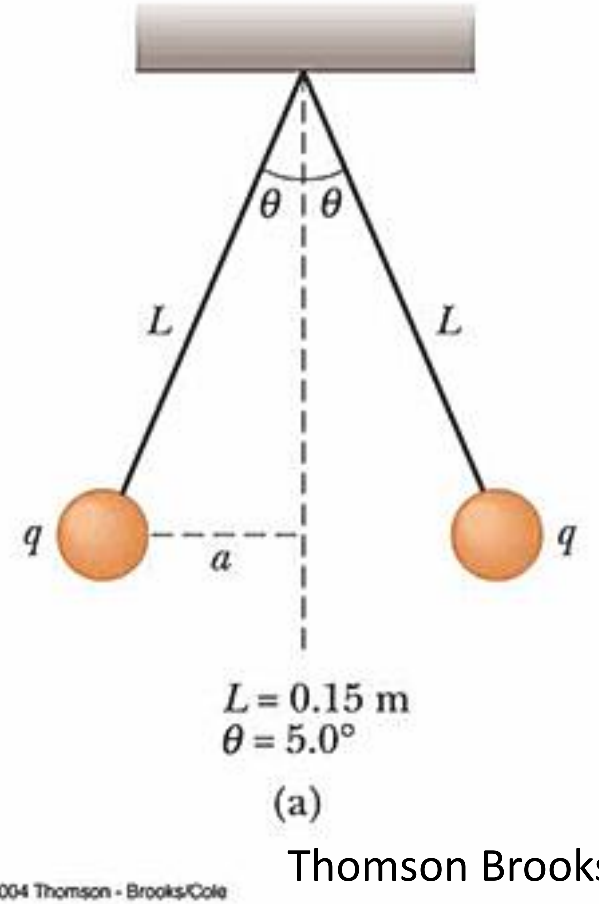
$$3.00x^2 + 8.00x - 8.00 = 0$$

$$x = 0.775\ \mathrm{m.}$$

Example 23.4 Find the Charge on the Spheres

- Two identical small charged spheres, each having a mass of 3×10^{-2} kg, hang in equilibrium
- The length of each string is 0.15m, and the angle θ is 5°
- Find the magnitude of the charge on each sphere

$\sin 5.0^\circ = 0.087$



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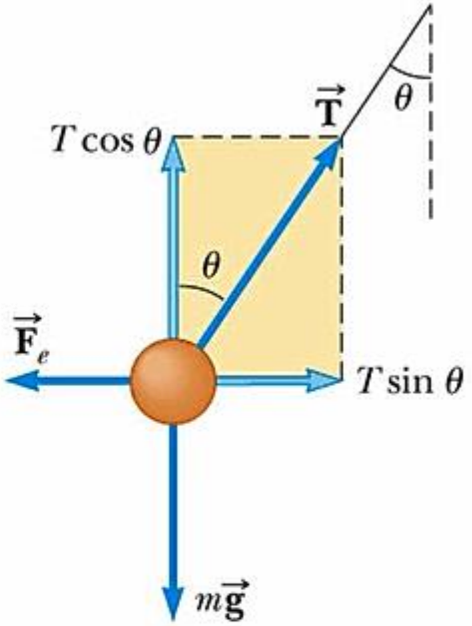
- The free body diagram includes the components of the tension, the electrical force, and the weight
- Solve for |*q*|
- You cannot determine the sign of *q*, only that they both have same sign

$$\sum F_x = T\sin\theta - F_e = 0$$

$$\sum F_y = T\cos\theta - mg = 0$$

$$F_e = k_e \frac{|q||q|}{r^2} = mg\tan\theta$$

Solve |*q*|!



$$F_e = mg\tan\theta = (3.0\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)\tan(5.0°)$$
$$= 2.6\times10^{-2}\ \text{N}$$

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$$\sin\theta = a/L$$

$$a = L\sin\theta = (0.15\ \text{m})\sin(5.0°) = 0.013\ \text{m}$$

The separation of the spheres is $2a = 0.026$ m.

$$|q|^2 = \frac{F_e r^2}{k_e} = \frac{(2.6\times10^{-2}\ \text{N})(0.026\ \text{m})^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.96\times10^{-15}\ \text{C}^2$$

$$|q| = 4.4\times10^{-8}\ \text{C}$$

sin5°=0.0875
tan5°=0.087

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Summary

- Charge is an intrinsic property of matter.
- Charge comes in two opposite senses, positive and negative.
- Mobil charges we will usually deal with are electrons, which can be removed from an atom to make positive charge, or added to an atom to make negative charge. A positively charged atom or molecule can also be mobil.
- There is a smallest unit of charge, e, which is e = 1.602×10^{-19} C. Charge can only come in units of e, so charge is quantized. The unit of charge is the Coulomb.
- Charge is conserved. Charge can be destroyed only in pairs (+e and –e can annihilate each other). Otherwise, it can only be moved from place to place.
- Like charges repel, opposite charges attract.
- The electric force is give by Coulomb's Law:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

- Materials can be either conductors or insulators.
- Conductors and insulators can both be charged by adding charge, but charge can also be induced.
- Spherical conductors act as if all of the charge on their surface were concentrated at their centers.