Physics 122: Electricity & Magnetism
Electric Field

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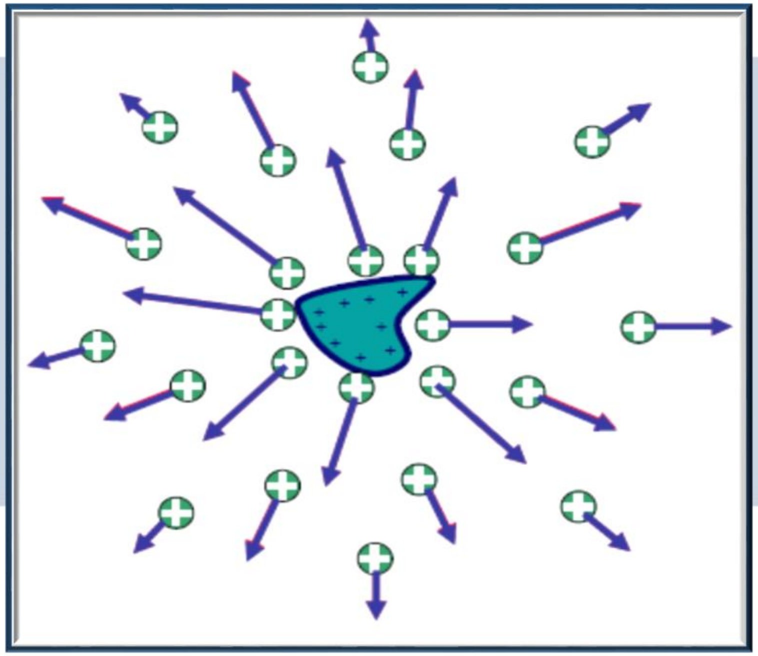
Electric Field

- Electric field is said to exist in the region of space around a charged object: The source charge.
- Concept of test charge:
  - Small and positive
  - Does not affect charge distribution
- Electric field: (SI unit: N/C)

$$\vec{E} = \frac{\vec{F}}{q}$$

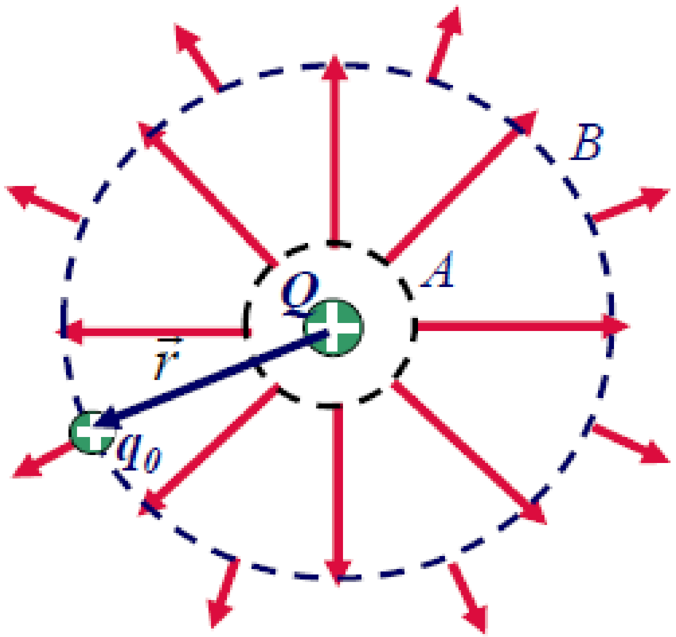


- Existence of an electric field is a property of its source.
- Presence of a test charge is not necessary for the field to exist.

Electric Field due to a Point Charge Q

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$



- Direction is radial: outward for +|Q|
  inward for -|Q|
- Magnitude: constant on any spherical shell
- Flux through any shell enclosing Q is the same: $E_A A_A = E_B A_B$

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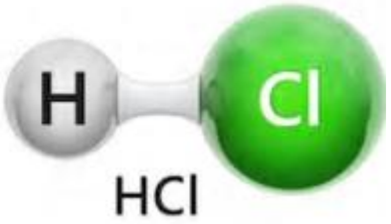
The electric dipole is a good model of many molecules, such as hydrochloric acid (HCl).



Example 23.6 Electric Field Due to Two Charges

Charges q_1 and q_2 are located on the *x* axis, at distances *a* and *b*, respectively, from the origin. (a) Find the components of the net electric field at the point P, which is at position (0, y).

$$E_1 = k_e \frac{|q_1|}{r_1^2} = k_e \frac{|q_1|}{a^2 + y^2}$$

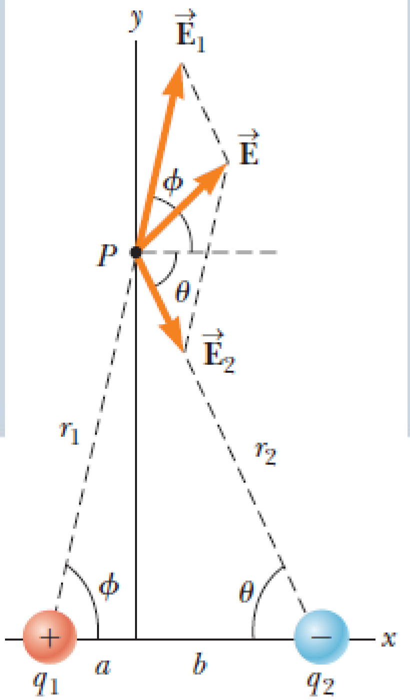
$$E_2 = k_e \frac{|q_2|}{r_2^2} = k_e \frac{|q_2|}{b^2 + y^2}$$

$$\vec{\mathbf{E}}_1 = k_e \frac{|q_1|}{a^2 + y^2} \cos\phi\,\hat{\mathbf{i}} + k_e \frac{|q_1|}{a^2 + y^2} \sin\phi\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}_2 = k_e \frac{|q_2|}{b^2 + y^2} \cos\theta\,\hat{\mathbf{i}} - k_e \frac{|q_2|}{b^2 + y^2} \sin\theta\,\hat{\mathbf{j}}$$

$$E_x = E_{1x} + E_{2x} = k_e \frac{|q_1|}{a^2 + y^2} \cos\phi + k_e \frac{|q_2|}{b^2 + y^2} \cos\theta$$

$$E_y = E_{1y} + E_{2y} = k_e \frac{|q_1|}{a^2 + y^2} \sin\phi - k_e \frac{|q_2|}{b^2 + y^2} \sin\theta$$



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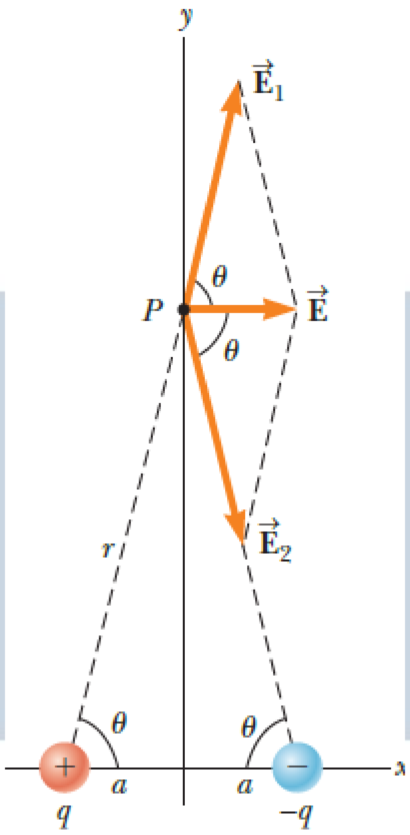
(B) Evaluate the electric field at point *P* in the special case that q1=q2 and a=b.

$$E_x = k_e \frac{q}{a^2 + y^2} \cos\theta + k_e \frac{q}{a^2 + y^2} \cos\theta = 2k_e \frac{q}{a^2 + y^2} \cos\theta$$

$$E_y = k_e \frac{q}{a^2 + y^2} \sin\theta - k_e \frac{q}{a^2 + y^2} \sin\theta = 0$$

$$\cos\theta = \frac{a}{r} = \frac{a}{(a^2 + y^2)^{1/2}}$$

$$E_x = 2k_e \frac{q}{a^2 + y^2} \left[\frac{a}{(a^2 + y^2)^{1/2}}\right] = k_e \frac{2aq}{(a^2 + y^2)^{3/2}}$$



(C) Find the electric field due to the electric dipole when point *P* is a distance *y* >> *a* from the origin.

$$E \approx k_e \frac{2aq}{y^3}$$

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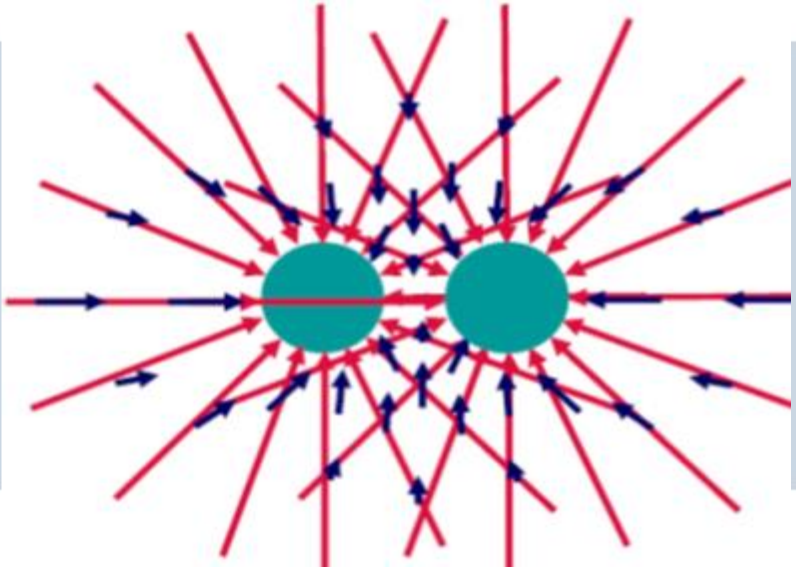
Electric Field due to a Group of Individual Charge

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \ldots + \vec{F}_{0n}$$

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \ldots \frac{\vec{F}_{0n}}{q_0}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots \vec{E}_n$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i^2} \hat{r}_i$$



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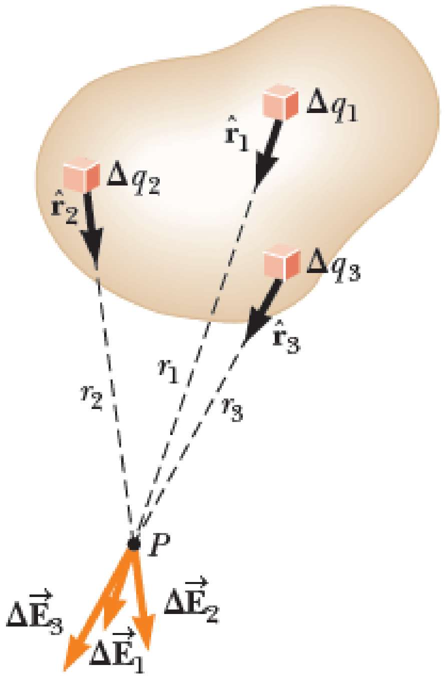
Electric Field of a Continuous Charge Distribution



$$\Delta\vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2}\hat{\mathbf{r}}$$

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^{\,2}}\hat{\mathbf{r}}_i$$

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

In many cases, we have a continuous distribution of charge rather than a collection of discrete charges.

- Find an expression for dq:
  - $dq = \lambda dl$ for a line distribution
  - $dq = \sigma dA$ for a surface distribution
  - $dq = \rho dV$ for a volume distribution
- Represent field contributions at P due to point charges dq located in the distribution. Use symmetry,

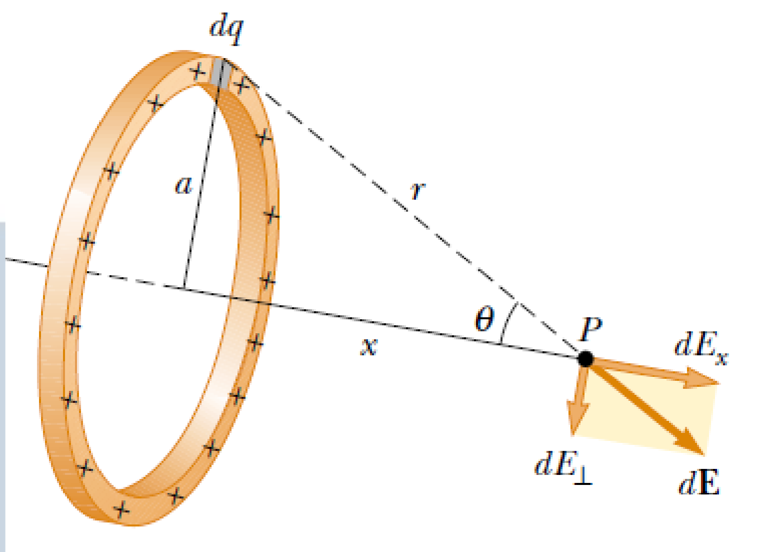
$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\hat{r}$$

- Add up (integrate the contributions) over the whole distribution, varying the displacement as needed,

$$\vec{E} = \int dE$$

Example 23.8 The Electric Field of a Uniform Ring of Charge

A ring of radius *a* carries a uniformly distributed positive total charge *Q*. Calculate the electric field due to the ring at a point *P* lying a distance *x* from its center. Calculate the electric field due to the ring at a point *P* lying a distance *x* from its center along the central axis perpendicular to the plane of the ring.



$$dE_x = dE\cos\theta = \left(k_e \frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2 + a^2)^{3/2}}\,dq$$

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\,dq = \frac{k_e x}{(x^2 + a^2)^{3/2}}\int dq$$

$$= \frac{k_e x}{(x^2 + a^2)^{3/2}}\,Q$$

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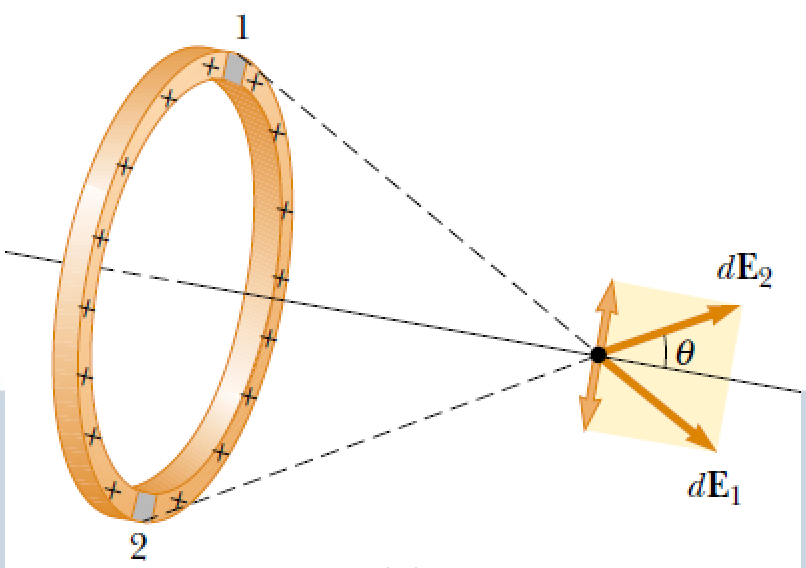

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The perpendicular component of the field at *P* due to segment 1 is canceled by the perpendicular component due to segment 2.

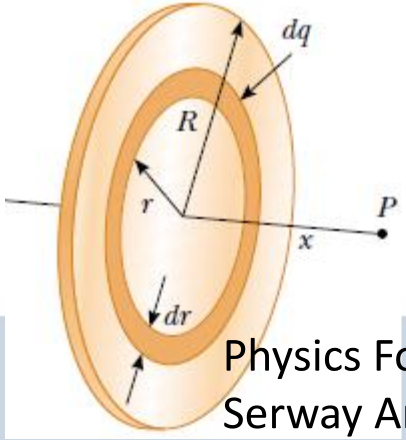
What If? **Suppose a negative charge is placed at the center of the ring in Figure 23.18 and displaced slightly by a distance $x \ll a$ along the x axis. When released, what type of motion does it exhibit?**

let $x \ll a$, $$E_x = \frac{k_e Q}{a^3} x \qquad F_x = -\frac{k_e q Q}{a^3} x$$

Because this force has the form of Hooke's law, the motion will be *simple harmonic*!

Example 23.9 The Electric Field of a Uniformly Charged Disk

A disk of radius R has a uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance *x* from the center of the disk.



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$$dq = 2\pi\sigma r\, dr$$

$$dE_x = \frac{k_e x}{(x^2 + r^2)^{3/2}}(2\pi\sigma r\, dr)$$

To obtain the total field at *P*, we integrate this expression over the limits *r*=0 to *r* = *R*, noting that *x* is a constant.

$$E_x = 2\pi k_e \sigma = \frac{\sigma}{2\epsilon_0}$$

$$E_x = k_e x\pi\sigma \int_0^R \frac{2r\, dr}{(x^2 + r^2)^{3/2}}$$

$$= k_e x\pi\sigma \int_0^R (x^2 + r^2)^{-3/2}\, d(r^2)$$

$$= k_e x\pi\sigma \left[\frac{(x^2 + r^2)^{-1/2}}{-1/2}\right]_0^R$$

$$= 2\pi k_e \sigma \left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)$$

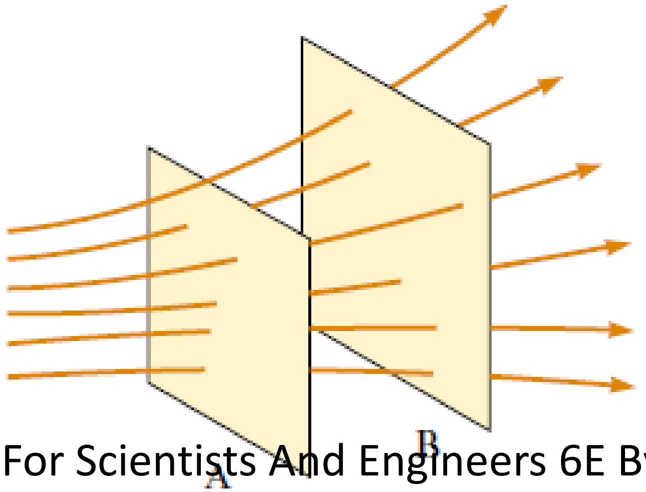
What If *R* >> *x* ?

The expression in parentheses reduces to unity to give us the near-field approximation:

$$E_x = 2\pi k_e \sigma = \frac{\sigma}{2\epsilon_0}$$

In the next chapter we shall obtain the same result for the field created by a uniformly charged infinite sheet.

Electric Field Lines



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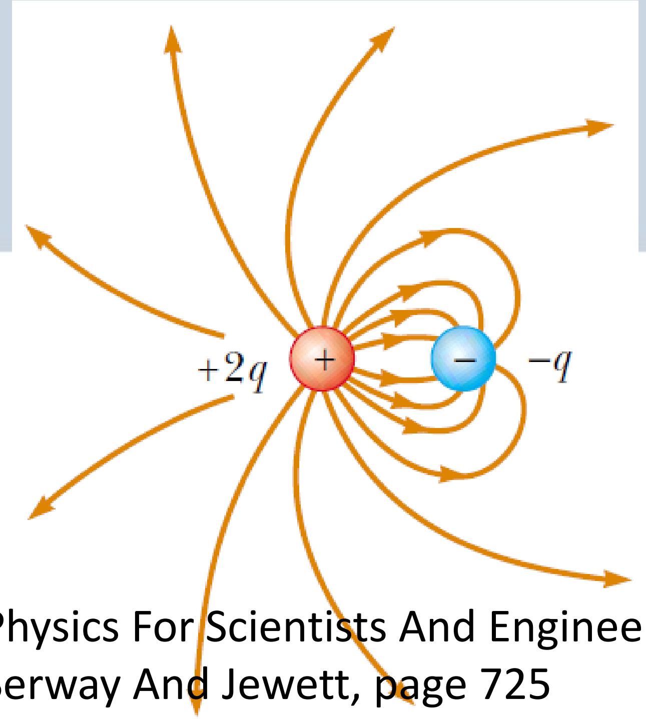
- The density of lines through surface A is greater than the density of lines through surface B. Therefore, the magnitude of the electric field is larger on surface A than on surface B.
- Furthermore, the fact that the lines at different locations point in different directions indicates that the field is nonuniform.



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- The number of lines leaving +2q is twice the number terminating at -q. Hence, only half of the lines that leave the positive charge reach the negative charge. The remaining half terminate on a negative charge we assume to be at infinity.
- At distances that are much greater than the charge separation, the electric field lines are equivalent to those of a single charge +q.

Example 23.10 An Accelerating Positive Charge

A positive point charge *q* of mass *m* is released from rest in a uniform electric field E directed along the *x* axis. Describe its motion.

Solution: We can apply the equations of kinematics in one dimension

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i) \Rightarrow v_f^2 = 2ax_f = \left(\frac{2qE}{m}\right)x_f$$

$$x_f = \frac{1}{2}at^2 = \frac{qE}{2m}t^2$$

The speed of the particle is given by,

$$v_f = at = \frac{qE}{m}t$$

The kinetic energy of the charge after it has moved a distance Δx:

$$K = \frac{1}{2}mv_f^2 = \frac{1}{2}m\left(\frac{2qE}{m}\right)\Delta x = qE\Delta x$$

II. Way: We can also obtain this result from the work–kinetic energy theorem because the work done by the electric force is:

$$F_e\Delta x = qE\Delta x \text{ and } W = \Delta K$$

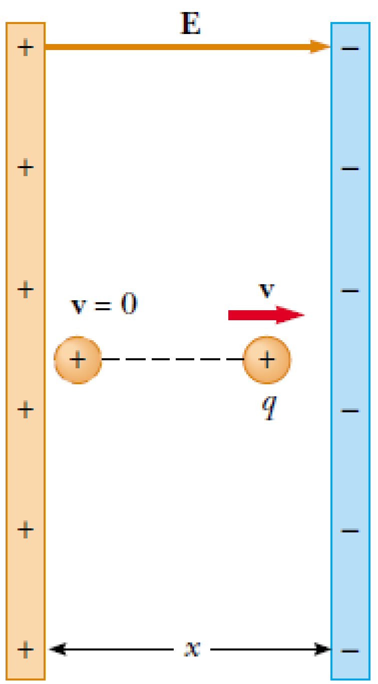

Figure 23.25 (Example 23.10) A positive point charge *q* in a uniform electric field **E** undergoes constant acceleration in the direction of the field.

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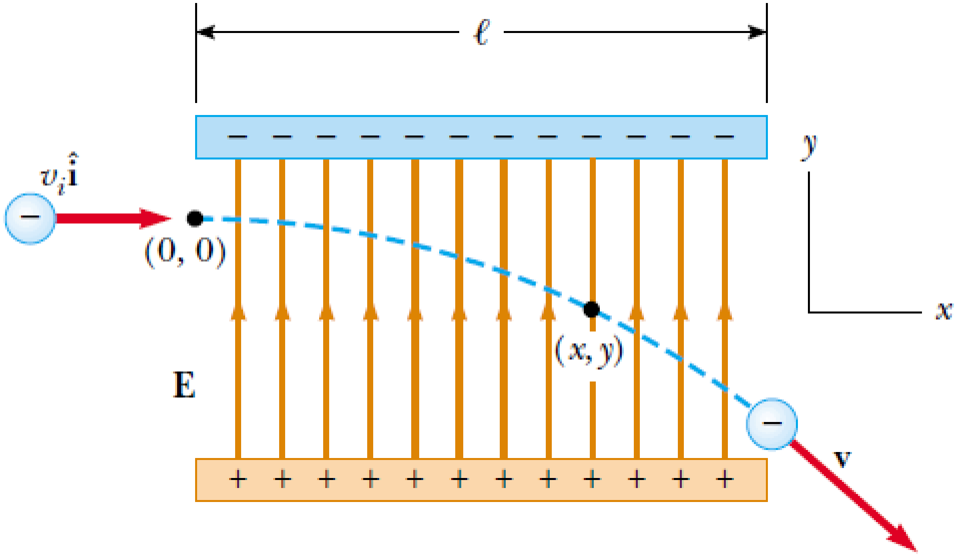
Suppose an electron of charge *e* is projected horizontally into the region between two oppositely charged flat metallic plates.

$$\mathbf{a} = -\frac{eE}{m_e}\hat{\mathbf{j}}$$



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$$v_x = v_i = \text{constant}$$

$$v_y = a_y t = -\frac{eE}{m_e}t$$

Its position coordinates at time *t* are

$$x_f = v_i t \Rightarrow t = x_f / v_i$$

$$y_f = \frac{1}{2}a_y t^2 = -\frac{1}{2}\frac{eE}{m_e}t^2 \Rightarrow y_f = -\frac{1}{2}\frac{eE}{m}\frac{x_f^2}{\vartheta_i^2}$$

Hence, the trajectory is a parabola.

After the electron leaves the field, the electric force vanishes and the electron continues to move in a straight line in the direction of ϑ.

Note that we have neglected the gravitational force acting on the electron. This is a good approximation when we are dealing with atomic particles.

Example 23.11 An Accelerated Electron

An electron enters the region of a uniform electric field as shown in Figure with $\vartheta_i = 3 \times 10^6 \frac{m}{s}$ *and* $E = 200\ N/C$.

The horizontal length of the plates is *l*=0.1m.

(A) Find the acceleration of the electron while it is in the electric field.

$$\mathbf{a} = -\frac{eE}{m_e}\hat{\mathbf{j}} = -\frac{(1.60 \times 10^{-19}\ \mathrm{C})(200\ \mathrm{N/C})}{9.11 \times 10^{-31}\ \mathrm{kg}}\hat{\mathbf{j}}$$

$$= -3.51 \times 10^{13}\hat{\mathbf{j}}\ \mathrm{m/s^2}$$

(B) If the electron enters the field at time *t*=0, find the time at which it leaves the field.

$$t = \frac{\ell}{v_i} = \frac{0.100\ \mathrm{m}}{3.00 \times 10^6\ \mathrm{m/s}} = 3.33 \times 10^{-8}\ \mathrm{s}$$

(C) If the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

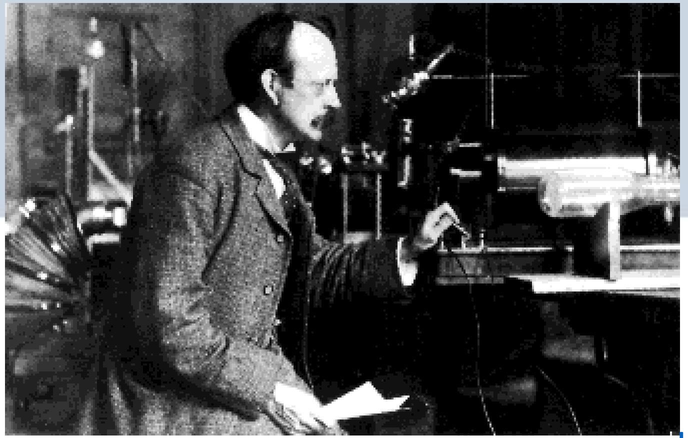
$$y_f = \tfrac{1}{2}a_y t^2 = -\tfrac{1}{2}(3.51 \times 10^{13}\ \mathrm{m/s^2})(3.33 \times 10^{-8}\ \mathrm{s})^2$$

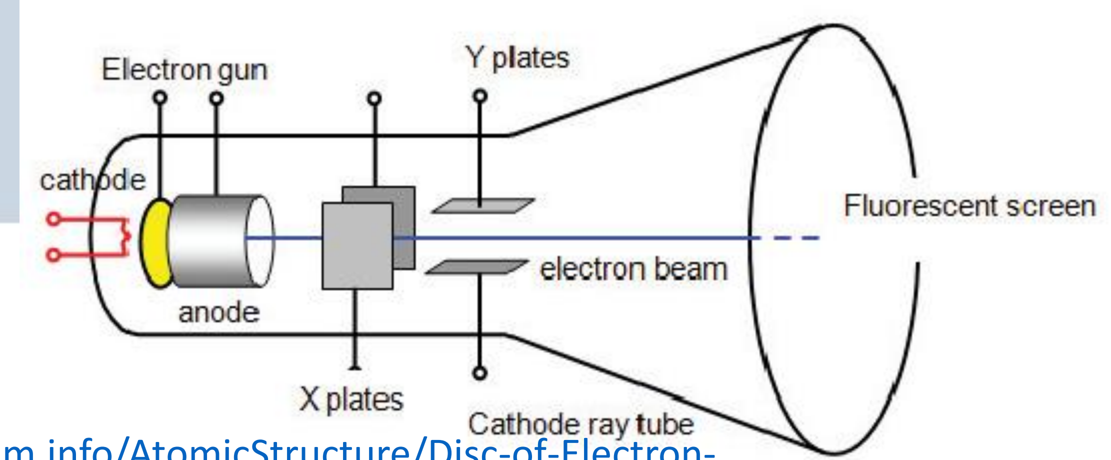
$$= -0.019\,5\ \mathrm{m} = -1.95\ \mathrm{cm}$$

The Cathode Ray Tube

The cathode ray tube (CRT) is commonly used to obtain a visual display of electronic information in oscilloscopes, radar systems and old television systems. The CRT is a vacuum tube in which a beam of electrons is accelerated and deflected under the influence of electric or magnetic fields.

In an oscilloscope the electrons are deflected in various directions by two sets of plates placed at right angle to each other in the neck of the tube. The placing of positive charge on one X or Y plates and negative charge on the other creates an electric field between the plates and allows the beam to be steered from side to side.





https://www.chemteam.info/AtomicStructure/Disc-of-Electron-Images.html

J.J. Thomson used results from cathode ray tube (commonly abbreviated CRT) experiments to discover the electron.

Summary - Continued

- At a distance *r* from a point charge *q*, the electric field due to the charge is

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\,\hat{\mathbf{r}}$$

where $\hat{r}$ is a unit vector directed from the charge toward the point in question. The directed radially outward from a positive charge and radially inward toward a negative charge.

- The electric field due to a group of point charges can be obtained by using the superposition principle. That is, the total electric field at some point equals the vector sum of the electric fields of all the charges:

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{r_i^{\,2}}\,\hat{\mathbf{r}}_i$$

- The electric field at some point due to continuous charge distribution is:

$$\vec{\mathbf{E}} = k_e \int \frac{dq}{r^2}\,\hat{\mathbf{r}}$$

where *dq* is the charge on one element of the charge distribution and *r* is the distance from the element to the point in question.