Physics-2: Electricity & Magnetism

Electric Potential

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The concept of **potential energy** is also of great value in the study of **electricity** (Remember elastic potential energy, gravitational potential energy, …you have seen in *Mechanics.*)

Potential Energy, Work and Conservative Force

- Start

$$W_g = \vec{F}.\overrightarrow{\Delta r} = -mg\hat{j}.[(y_f - y_i)\hat{j}$$
$$= mg(y_i - y_f)$$

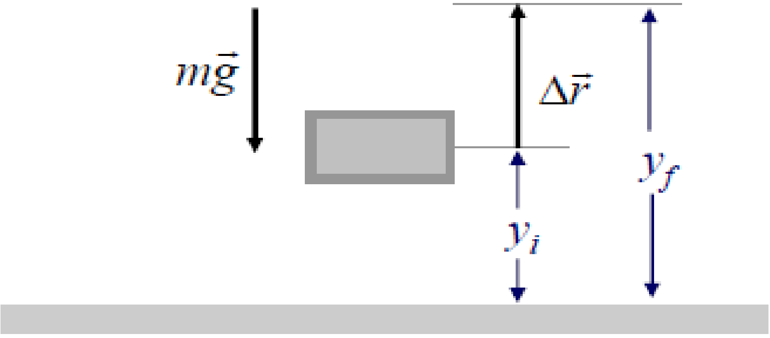
- Then

$$U_g \equiv mgy$$

- So

$$W_g \equiv U_i - U_f = -\Delta U$$

$$\boxed{\Delta U = U_f - U_i = -W_g}$$



- The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
- The work done by a conservative force on a particle moving through any closed path is zero.

Electric Potential Energy

- The potential energy of the system

$$\Delta U = U_f - U_i = -W$$

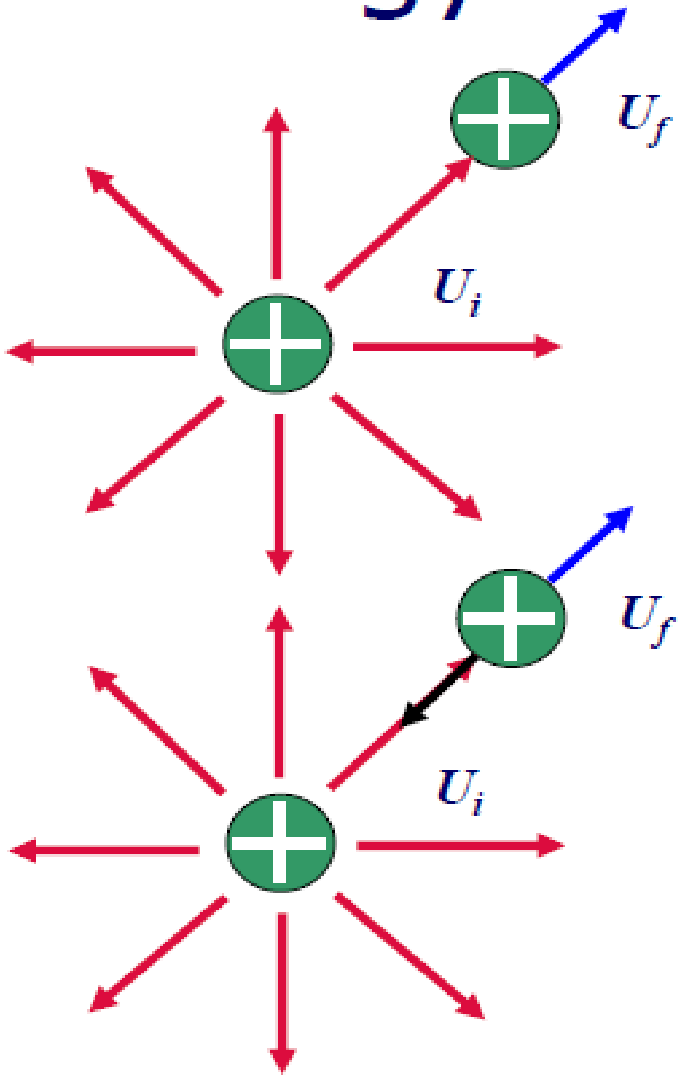
- The work done by the electrostatic force is path independent.
- Work done by a electric force or "field"

$$W = \vec{F} \cdot \Delta\vec{r} = q\vec{E} \cdot \Delta\vec{r}$$

- Work done by an Applied force

$$\Delta K = K_f - K_i = W_{app} + W$$

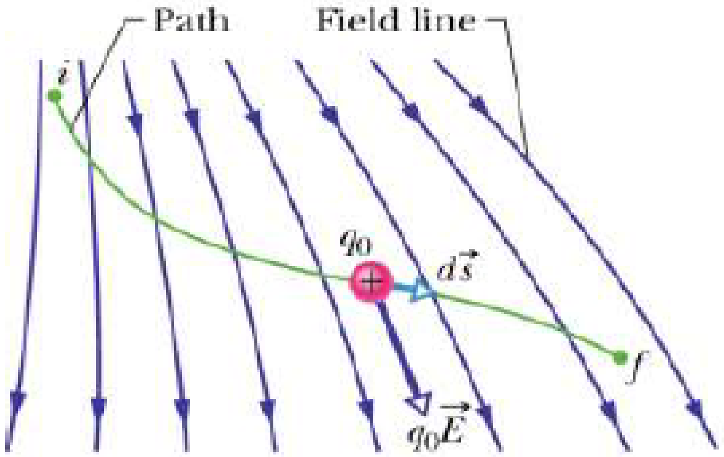
$$W_{app} = -W \qquad \Delta U = U_f - U_i = W_{app}$$



Electric Potential

- The electric potential energy
  - Start $dW = \vec{F} d\vec{s}$
  - Then $dW = q_0 \vec{E} d\vec{s}$
  - So $W = q_0 \int_i^f \vec{E} d\vec{s}$

$$\Delta U = U_f - U_i = -W = -q_0 \int_i^f \vec{E} \cdot d\vec{s}$$



- The electric potential $V = \frac{U}{q}$

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}$$

$$\Delta V \equiv \frac{\Delta U}{q} = -\int_i^f \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Dividing the potential energy by the test charge gives a physical quantity that depends only on the source charge distribution (U/q_0 is called the electric potential).
The fact that potential energy is a scalar quantity means that electric potential also is a scalar quantity.

Because the force q_0E is conservative, this line integral does not depend on the path taken from ***i*** to ***f***

Electric Potential

- Just as with potential energy, only *differences* in electric potential are meaningful.
  - Relative reference: choose arbitrary zero reference level for ΔU or ΔV.
  - Absolute reference: start with all charge infinitely far away and set $U_i = 0$, then we have $U = -W_\infty$ and $V = -W_\infty/q$ at any point in an electric field, where W_∞ is the work done by the electric field on a charged particle as that particle moves in from infinity to point f.

- SI Unit of electric potential: Volt (V)

1 volt = 1 joule per coulomb

1 J = 1 VC and 1 J = 1 N m

1 J of work should be done to move a 1 C of charge through a ΔV of 1 V.

- Electric field: 1 N/C = (1 N/C)(1 VC/J)(1 J/Nm) = 1 V/m
- Electric energy: 1 eV = e(1 V)

$= (1.60\times10^{-19}\ \text{C})(1\ \text{J/C}) = 1.60\times10^{-19}\ \text{J}$

Electron volt, unit of **energy** commonly used in atomic and nuclear **physics**, equal to the **energy** gained by an **electron** (a charged particle carrying unit electronic charge) when the electrical **potential** at the **electron** increases by one **volt**. The **electron volt** equals 1.602×10^{-12} erg, or 1.602×10^{-19} joule.

25.2 Potential Differences in a Uniform Electric Field

Let us calculate the potential difference between two points *A* and *B.*

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\int_A^B (E\cos 0°)\,ds = -\int_A^B E\,ds$$

$$\Delta V = -E\int_A^B ds = -Ed$$

$V_B < V_A$

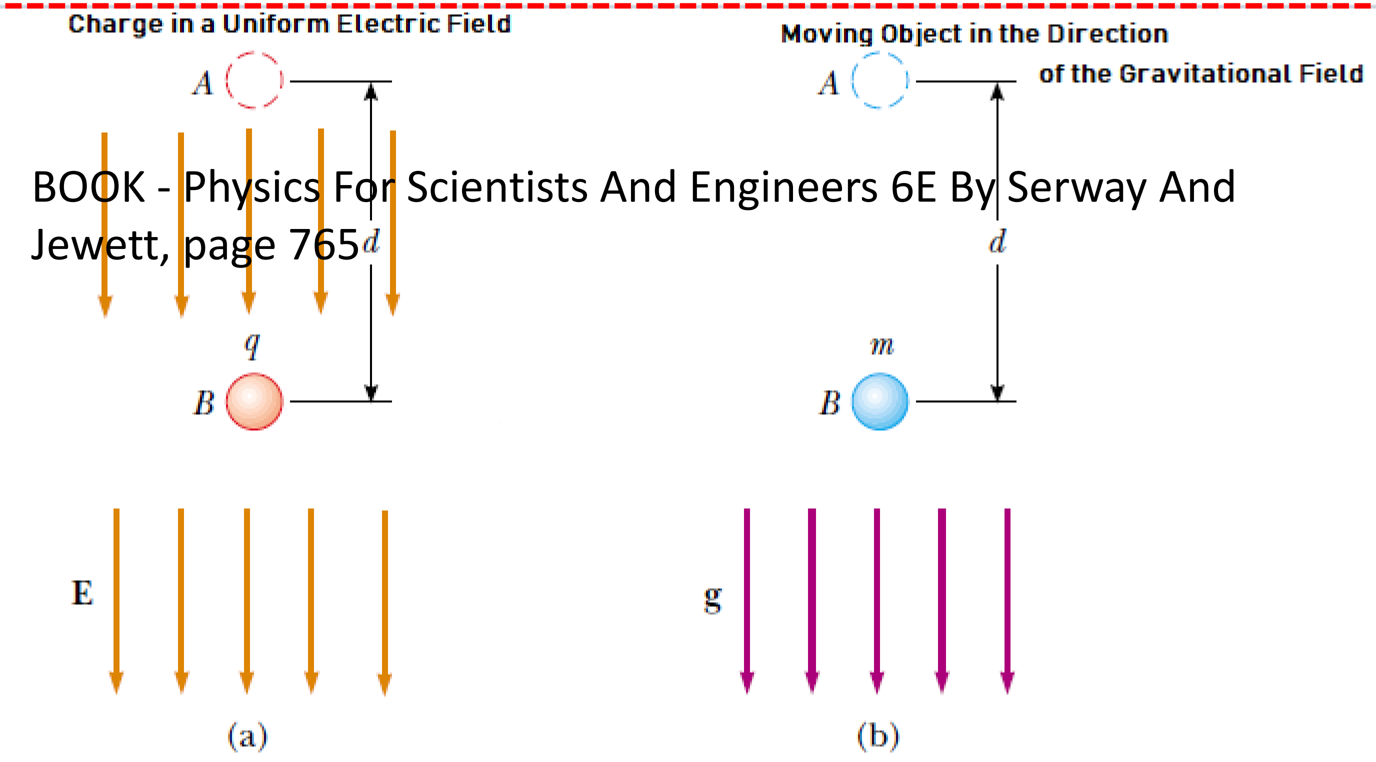


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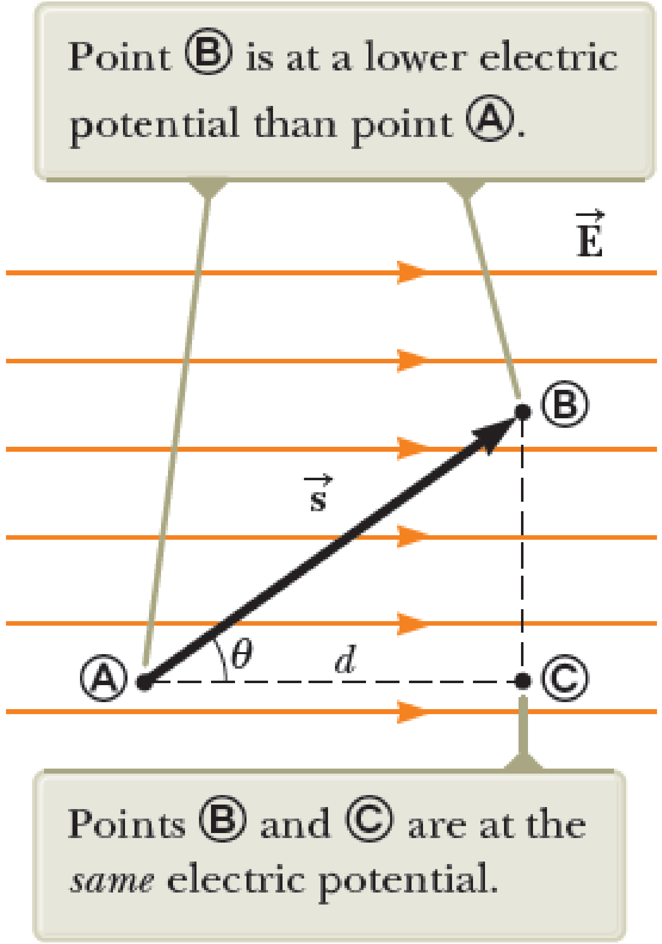
Figure 25.2 (a) When the electric field **E** is directed downward, point *B* is at a lower electric potential than point *A*. When a positive test charge moves from point *A* to point *B*, the charge–field system loses electric potential energy. (b) When an object of mass *m* moves downward in the direction of the gravitational field **g**, the object–field system loses gravitational potential energy.

q accelerates downward, gaining kinetic energy.
As the charged particle gains kinetic energy, the charge–field system loses an equal amount of potential energy (vice-versa for –*q*).

As

If the charge moves in an angular path with *E*,



$$\Delta V = -\int_A^B \mathbf{E}\cdot d\mathbf{s} = -\mathbf{E}\cdot\int_A^B d\mathbf{s} = -\mathbf{E}\cdot\mathbf{s}$$

$$\Delta U = q\Delta V = -q\vec{\mathbf{E}}\cdot\vec{\mathbf{s}}$$

Same electric potential energy as the charge moves A to C:

$$\Delta U_{AC} = \Delta U_{AB}$$

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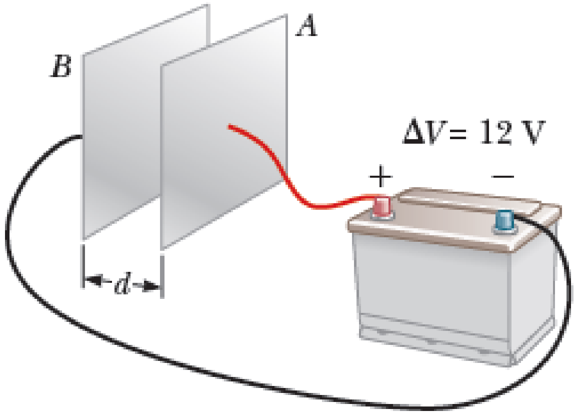
Example 25.1 The Electric Field Between Two Parallel Plates of Opposite Charge



Problem A ΔV = 12 V battery is connected between two parallel plates as in Figure 25.5. The separation between the plates is 0.30cm, and the electric field is assumed to be uniform. (This simplification model is reasonable if the plate separation is small relative to the plate size and if we do not consider points near the edges of the plates.) Find the magnitude of the electric field between the plates.

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$$E = \frac{|V_B - V_A|}{d} = \frac{12\ \text{V}}{0.30 \times 10^{-2}\ \text{m}} = 4.0 \times 10^3\ \text{V/m}$$

The configuration of plates in Figure 25.5 is called a *parallel-plate capacitor*, and is examined in greater detail in Chapter 26.

Example 25.2 Motion of a Proton in a Uniform Electric Field

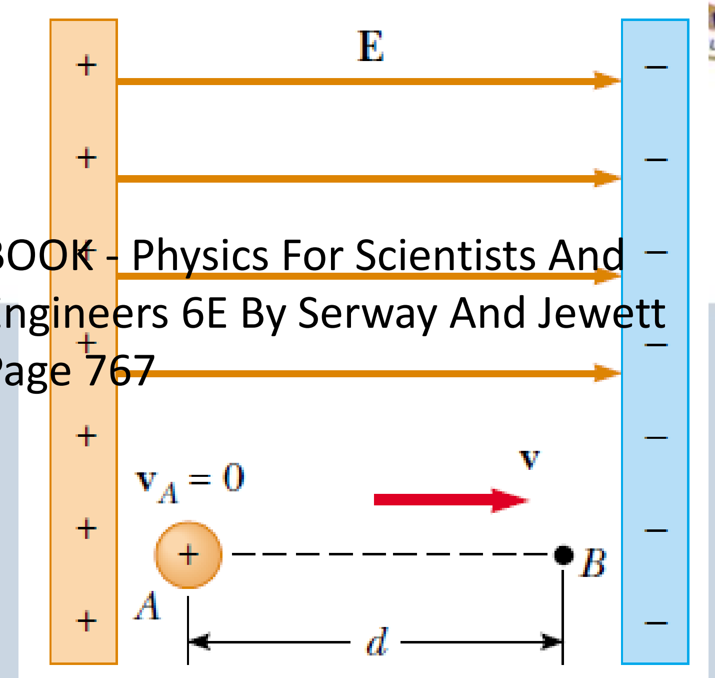
A proton is released from rest in a uniform electric field that has a magnitude of 8×10^4 V/m. The proton undergoes a displacement of 0.50 m in the direction of E.

(A) Find the change in electric potential between points A and B.

(B) Find the change in potential energy of the proton–field system for this displacement.

(C) Find the speed of the proton after completing the 0.50 m displacement in the electric field.

Solution:



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(A) $\Delta V = -Ed = -(8.0\times10^4\ \text{V/m})(0.50\ \text{m}) = -4.0\times10^4\ \text{V}$

(B)
$$\Delta U = q_0\,\Delta V = e\,\Delta V$$
$$= (1.6\times10^{-19}\ \text{C})(-4.0\times10^4\ \text{V})$$
$$= -6.4\times10^{-15}\ \text{J}$$

(C)
$$\Delta K + \Delta U = 0$$
$$(\tfrac{1}{2}mv^2 - 0) + e\,\Delta V = 0$$
$$v = \sqrt{\frac{-(2e\,\Delta V)}{m}}$$
$$= \sqrt{\frac{-2(1.6\times10^{-19}\ \text{C})(-4.0\times10^4\ \text{V})}{1.67\times10^{-27}\ \text{kg}}}$$
$$= 2.8\times10^6\ \text{m/s}$$

Potential Due to a Point Charge

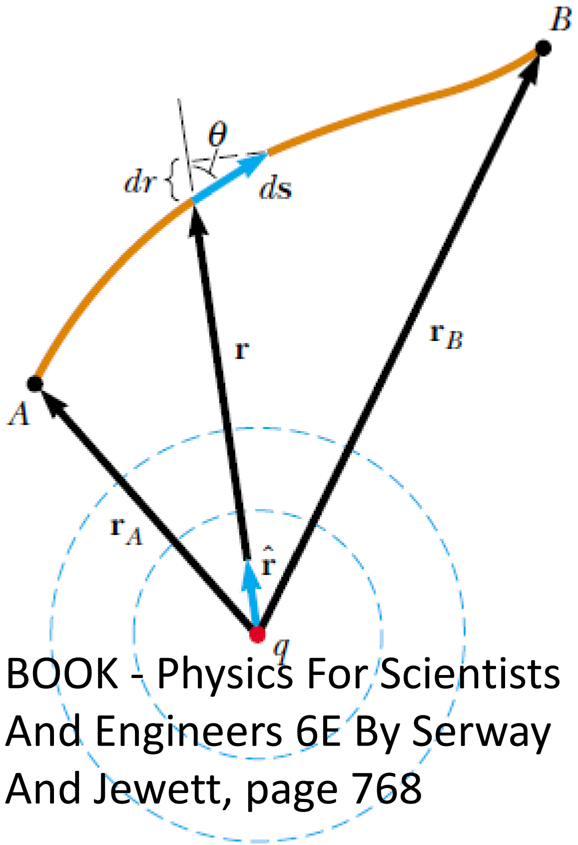
$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\vec{\mathbf{s}}$$

$$V_{Ⓑ} - V_{Ⓐ} = -k_e q \int_{r_{Ⓐ}}^{r_{Ⓑ}} \frac{dr}{r^2} = k_e \frac{q}{r} \Bigg|_{r_{Ⓐ}}^{r_{Ⓑ}}$$

$$V_{Ⓑ} - V_{Ⓐ} = -k_e q \int_{r_{Ⓐ}}^{r_{Ⓑ}} \frac{dr}{r^2} = k_e \frac{q}{r} \Bigg|_{r_{Ⓐ}}^{r_{Ⓑ}}$$

$$V_{Ⓑ} - V_{Ⓐ} = k_e q \left[\frac{1}{r_{Ⓑ}} - \frac{1}{r_{Ⓐ}} \right]$$



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It is customary to choose the reference of electric potential for a point charge to be $V = 0$ at $r_A = \infty$. With this reference choice,

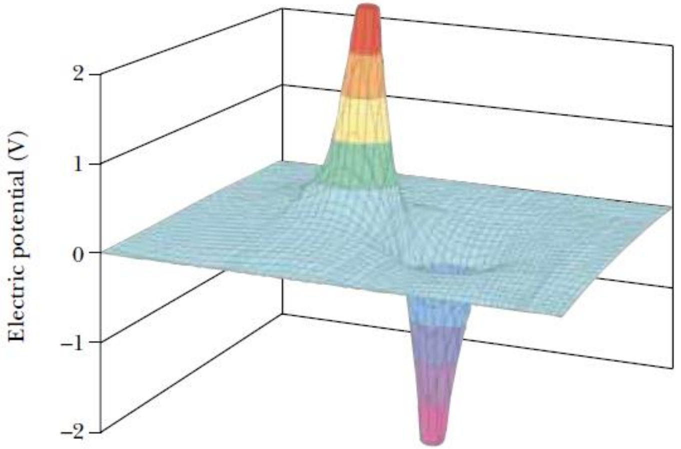
$$V = k_e \frac{q}{r}$$

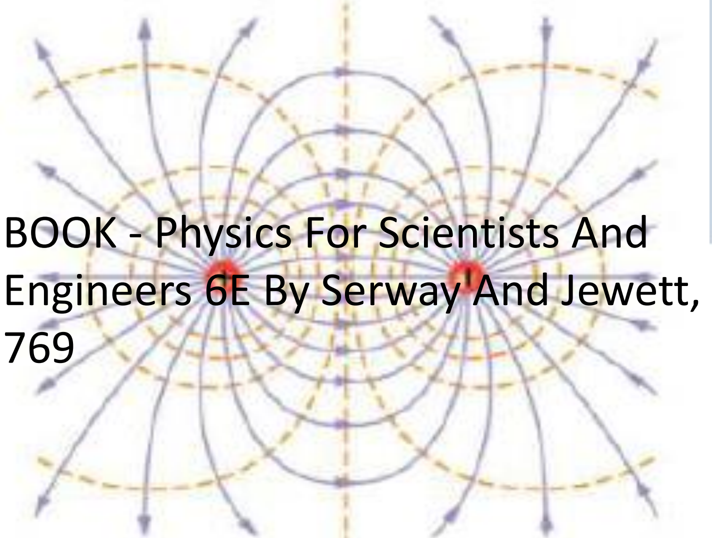
Potential due to a group of point charges

- Use superposition
- For point charges

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

- The sum is an algebraic sum, not a vector sum.
- E may be zero where V does not equal to zero.
- V may be zero where E does not equal to zero.





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The electric potential in the plane containing a dipole.

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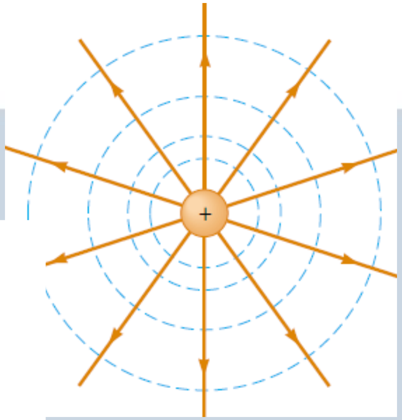
25.4 Obtaining the Value of the Electric Field from the Electric Potential

The potential difference *dV* between two points a distance *ds* apart is: $dV = -\mathbf{E} \cdot d\mathbf{s}$

If the electric field has only one component E_x, then $E_x = -\dfrac{dV}{dx}$



- If we know V(x, y, z),

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

If the charge distribution has spherical symmetry, $E_r = -\dfrac{dV}{dr}$

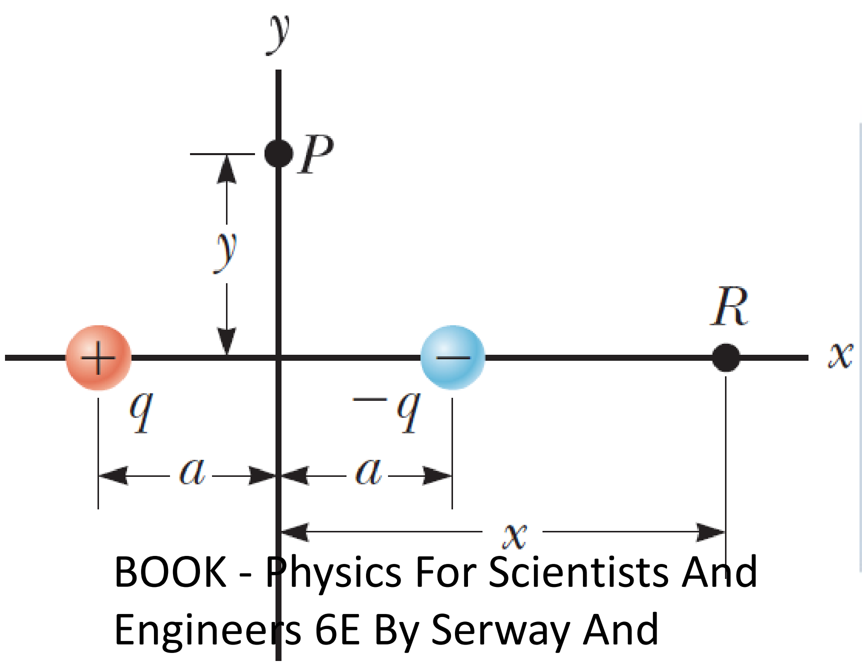
Experimentally, electric potential and position can be measured easily with a voltmeter. Consequently, an electric field can be determined by measuring the electric potential at several positions in the field and making a graph of the results. When a test charge undergoes a displacement ds along an equipotential surface, then dV = 0 because the potential is constant along an equipotential surface. $dV = -\mathbf{E} \cdot d\mathbf{s} = 0$
Thus, ***E*** must be perpendicular to the displacement.

Example 25.4 The Electric Potential at point P on y-axis due to a dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance 2*a* as shown in Figure 25.13. The dipole is along the *x* axis and is centered at the origin.
Calculate the electric potential at point *P* on the *y* axis.

$$V_P = k_e \sum_i \frac{q_i}{r_i}$$

$$= k_e\left(\frac{q}{\sqrt{a^2+y^2}} + \frac{-q}{\sqrt{a^2+y^2}}\right) = 0$$



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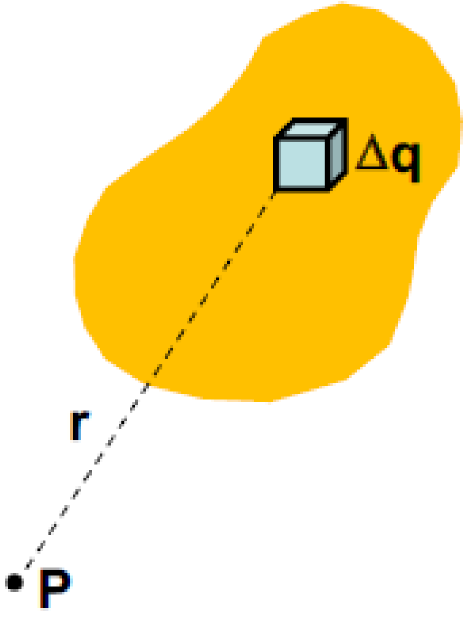
Pay attention that electric potential is a scalar quantity, therefore, it has no direction.
We just simply add the potential at P due to +q and potential at P due to –q to find the net potential at point P.

Potential due to a Continuous Charge Distribution



- Find an expression for dq:
  - $dq = \lambda dl$ for a line distribution
  - $dq = \sigma dA$ for a surface distribution
  - $dq = \rho dV$ for a volume distribution

- Represent field contributions at P due to point charges dq located in the distribution.

$$dV = k_e \frac{dq}{r}$$

- Integrate the contributions over the whole distribution, varying the displacement as needed,

$$V = k_e \int \frac{dq}{r}$$

Example 25.7 Electric Potential Due to a Finite Line of Charge

A rod of length *l*, located along the *x* axis has a total charge *Q* and a uniform linear charge density λ. Find the electric potential at a point *P* located on the *y* axis a distance ***a*** from the origin (Fig. 25.16).

Solution:

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}}$$

$$V = \int_0^{\ell} k_e \frac{\lambda\, dx}{\sqrt{a^2 + x^2}}$$

$$V = k_e\lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2 + x^2}} = k_e \frac{Q}{\ell} \ln\left(x + \sqrt{a^2 + x^2}\right)\Big|_0^{\ell}$$

$$\lambda = Q/\ell$$

$$V = k_e \frac{Q}{\ell}\left[\ln\left(\ell + \sqrt{a^2 + \ell^2}\right) - \ln a\right]$$

$$V = k_e \frac{Q}{\ell}\left(\ln \frac{\left(\ell + \sqrt{a^2 + \ell^2}\right)}{a} \right)$$

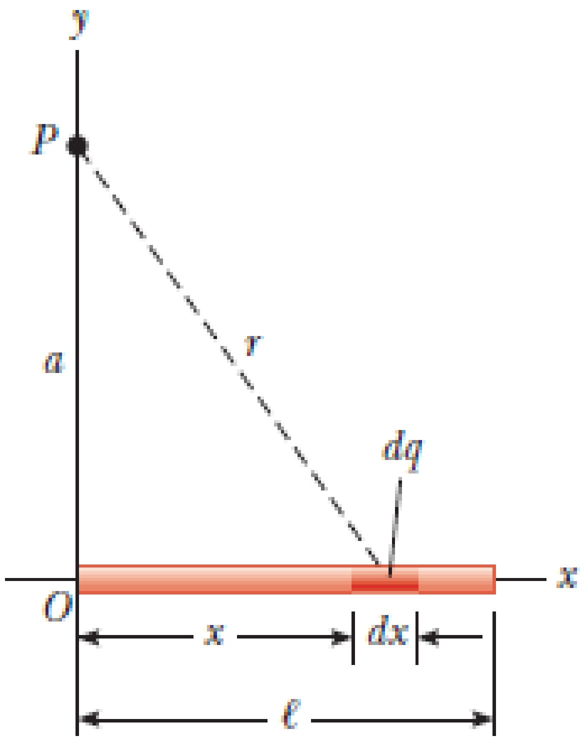
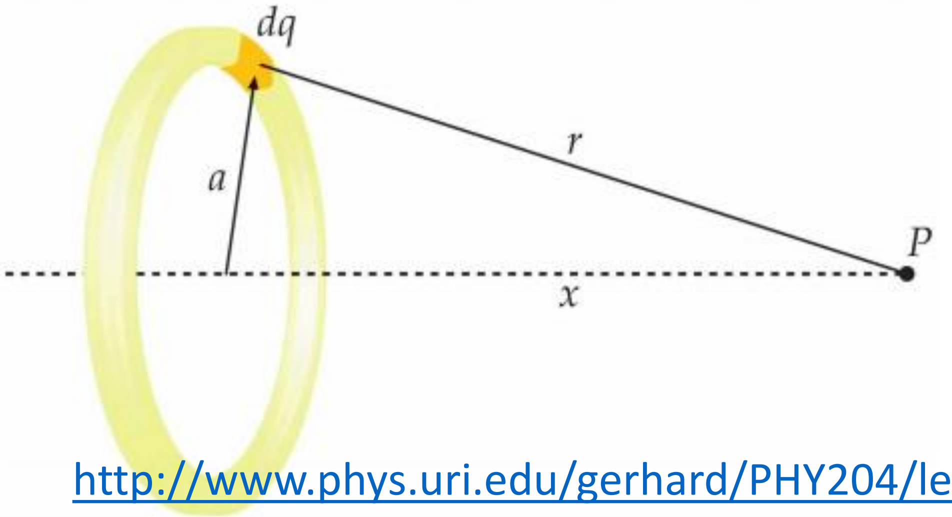


Figure 25.16 A uniform line charge of length *l* located along the *x* axis

Electric Potential of Charged **Ring**

- Total charge on ring: Q
- Charge per unit length: $\lambda = Q/2\pi a$
- Charge on arc: dq



http://www.phys.uri.edu/gerhard/PHY204/lecture9.pdf

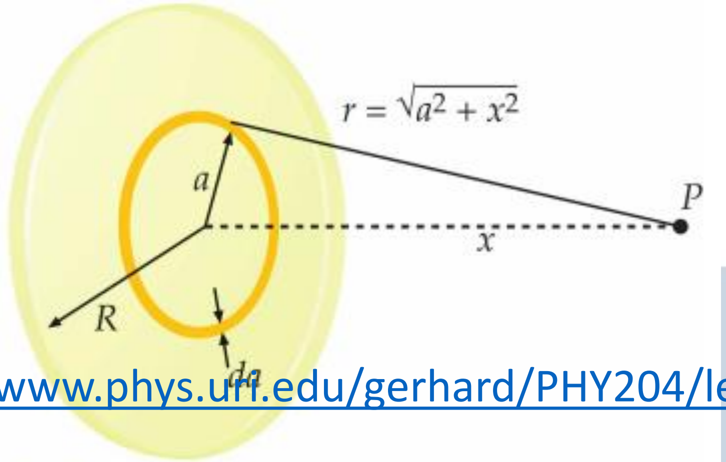
Find the electric potential at point P on the axis of the ring.

- $dV = k\dfrac{dq}{r} = \dfrac{k\,dq}{\sqrt{x^2+a^2}}$

- $V(x) = k\displaystyle\int \frac{dq}{\sqrt{x^2+a^2}} = \frac{k}{\sqrt{x^2+a^2}}\int dq = \frac{kQ}{\sqrt{x^2+a^2}}$

Electric Potential of Charged Disk

- Area of ring: $2\pi a da$
- Charge on ring: $dq = \sigma(2\pi a da)$
- Charge on disk: $Q = \sigma(\pi R^2)$



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Find the electric potential at point P on the axis of the disk.

- $dV = k\dfrac{dq}{\sqrt{x^2+a^2}} = 2\pi\sigma k\dfrac{ada}{\sqrt{x^2+a^2}}$
- $V(x) = 2\pi\sigma k\displaystyle\int_0^R \frac{ada}{\sqrt{x^2+a^2}} = 2\pi\sigma k\left[\sqrt{x^2+a^2}\right]_0^R = 2\pi\sigma k\left[\sqrt{x^2+R^2} - |x|\right]$

Electric potential at large distances from the disk ($|x| \gg a$):

$$V(x) = 2\pi\sigma k|x|\left[\sqrt{1+\frac{R^2}{x^2}} - 1\right] \simeq 2\pi\sigma k|x|\left[1+\frac{R^2}{2x^2} - 1\right] = \frac{k\sigma\pi R^2}{|x|} = \frac{kQ}{|x|}$$

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Electric Potential of a Uniformly Charged Spherical **Shell**

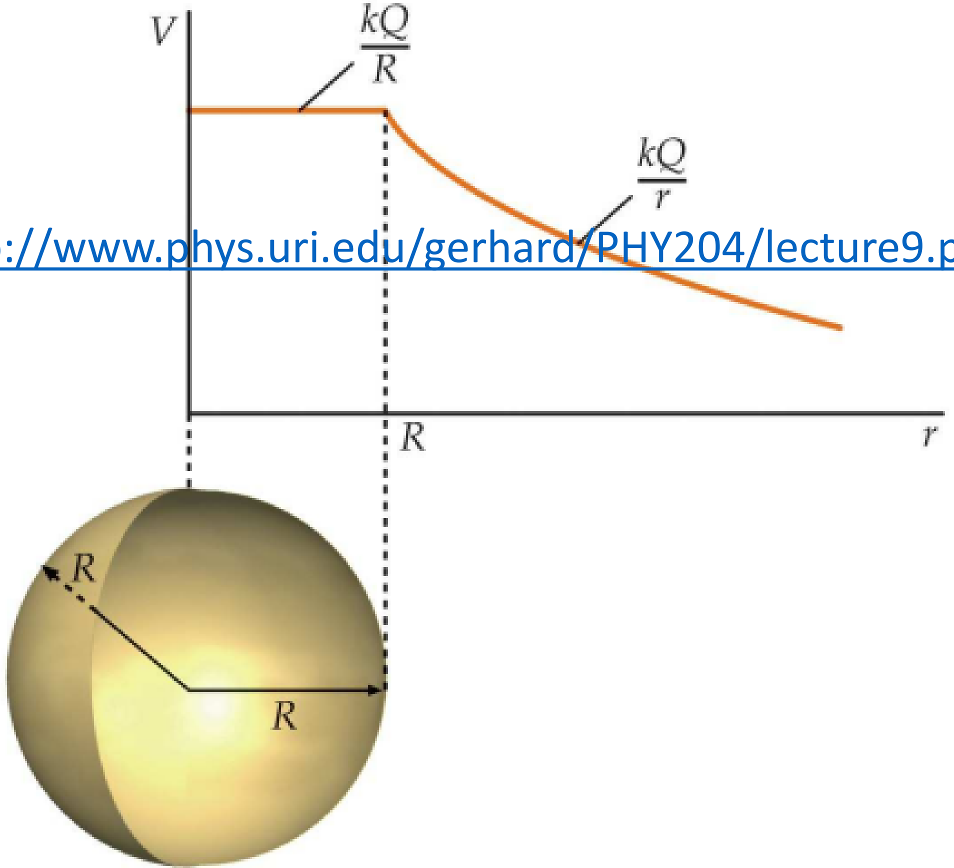
- Electric charge on shell: $Q = \sigma A = 4\pi\sigma R^2$
- Electric field at $r > R$: $E = \frac{kQ}{r^2}$
- Electric field at $r < R$: $E = 0$
- Electric potential at $r > R$:

$$V = -\int_{\infty}^{r} \frac{kQ}{r^2}\,dr = \frac{kQ}{r}$$

- Electric potential at $r < R$:

$$V = -\int_{\infty}^{R} \frac{kQ}{r^2}\,dr - \int_{R}^{r} (0)dr = \frac{kQ}{R}$$

- Here we have used $r_0 = \infty$ as the reference value of the radial coordinate.



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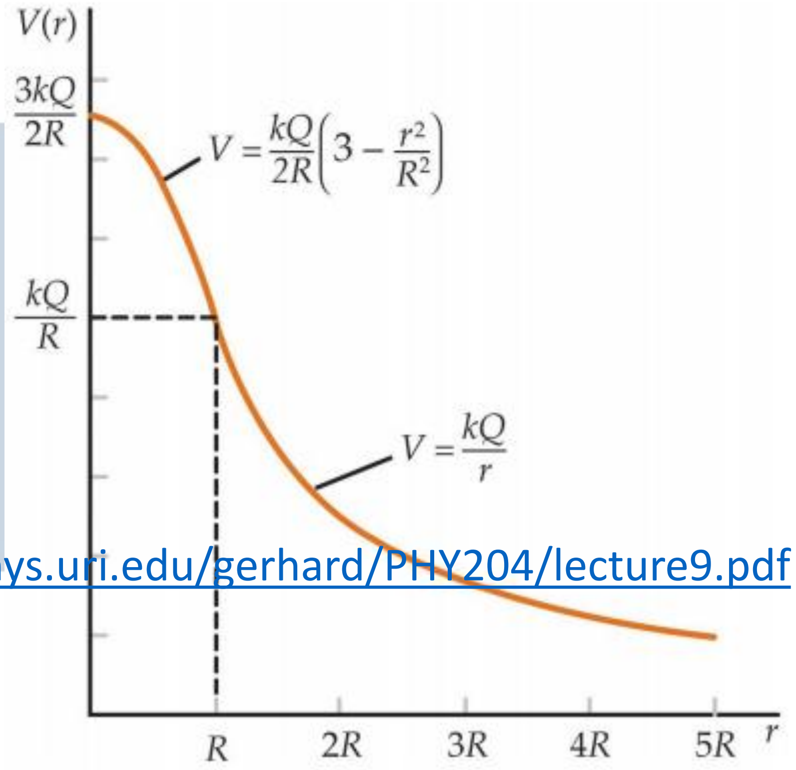
- Electric charge on sphere: $Q = \rho V = \frac{4\pi}{3}\rho R^3$
- Electric field at $r > R$: $E = \frac{kQ}{r^2}$
- Electric field at $r < R$: $E = \frac{kQ}{R^3}\, r$
- Electric potential at $r > R$:

$$V = -\int_{\infty}^{r} \frac{kQ}{r^2}\, dr = \frac{kQ}{r}$$

- Electric potential at $r < R$:

$$V = -\int_{\infty}^{R} \frac{kQ}{r^2}\, dr - \int_{R}^{r} \frac{kQ}{R^3}\, r\, dr$$

$$\Rightarrow V = \frac{kQ}{R} - \frac{kQ}{2R^3}\left(r^2 - R^2\right) = \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right)$$



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A Cavity Within a Conductor

Every point on the conductor is at the same electric potential. Therefore,

any two points *A* and *B* on the surface of the cavity must be at the same potential.

Imagine ***E*** exists in the cavity. The potential difference is:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

Because $V_A = V_B$, the integral is zero for all paths.

This means that ***E*** is zero in the cavity.

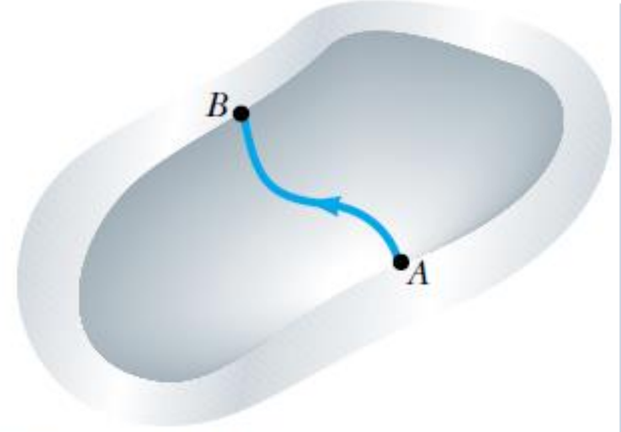

Figure 25.26 A conductor in electrostatic equilibrium containing a cavity. The electric field in the cavity is zero, regardless of the charge on the conductor.

A cavity surrounded by conducting walls is a field-free region

as long as no charges are inside the cavity.

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Corona Discharge

A phenomenon known as corona discharge is often observed near a conductor such as a high-voltage power line. A corona discharge is an electrical discharge brought on by the ionization of a fluid such as air surrounding a conductor that is electrically charged.

If a conductor has an irregular shape, the electric field can be very high near sharp points or edges of the conductor; consequently, the ionization process and corona discharge are most likely to occur around such points.

Corona discharge is used in the electrical transmission industry to locate broken or faulty components. For example, a broken insulator on a transmission tower has sharp edges where corona discharge is likely to occur.

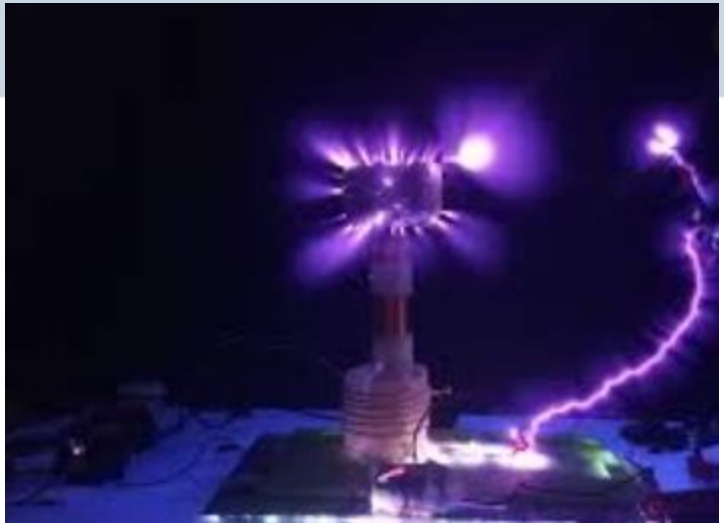

Figure from Wikimedia Commons
Corona discharge of a homemade tesla coil

Summary

- Electric Potential Energy: a point charge moves from i to f in an electric field, the change in electric potential energy is $$\Delta U = U_f - U_i = -W$$
- Electric Potential Difference between two points i and f in an electric field: $$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}$$
- Equipotential surface: the points on it all have the same electric potential. No work is done while moving charge on it. The electric field is always directed perpendicularly to corresponding equipotential surfaces.
- Finding V from E: $$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_i^f \vec{E} \cdot d\vec{s}$$
- Potential due to point charges: $$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$
- Potential due to a collection of point charges: $$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$
- Potential due to a continuous charge distribution: $$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$
- Potential of a charged conductor is constant everywhere inside the conductor and equal to its value to its value at the surface.
- Calculatiing E from V: $E_s = -\frac{\partial V}{\partial s}$ $E_x = -\frac{\partial V}{\partial x}$ $E_y = -\frac{\partial V}{\partial y}$ $E_z = -\frac{\partial V}{\partial z}$
- Electric potential energy of system of point charges: $$U = q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$