Physics 2: Electricity & Magnetism

Chapter 27. Current and Resistance

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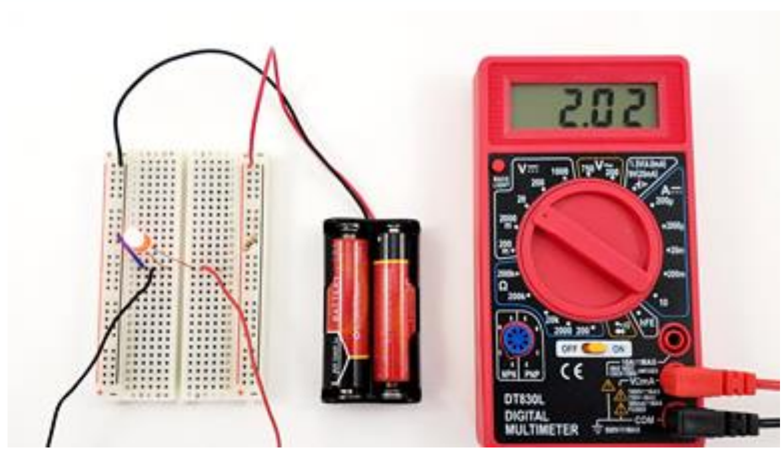

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Chapter 25 – Current, Resistance and Electromotive Force

- Current
- Resistivity
- Resistance
- Electromotive Force and Circuits
- Theory of Metallic Conduction
- Energy and Power in Electric Circuits

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1. Current

Electric current: Charges in motion from one region to another.

Electric circuit: Conducting path that forms a closed loop in which charges move. In these circuits, energy is conveyed from one place to another.

Electrostatics: E = 0 within a conductor -> Current (I) = 0, but not all charges are at rest, free electrons can move ($\vartheta \sim 10^6$ m/s). Electrons are attracted to + ions in material. Therefore they can not escape.
Electron motion is random -> no net charge flow

Non-electrostatic: E ≠ 0 inside conductor -> $\vec{F} = q\,\vec{E}$

Charged particle moving in vacuum -> steady acceleration // F

Charged particle moving in a conductor -> collisions with "nearly" stationary massive ions in material change random motion of charged particles.
Due to **E**, superposition of random motion of charge + slow net motion **(drift)** of charged particles as a group in direction of $\vec{F} = q\,\vec{E}_{net}$ current in conductor

Drift velocity (v_d) = 10^{-4} m/s (slow)

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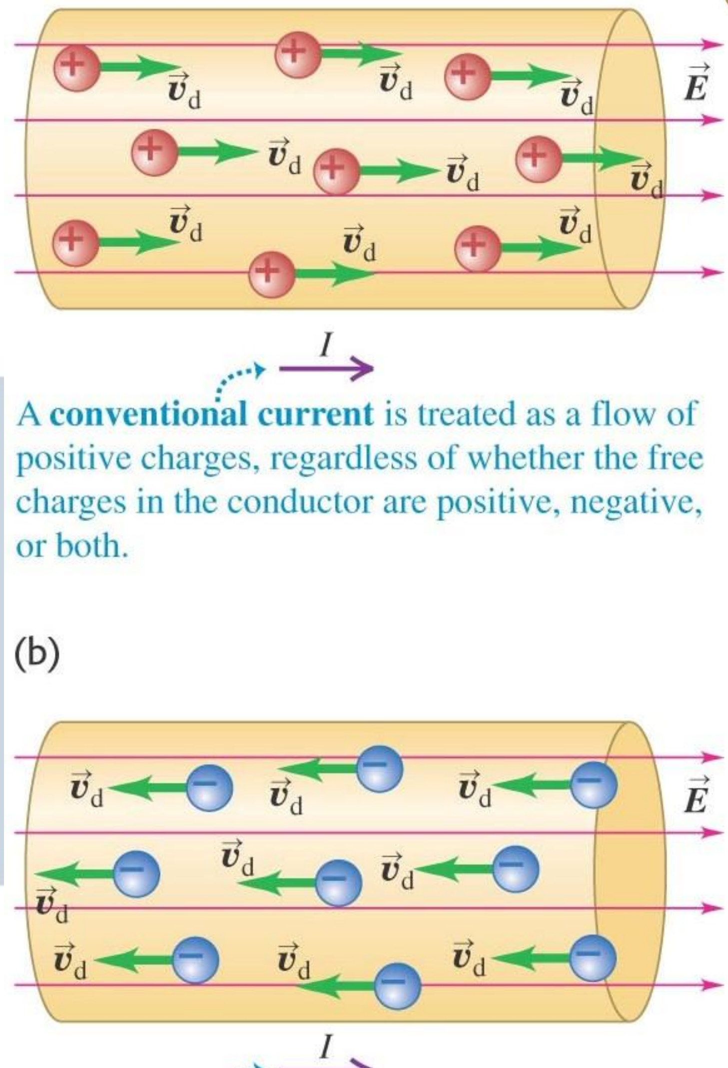
- Positive charges would move with the electric field, electrons move in opposition.

- The motion of electrons in a wire is analogous to water coursing through a river.

Conventional current (I): direction in which there is a flow of positive charge.
This direction is not necessarily the same as the direction in which charged particles are actually moving.

Current: $$I = \frac{dQ}{dt}$$



A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

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- Current is not a vector! → no single vector can describe motion along curved path.

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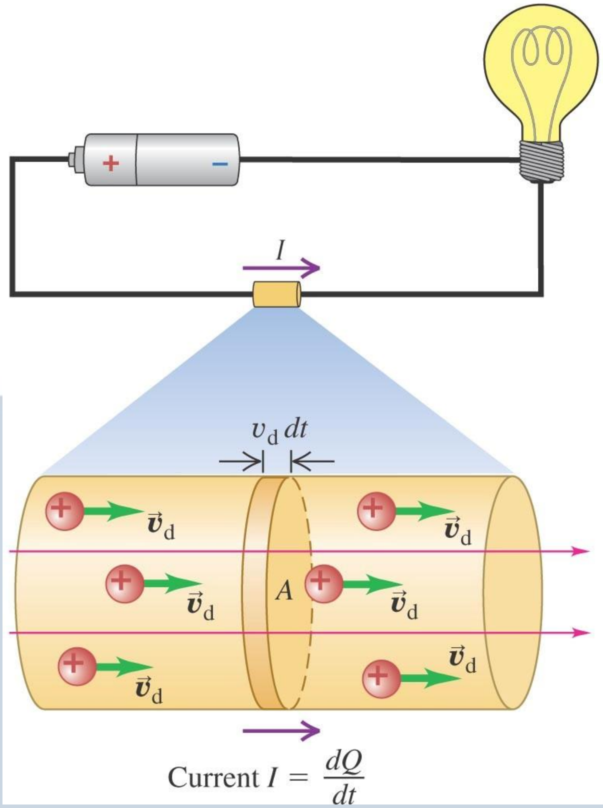
Current units: 1 A = 1 C/s

Current (I) is the time rate of charge transfer through a cross sectional area.

The random component of each moving charged particle's motion averages to zero → I in same direction as E.



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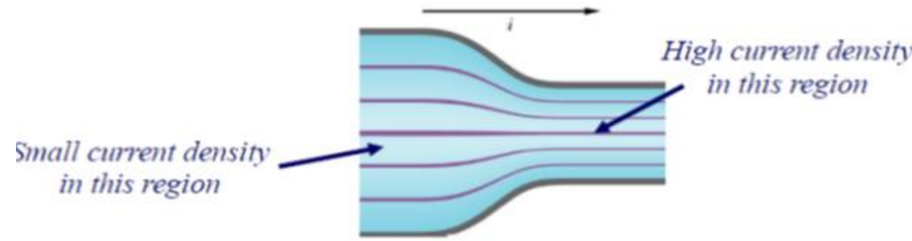
Current, Drift Velocity and Current Density:

$$I = \frac{dQ}{dt} = n|q|v_d A$$

n = concentration of charged particles

v_d = drift velocity



Current Density (J):

$$J = \frac{I}{A} = n|q|v_d$$

$$\vec{J} = nq\vec{v}_d$$

J is a vector, describes how charges flow at a certain point.

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Example 27.1 The drift speed in a copper wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of $3.31 \times 10^{-6}\ \mathrm{m^2}$. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. Take the density of copper as $8.95\ \mathrm{g/cm^3}$.

Solution From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Knowing the density of copper enables us to calculate the volume occupied by 1 mol of copper:

$$V = \frac{M}{\rho} = \frac{63.5\ \mathrm{g/mol}}{8.95\ \mathrm{g/cm^3}} = 7.09\ \mathrm{cm^3/mol}$$

Recall that one mole of any substance contains Avogadro's number of atoms, 6.02×10^{23} atoms. Because each copper atom contributes one free electron to the current, the density of charge carriers is

$$n = \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ \text{cm}^3}\left(\frac{1.00 \times 10^{6}\ \text{cm}^3}{1\ \text{m}^3}\right)$$
$$= 8.49 \times 10^{28}\ \text{electrons/m}^3$$

From Equation 21.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$
$$= \frac{10.0\ \text{C/s}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})(3.31 \times 10^{-6}\ \text{m}^2)}$$
$$= 2.22 \times 10^{-4}\ \text{m/s}$$

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Steady current (closed circuit): total charge in every segment of conductor is constant →equal rate of flow of charge in and out of segment.

Direct current: direction of current is always the same.

Alternating current: current continuously changes direction.

2. Resistivity

Ohm's law → $\vec{J}$ directly proportional to $\vec{E}$.

(Intrinsic material property)

1 Ohm = 1 Ω = V/A

Resistivity: $$\rho = \frac{E}{J}$$

Units: Ω m = (V/m)/(A/m²) = (V/A) m

Substance		$\rho(\Omega \cdot m)$	Substance	$\rho(\Omega \cdot m)$
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

Table 25.2 Temperature Coefficients of Resistivity (Approximate Values Near Room Temperature)

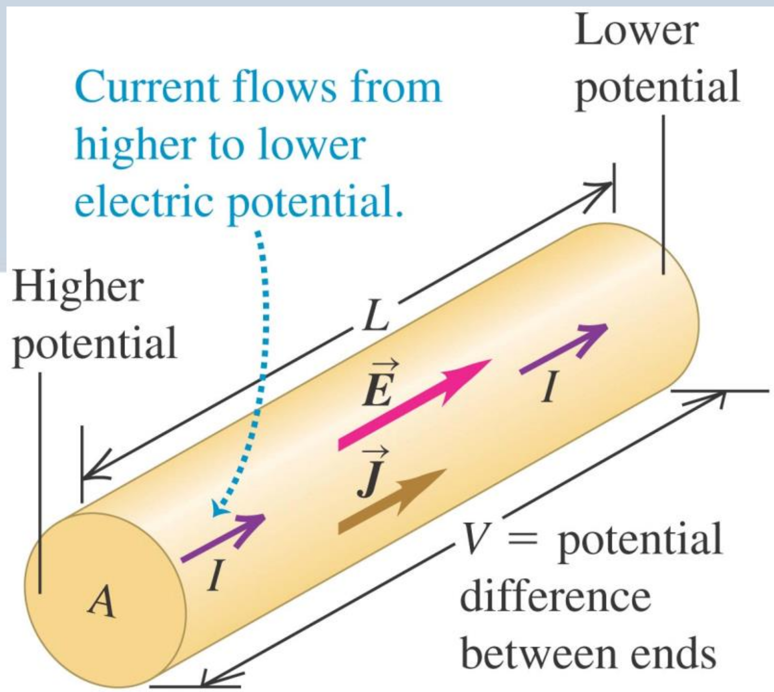
Material	$\alpha\ [(^{\circ}C)^{-1}]$	Material	$\alpha\ [(^{\circ}C)^{-1}]$
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

3. Resistance

$$E = \rho \cdot J$$

Ohm's law → ρ = constant

Figure from http://slideplayer.com/slide/14338811/



Current direction: from higher V end to lower V end. Follows E direction, independent of sign of moving charges.

- As the current flows through a potential difference, electric potential energy is lost. This energy is transferred to the ions of conducting material during collisions.

$$I = J \cdot A$$

$$V = E \cdot L$$

R = resistance

$$E = \frac{V}{L} = \rho \cdot J = \rho \frac{I}{A} \quad \rightarrow \quad V = \boxed{\frac{\rho \cdot L}{A}} I$$

Resistance:

$$R = \frac{V}{I} = \frac{\rho \cdot L}{A}$$

$$\boxed{V = I \cdot R}$$

Ohm's law (conductors)

Units: Ohm = Ω = 1 V/A

$$\boxed{R(T) = R_0[1 + \alpha(T - T_0)]}$$

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Resistor: circuit device with a fixed R between its ends.

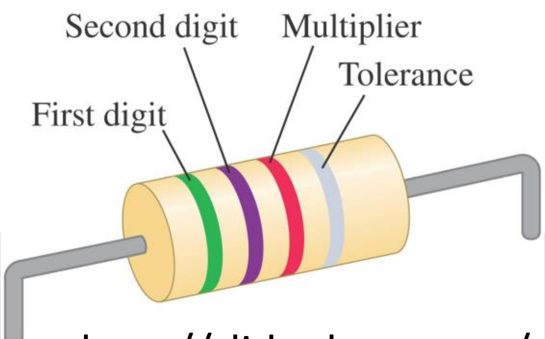


Figure from http://slideplayer.com/slide/14338811/

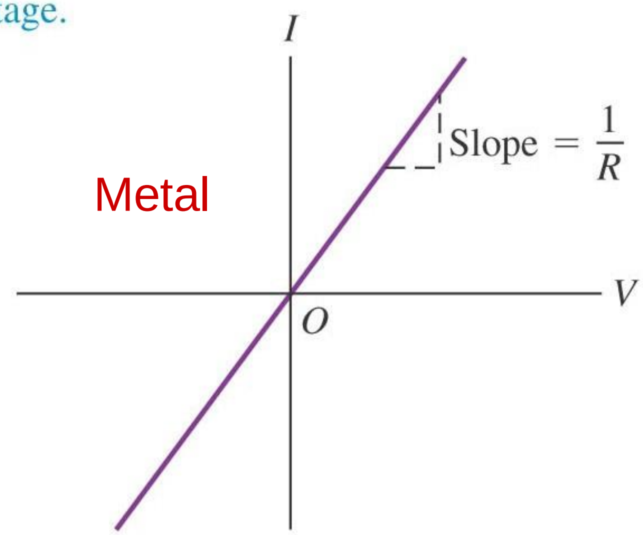
Ex: 5.7 kΩ = green (5) violet (7) red multiplier (100)

Table 25.3 Color Codes for Resistors

Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9

Current-voltage curves

Ohmic resistor (e.g., typical metal wire): At a given temperature, current is proportional to voltage.



Semiconductor diode: a nonohmic resistor

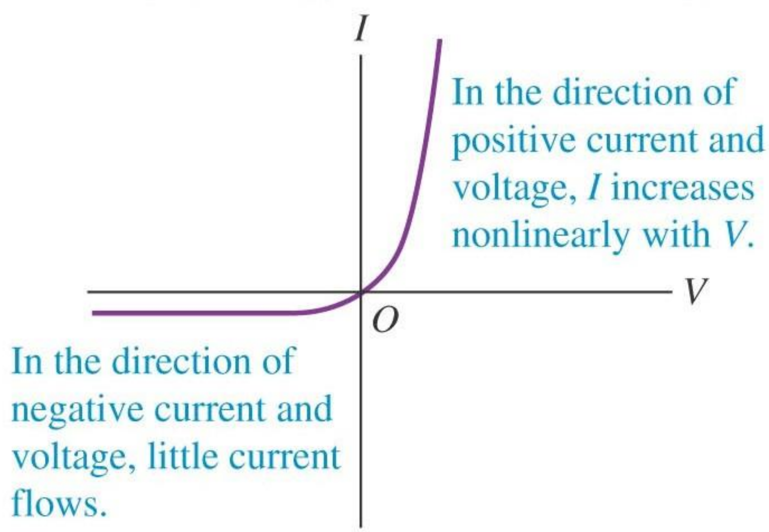


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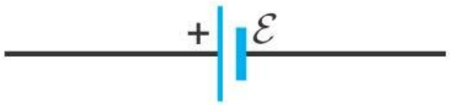
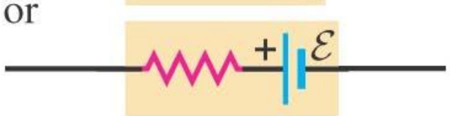
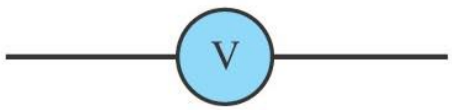
Table 25.4 Symbols for Circuit Diagrams

	Conductor with negligible resistance
	Resistor
	Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)
 or 	Source of emf with internal resistance r (r can be placed on either side)
	Voltmeter (measures potential difference between its terminals)
	Ammeter (measures current through it)

- The meters do not disturb the circuit in which they are connected.

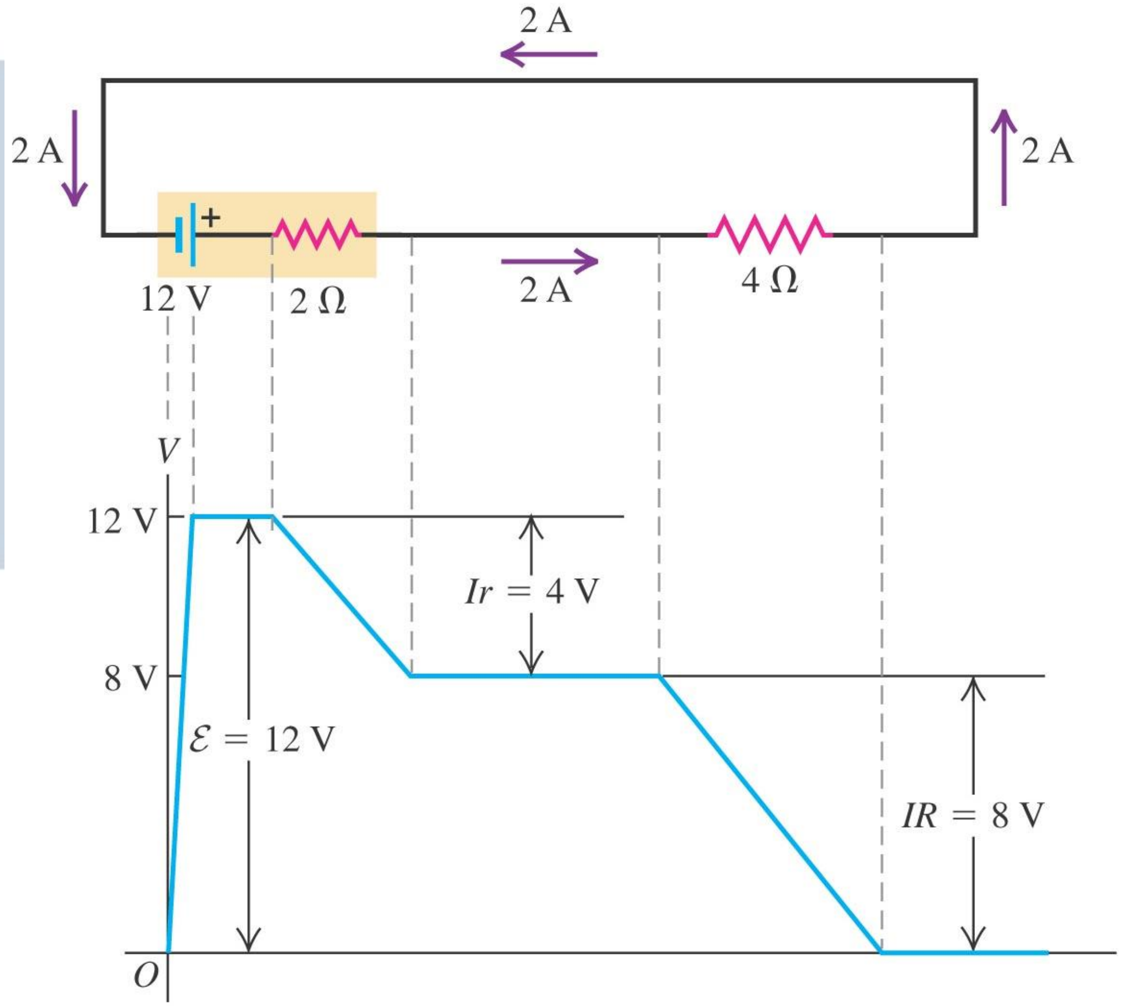
- Voltmeter → infinite resistance → I = V/ R → I =0 (measures V)

- Ammeter → zero resistance → V = I R = 0 (measures I)

Potential changes around a circuit

-The net change in potential energy for a charge *q* making a round trip around a complete circuit must be zero.

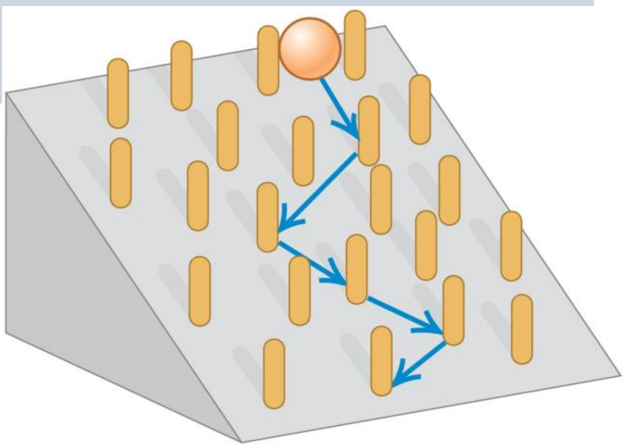
- Local differences in potential occur.



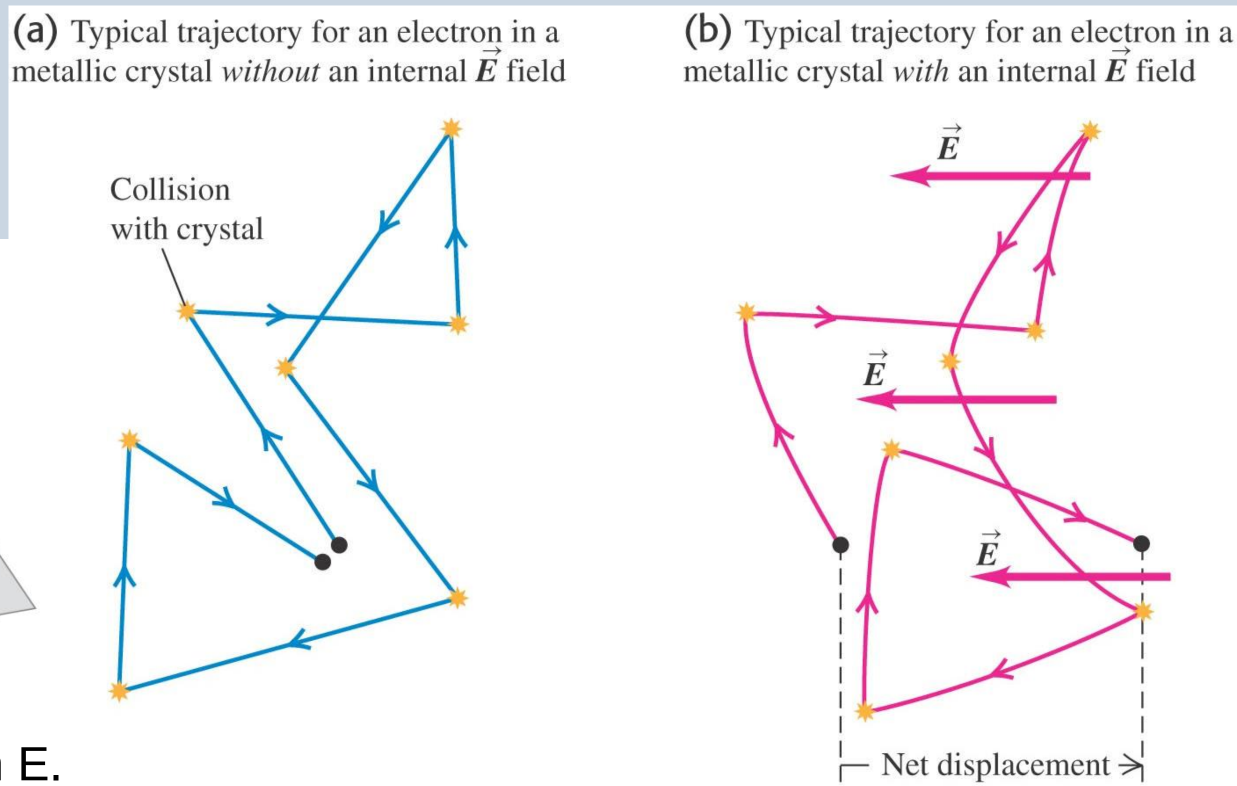
6. Theory of Metallic Conduction

- If no E → free e^- move in straight lines between collisions with + ions → random velocities, in average, no net displacement.
- If E → e^- path curves due to acceleration caused by F_e → drift speed.

Mean free time (τ): average time between collisions.



Analogy to motion of e- with E.



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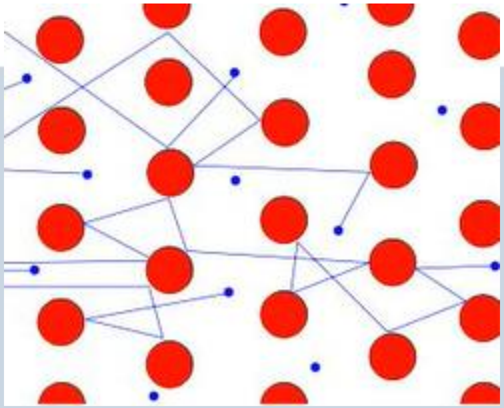
A Model for Electrical Conduction

Electrical conduction in metals, Paul **Drude** in 1900
Although has limitations, it introduces concepts that are applied in more elaborate treatment

Conductor: Array of atoms + free electrons
When $E = 0$, conduction electrons move in random directions
Electrons shown here in blue, stationary crystal ions in red

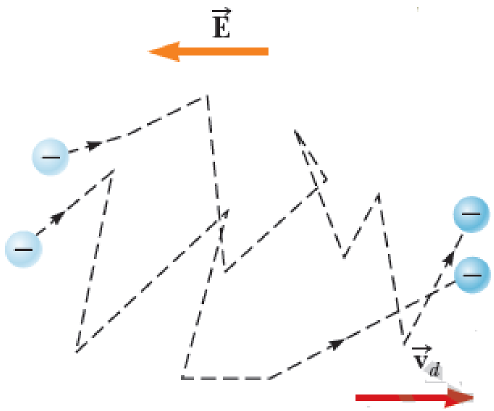


When $E > 0$, the electrons experience force. Free electrons drift slowly in a direction opposite that of the E.

$$a = \frac{qE}{m_e} \qquad \vartheta_f = \vartheta_i + \frac{qE}{m_e}t$$

Let's now take the average. Assuming random motion,

$$< \vartheta_i > = 0$$



τ is the average time between subsequent collisions.

$$< \vartheta_f > = \frac{qE}{m_e}\tau$$

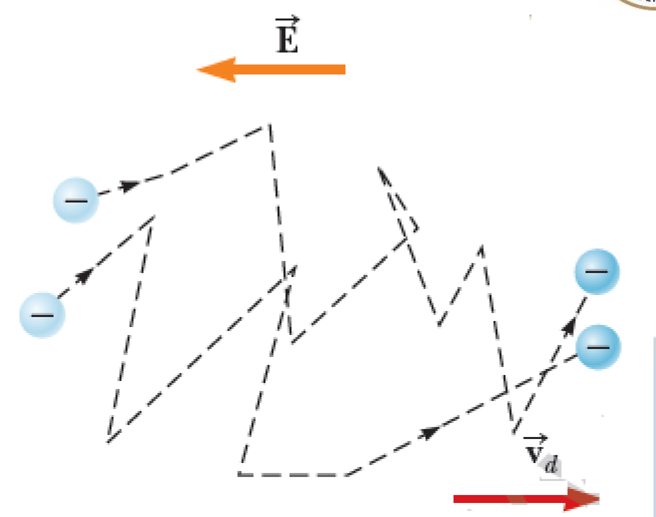
$$I_{avg} = \frac{\Delta Q}{\Delta t} = nq\vartheta_d A$$

$$I_{avg} = \frac{\Delta Q}{\Delta t} = nq\left(\frac{qE}{m_e}\tau\right)A$$

$$I_{avg} = \frac{\Delta Q}{\Delta t} = \frac{nq^2 E\tau A}{m_e}$$

$$J = \frac{I_{avg}}{A} = \frac{nq^2\tau E}{m_e} = \sigma E$$

$$\sigma = \frac{nq^2\tau}{m_e}$$



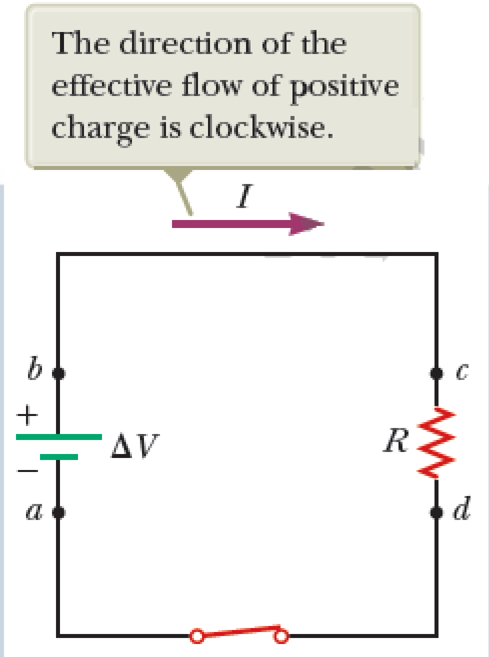
According to this classical model, σ does not depend on the strength of the electric field.

This model does not correctly predict the values of resistivity with temperature.
The classical model modified with the **wave-like character of electrons** results in good predictions of resistivity, in agreement with measured values.

5. Energy and Power in Circuits

Let's now investigate the rate at which the electric potential energy of the system decreases as the charge Q passes through the resistor.

$$\frac{dU}{dt} = \frac{d}{dt}(Q\,\Delta V) = \frac{dQ}{dt}\Delta V = I\,\Delta V$$

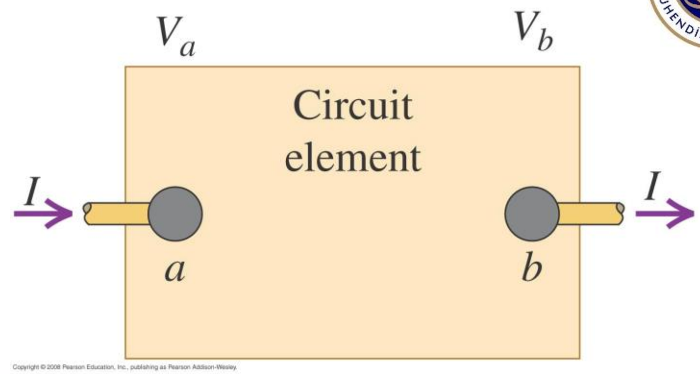


The system regains this potential energy when the charge passes through the battery, at the expense of chemical energy in the battery.

The rate at which the potential energy of the system decreases as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor. Therefore, the power P, representing the rate at which energy is delivered to the resistor, is

$$P = I\Delta V$$

Power: rate at which energy is delivered to or extracted from a circuit element.



$$P = V_{ab} I = (V_a - V_b)\ I$$

Units: 1 Watt = W = V A = (J/C) (C/s) = J/s

Potential Input to a Pure Resistance

$$P = V_{ab} I = I^2 R = \frac{V_{ab}^2}{R}$$

Rate of transfer of electric potential energy into the circuit ($V_a > V_b$) → energy dissipated (heat) in resistor at a rate $I^2 R$.

Potential Output of a source

$$P = V_{ab} I = (\varepsilon - Ir) I = \varepsilon \cdot I - I^2 r$$

ε I = rate at which the emf source converts nonelectrical to electrical energy.

I^2 r = rate at which electric energy is dissipated at the internal resistance of source.

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Problem

A 500 W heating coil designed to operate from 110 V is made of nichrome wire 0.500 mm in diameter. (a) Assuming that the resistivity of the Nichrome remains constant at its 20.0 °C value, find the length of wire used. (b) What If? Now consider the variation of resistivity with temperature. What power will the coil of part (a) actually deliver when it is heated to 1200 °C? $\alpha = 0.4.10^{-3}$

Solution: (a) The power delivered by the wire is;

$$P = I\Delta V = \frac{(\Delta V)^2}{R}$$

$$R = \frac{(\Delta V)^2}{P} = \frac{(110)^2}{500} = 24.2\Omega$$

Using the resistance we can get the length of the wire:

$$R = \rho \frac{l}{A} => l = \frac{RA}{\rho} = \frac{R\pi r^2}{\rho} = \frac{24.2\pi(2.5.10^{-4})^2}{1.5.10^{-6}} = 3.17m$$

(b) The resistance of the wire at 1200 °C is:

$$R = R_0[1+\alpha(T - T_0)]$$

$$R = R_0[1+\alpha(T - T_0)]$$

$$R = 24.2[1+0.4.10^{-3}(1200-20)] = 35.6\Omega$$

The power delivered at 1200 °C is:

$$P = \frac{(\Delta V)^2}{R} = \frac{(110)^2}{35.6} = 340\ W$$