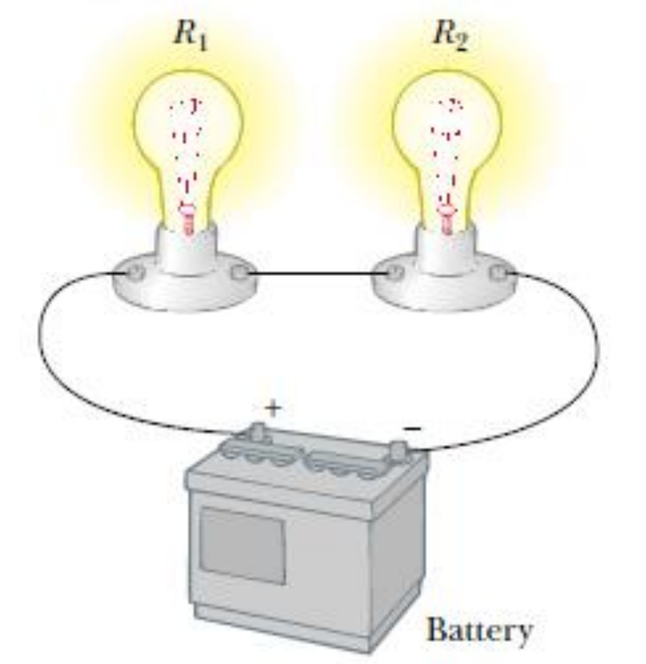
Physics 2: Electricity & Magnetism
Direct-Current Circuits



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Outline

- Electromotive Force (shortly)
- Resistors in Series and Parallel
- Kirchhoff’s Rules

The analysis of more complicated circuits is simplified using two rules known as *Kirchhoff’s rules*

- R-C Circuits

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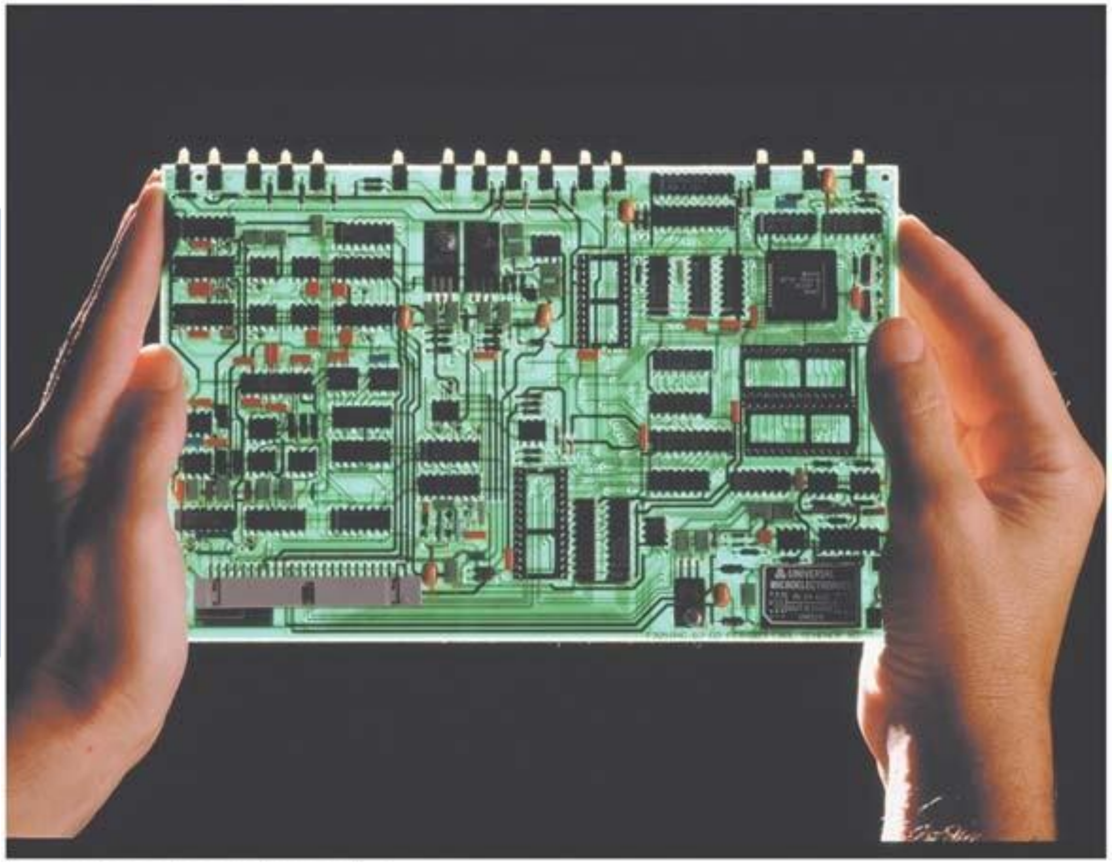
Introduction

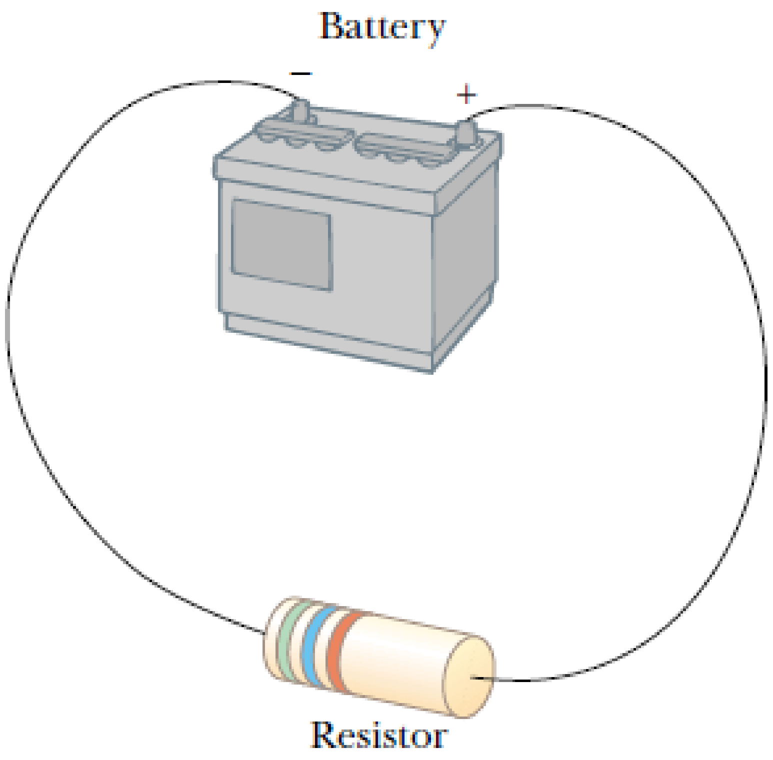
- In the last chapter, we gained insight about how current flows through a resistor in simple examples like a light bulb attached to a battery.

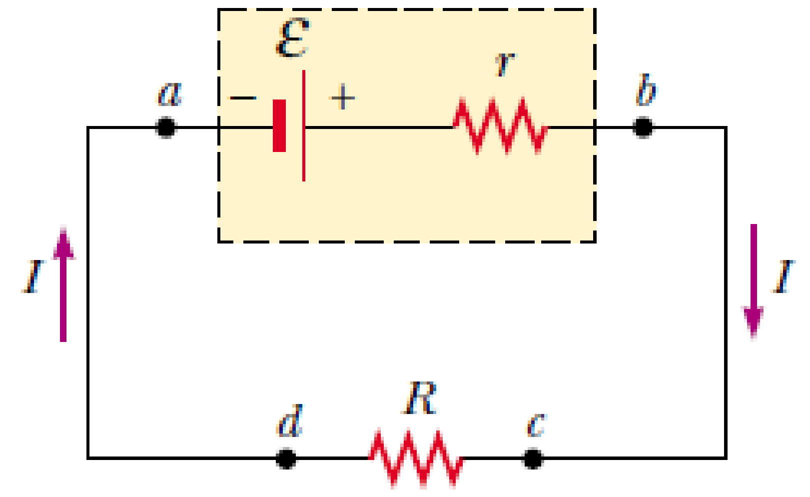
- Now, imagine many thousands of circuits wired onto flat wafers with structure so tiny that microscopy would be necessary to view their patterns. Understanding the next step and mastering more complex circuit patterns is the goal for Chapter 28.

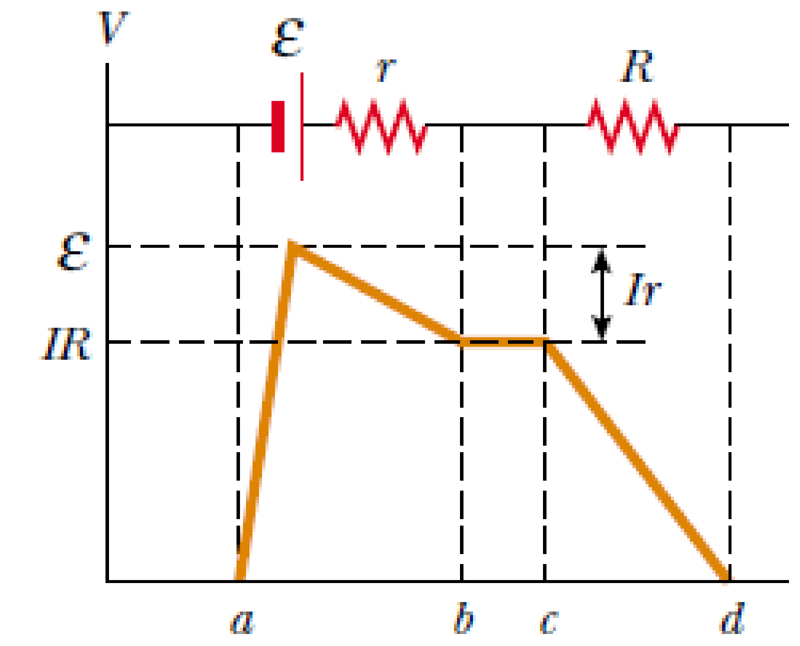


Electromotive Force

The potential difference at the battery terminals is constant in a particular circuit, the current in the circuit is constant in magnitude and direction and is called *direct current*. A battery is called either a source of electromotive force or, more commonly, a source of emf. (The phrase electromotive force is an unfortunate historical term, describing not a force but rather a potential difference in volts.) The emf of a battery is the maximum possible voltage that the battery can provide between its terminals. An idealized battery with zero internal resistance, the potential difference across the battery (called its *terminal voltage*) equals its emf.







- As we pass from the negative terminal to the positive terminal, the potential *increases* by an amount ε.
- As we move through the resistance *r*, the potential *decreases* by an amount *Ir*

$$\mathcal{E} = IR + Ir$$

- Multiply (1st Eq.) with I

$$I = \frac{\mathcal{E}}{R + r}$$

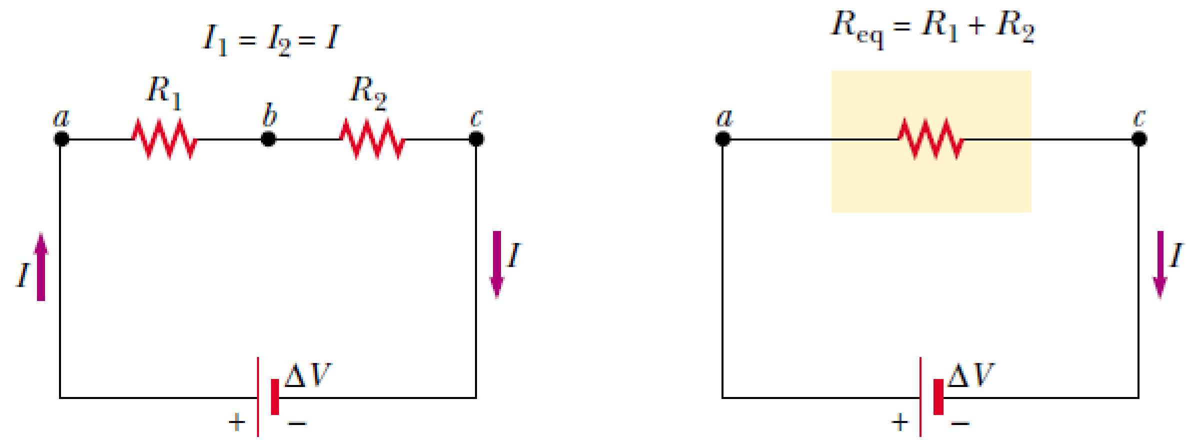
- The total power output *I* of the battery is delivered to the external load resistance in the amount I^2R and to the internal resistance in the amount I^2r.

$$I\mathcal{E} = I^2R + I^2r$$

Resistors in Series

- In a series connection, if an amount of charge Q exits resistor $R1$, charge Q must also enter the second resistor $R2$. Otherwise, charge will accumulate on the wire between the resistors. Thus, the same amount of charge passes through both resistors in a given time interval. Hence,

> for a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through R_1 must also pass through R_2 in the same time interval.



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$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

$$\Delta V = IR_{\text{eq}}$$

$$R_{\text{eq}} = R_1 + R_2$$

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots$$

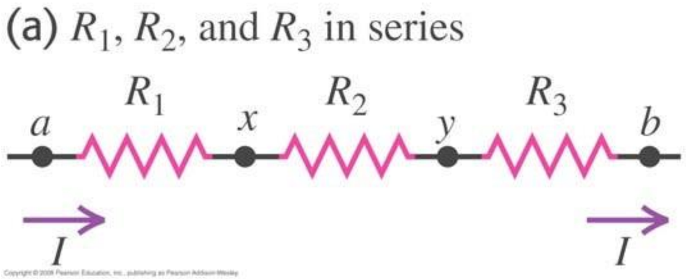
This relationship indicates that **the equivalent resistance of a series connection of resistors** is the numerical sum of the individual resistances and **is always greater than any individual resistance**.

Series resistors :

(1)Same current through all resistors = I

(2)Total voltage V = $V_1 + V_2 + V_2$

- $V = IR_1 + IR_2 + IR_3 = IR_{eq}$
- $R_{eq} = R_1 + R_2 + R_3$



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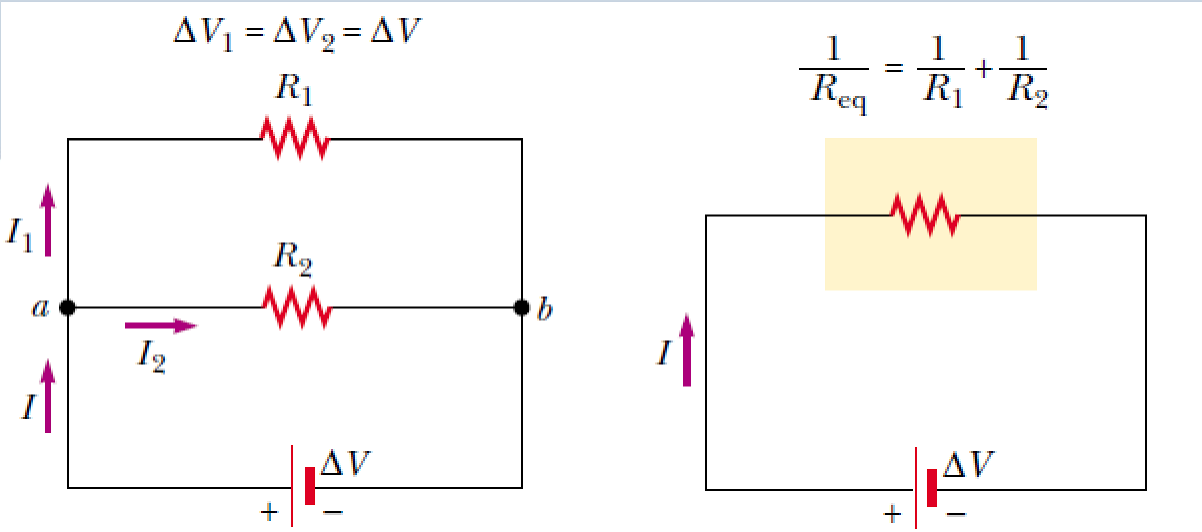
Resistors in Parallel

- When charges reach point *a* in Figure, called a *junction,* they split into two parts, with some going through *R*1 and the rest going through *R*2. A junction is any point in a circuit where a current can split. This split results in less current in each individual resistor than the current leaving the battery. Because electric charge is conserved, the current *I* that enters point *a* must equal the total current leaving that point:



$$I = I_1 + I_2$$

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- As can be seen from Figure 28.6, both resistors are connected directly across the terminals of the battery. Therefore, when resistors are connected in parallel, the potential differences across the resistors is the same.

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{\text{eq}}}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{\text{eq}} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

Parallel resistors :

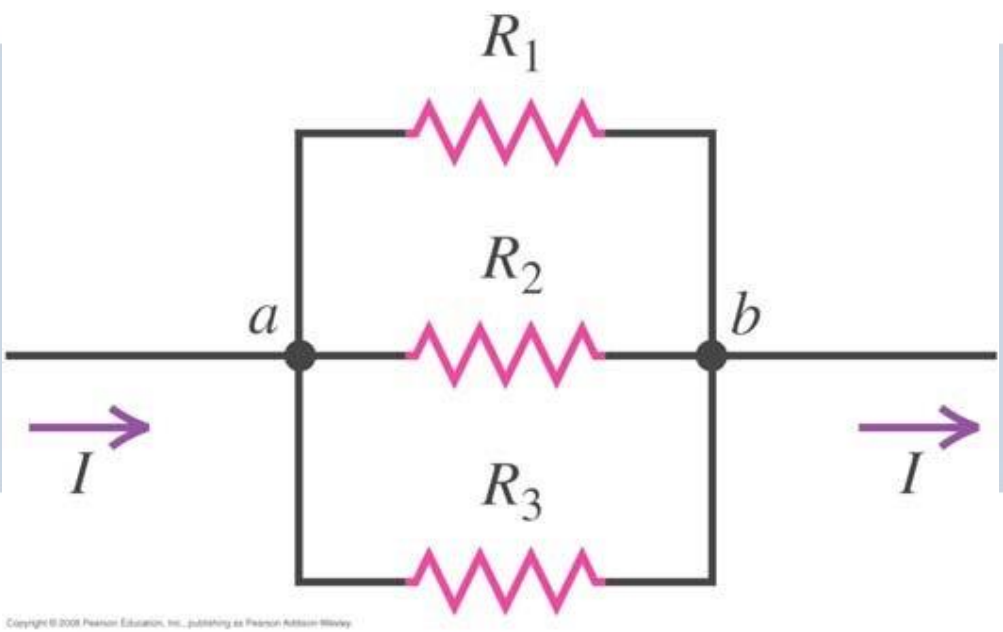
(1) Same voltage across all resistors = V

(2) Total current I = $I_1 + I_2 + I_2$

$$I = \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

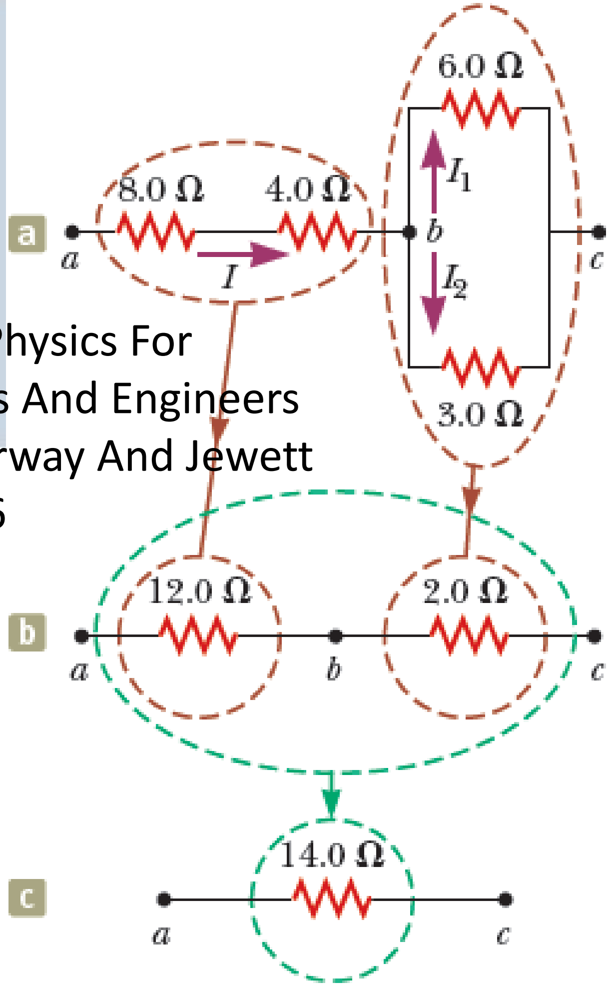
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

(b) R_1, R_2, and R_3 in parallel



Example 28.4 Find the Equivalent Resistance

Four resistors are connected as shown in Figure 28.9a. **(A)** Find the equivalent resistance between points *a* and *c*. **(B)** What is the current in each resistor if a potential difference of 42 V is maintained between *a* and *c*?



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Solution:

$$\frac{1}{R_{eq}} = \frac{1}{6.0\ \Omega} + \frac{1}{3.0\ \Omega} = \frac{3}{6.0\ \Omega}$$

$$R_{eq} = \frac{6.0\ \Omega}{3} = 2.0\ \Omega$$

$$R_{eq} = 12.0\ \Omega + 2.0\ \Omega = 14.0\ \Omega$$

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\ \text{V}}{14.0\ \Omega} = 3.0\ \text{A}$$ in the 8.0-Ω and 4.0-Ω resistors.

When this 3.0-A current enters the junction at *b*, it splits, with part passing through the 6.0 Ω resistor (*I*1) and part through the 3.0 Ω resistor (*I*2). Because the potential difference is ΔV_{bc} across each of these parallel resistors, we see that $(6.0\ \Omega)I_1 = (3.0\ \Omega)I_2$, or $I_2 = 2I_1$. Using this result and the fact that *I*1 + *I*2 = 3.0 A, we find that $I_1 = 1.0$ A and I_2=2.0 A.

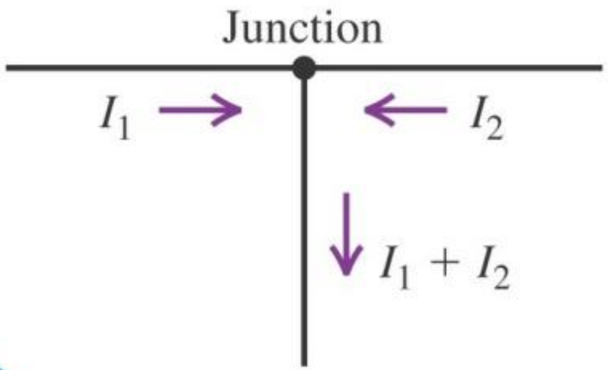
Kirchhoff’s Rules

Junction: point where three or more conductors meet (nodes, branch points).

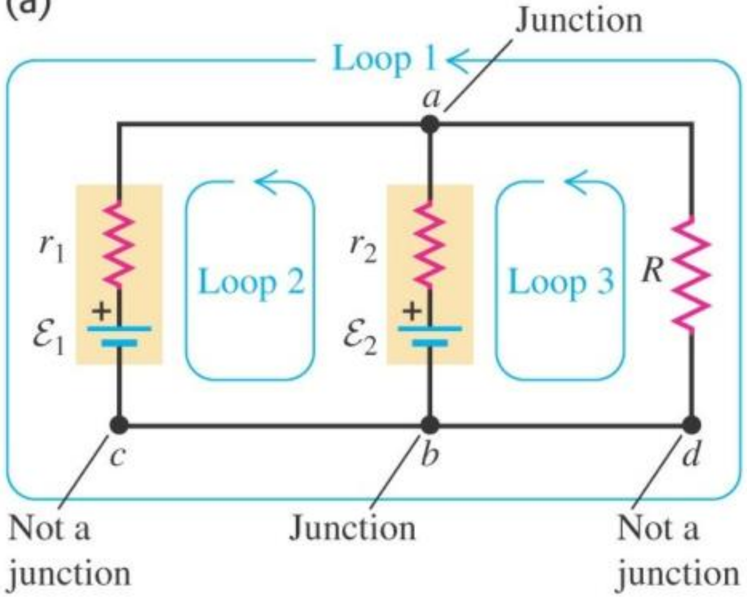
Loop: closed conducting path.

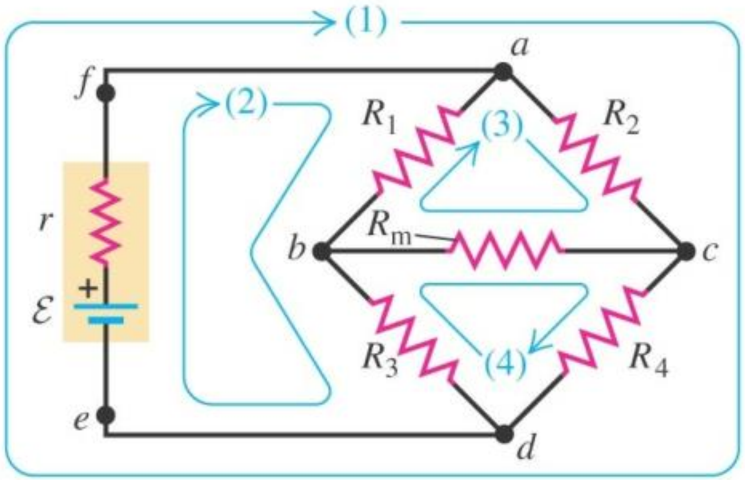
Kirchhoff’s junction rule: the algebraic sum of the currents into any junction is zero.

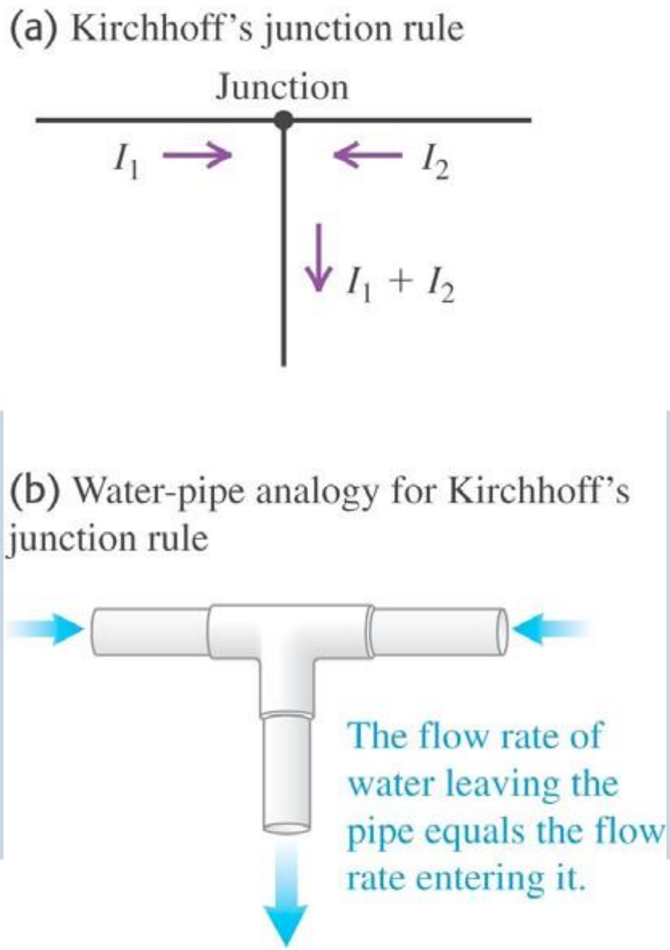
$$\Sigma I = 0$$



(a)







Note:(1) Junction rule = conservation of charge

(2) You may ASSUME the direction of the current; if your final answer is negative, then the actual current flows in the opposite direction.

-The junction rule is based on conservation of electric charge. No charge can accumulate at a junction ⇨ total charge entering the junction per unit time = total charge leaving.

Kirchhoff's loop rule: the algebraic sum of the potential difference in any loop, Including those associated with emfs and those of resistive elements, must equal zero.

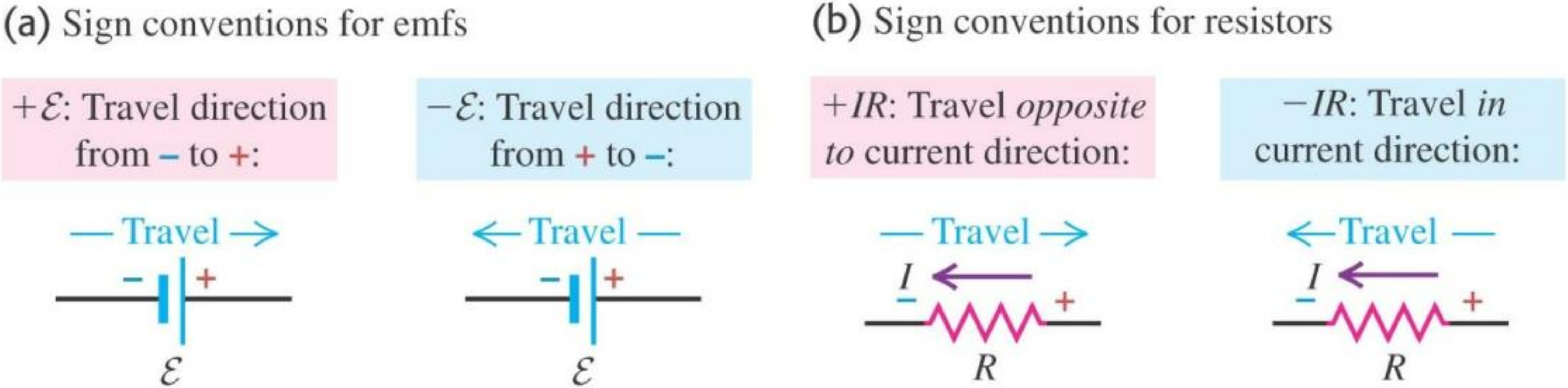
$$\Sigma V = 0$$ (electrostatic force is conservative)

Sign Conventions for Loop Rule:

emf source (ε) positive (travel from – to +)
negative (travel from + to -)

resistor (IR) negative (travel in same direction as I ⇨ decreasing V)
positive (travel in contrary direction to I ⇨ increasing V)

- "Travel" is the direction that we imagine going around the loop, not necessarily the direction of the current.



(a) Sign conventions for emfs
+ℰ: Travel direction from – to +:
Travel
–
+
ℰ
−ℰ: Travel direction from + to –:
Travel
–
+
ℰ
(b) Sign conventions for resistors
+IR: Travel opposite to current direction:
Travel
I
–
+
R
−IR: Travel in current direction:
Travel
I
–
+
R

Example 28.8 **A Single-Loop Circuit**

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A single-loop circuit contains two resistors and two batteries,as shown in Figure 28.16. (Neglect the internal resistances of the batteries.)

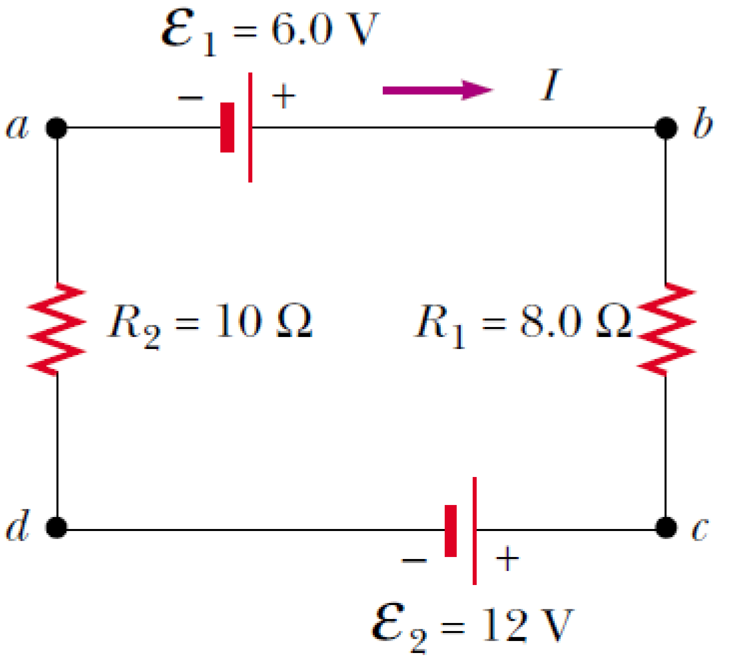
(A) Find the current in the circuit.

Solution:

$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\text{ V} - 12\text{ V}}{8.0\ \Omega + 10\ \Omega} = -0.33\text{ A}$$



The negative sign for *I* indicates that the direction of the current is opposite the assumed direction. Notice that the emfs in the numerator subtract because the batteries have opposite polarities in Figure 28.16

(B) What power is delivered to each resistor? What power is delivered by the 12-V battery?

Solution:

$$\mathcal{P}_1 = I^2R_1 = (0.33\text{ A})^2(8.0\ \Omega) = 0.87\text{ W}$$

$$\mathcal{P}_2 = I^2R_2 = (0.33\text{ A})^2(10\ \Omega) = 1.1\text{ W}$$

Hence, the total power delivered to the resistors is 2 W. The 12-V battery delivers power $I\mathcal{E}_2$ = 4.0 W. Half of this power is delivered to the two resistors, as we just calculated.

Example 28.9 **Applying Kirchhoff's Rules**

Find the currents I_1, I_2, and I_3 in the circuit shown in Figure 28.17.

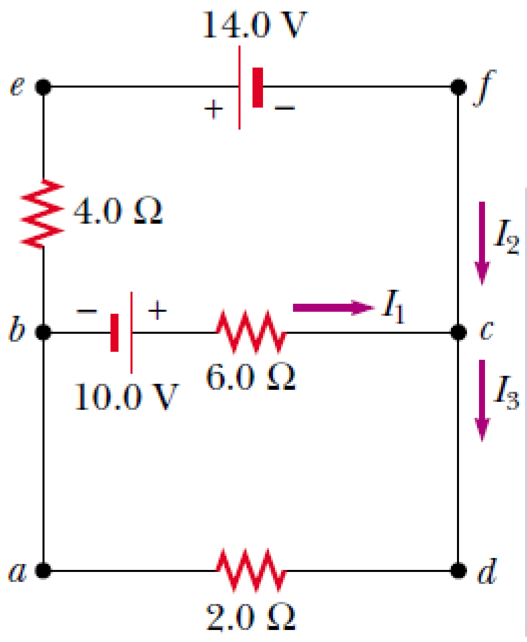


Figure 28.17

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Solution: Applying Kirchhoff's junction rule to junction *c* gives,

$$(1) \qquad I_1 + I_2 = I_3$$

We now have one equation with three unknowns—I_1, I_2, and I_3. There are three loops in the circuit—*abcda, befcb,* and *aefda.* Applying Kirchhoff's loop rule to loops abcda and befcb and traversing these loops clockwise, we obtain the expressions,

$$(2) \qquad abcda \quad 10.0\text{ V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)I_3 = 0$$

$$(3) \quad befcb \quad -14.0\text{ V} + (6.0\ \Omega)I_1 - 10.0\text{ V} - (4.0\ \Omega)\ I_2 = 0$$

Equation (1) into Equation (2) gives

$$10.0\ \text{V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)(I_1 + I_2) = 0$$

$$(4) \qquad 10.0\ \text{V} = (8.0\ \Omega)I_1 + (2.0\ \Omega)I_2$$

Dividing each term in Equation (3) by 2 and rearranging gives

$$(5) \qquad -12.0\ \text{V} = -(3.0\ \Omega)I_1 + (2.0\ \Omega)I_2$$

Subtracting Equation (5) from Equation (4) eliminates I_2, giving

$$22.0\ \text{V} = (11.0\ \Omega)I_1$$

$$I_1 = 2.0\ \text{A}$$

$$(2.0\ \Omega)I_2 = (3.0\ \Omega)I_1 - 12.0\ \text{V}$$
$$= (3.0\ \Omega)(2.0\ \text{A}) - 12.0\ \text{V} = -6.0\ \text{V}$$

$$I_2 = -3.0\ \text{A}$$

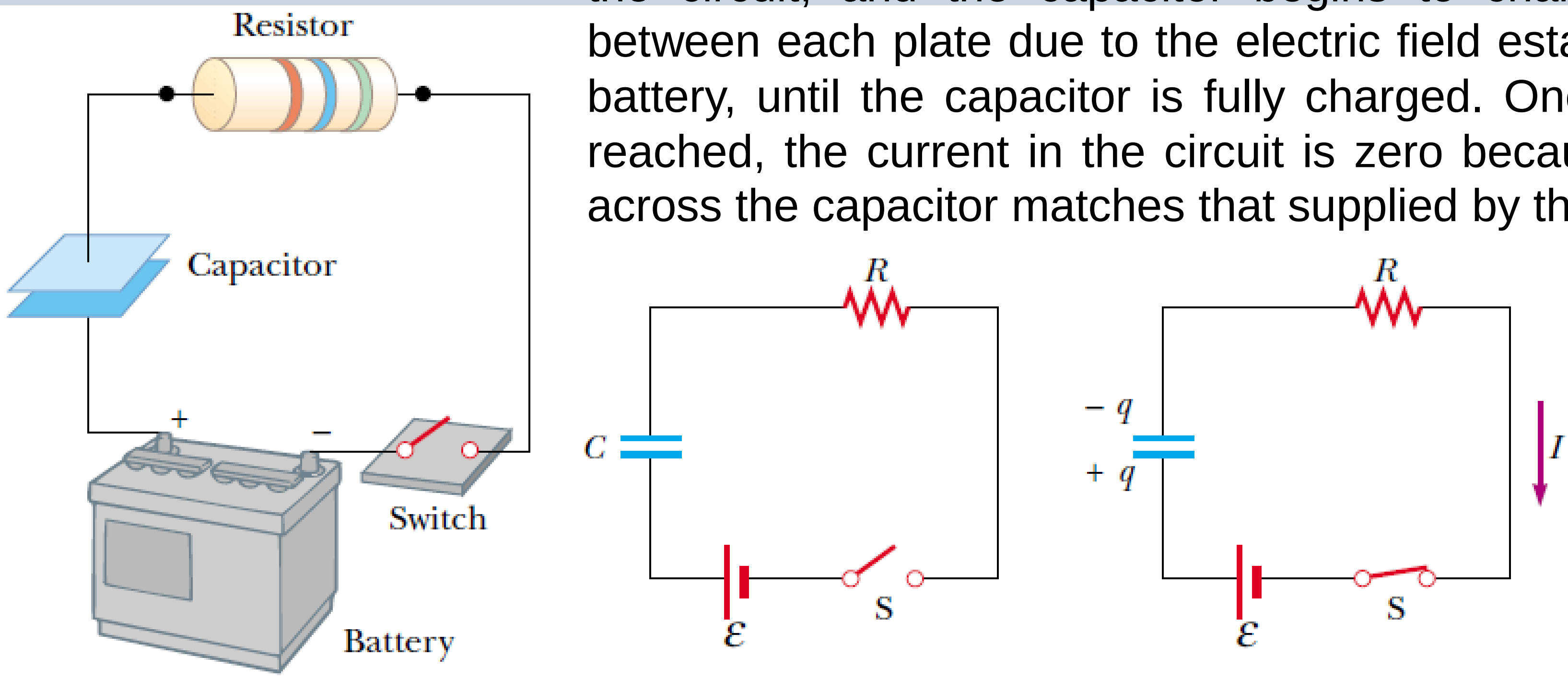
$$I_3 = I_1 + I_2 = -1.0\ \text{A}$$

28.4 RC Circuits

So far we have analyzed direct current (DC) circuits in which the current is constant. In DC circuits containing capacitors, the current is always in the same direction but may vary in time. A circuit containing a series combination of a resistor and a capacitor is called an RC circuit.

Charging a Capacitor

If the switch is closed at $t = 0$, charge begins to flow, setting up a current in the circuit, and the capacitor begins to charge. Charge is transferred between each plate due to the electric field established in the wires by the battery, until the capacitor is fully charged. Once the maximum charge is reached, the current in the circuit is zero because the potential difference across the capacitor matches that supplied by the battery.



(a) (b) $t < 0$ (c) $t > 0$

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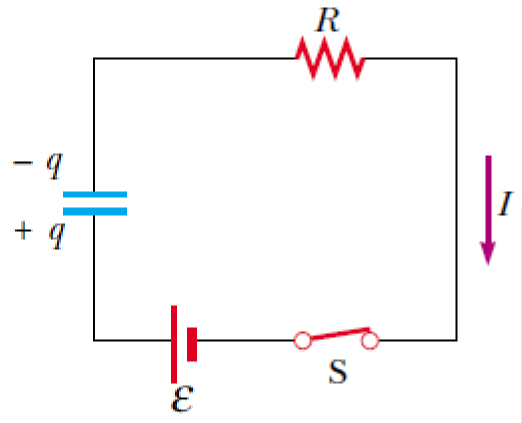
To analyze this circuit quantitatively, let us apply Kirchhoff's loop rule to the circuit after the switch is closed.

$$\varepsilon - \frac{q}{C} - IR = 0$$ Equation 28.11



The initial current I_0 in the circuit is a maximum and is equal to

$$I_0 = \frac{\varepsilon}{R}$$ (current at $t = 0$)

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Later, when the capacitor is charged to its maximum value *Q*, charges cease to flow, the current in the circuit is zero.

Substituting $I = 0$ into Equation 28.11 gives the charge on the capacitor at this time:

$$Q = C\varepsilon$$ (maximum charge)

Substitute $I = dq/dt$ into Equation 28.11 and rearrange the equation:

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC}$$

To find an expression for *q*, we solve this separable differential equation. We first combine the terms on the right-hand side:

$$\frac{dq}{dt} = \frac{C\varepsilon}{CR} - \frac{q}{RC} = -\frac{q - C\varepsilon}{RC}$$

$$\frac{dq}{q-C\varepsilon} = -\frac{1}{RC}\,dt$$

Integrating this expression, using the fact that *q* = 0 at *t* = 0, we obtain

$$\int_0^q \frac{dq}{q - C\varepsilon} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

$$q(t) = C\varepsilon(1 - e^{-t/RC})$$

We can find an expression for the charging current by differentiating Equation 28.14 with respect to time:

$$q(t) = C\varepsilon(1 - e^{-t/RC}) \quad 28.14 \qquad I(t) = \frac{\varepsilon}{R} e^{-t/RC} \quad 28.15$$

Plots of capacitor charge and circuit current versus time are shown in Figure 28.20.
Note that the charge is zero at $t = 0$ and approaches the maximum value $C\varepsilon$ as $t \to \infty$.

The quantity *RC*, which appears in the exponents of Equations 28.14 and 28.15, is called the time constant τ of the circuit. It represents the time interval during which the current decreases to 1/*e* of its initial value.

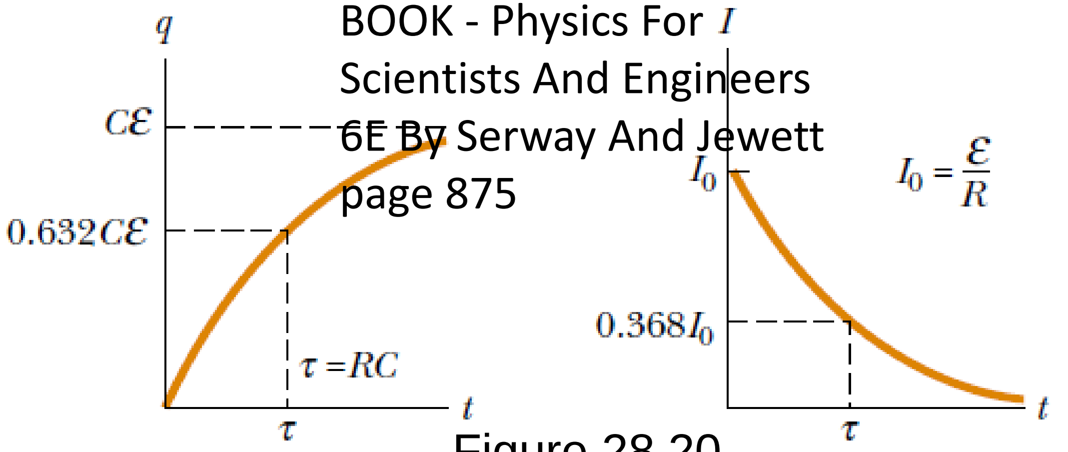

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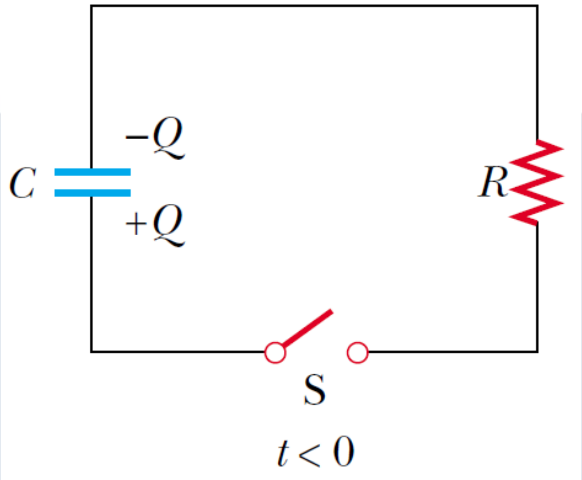
Figure 28.20

The energy output of the battery as the capacitor is fully charged is $Q\mathcal{E} = C\mathcal{E}^2$

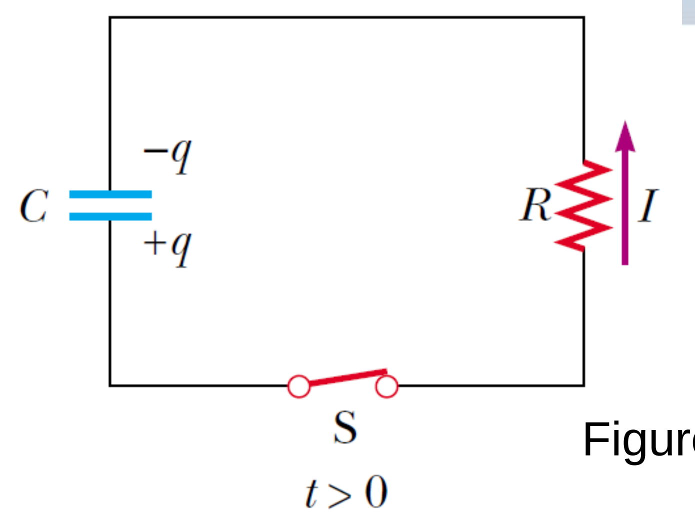
After the capacitor is fully charged, the energy stored in the capacitor is $\frac{1}{2}Q\mathcal{E} = \frac{1}{2}C\mathcal{E}^2$

Discharging a Capacitor

Now consider the circuit shown in Figure 28.21, which consists of a capacitor carrying an initial charge *Q* , a resistor, and a switch.



When the switch is open, a potential difference *Q* /*C* exists across the capacitor and there is zero potential difference across the resistor because *I* = 0.



If the switch is closed at *t* = 0, the capacitor begins to discharge through the resistor. At some time *t* during the discharge, the current in the circuit is *I* and the charge on the capacitor is *q.*

Figure 28.21

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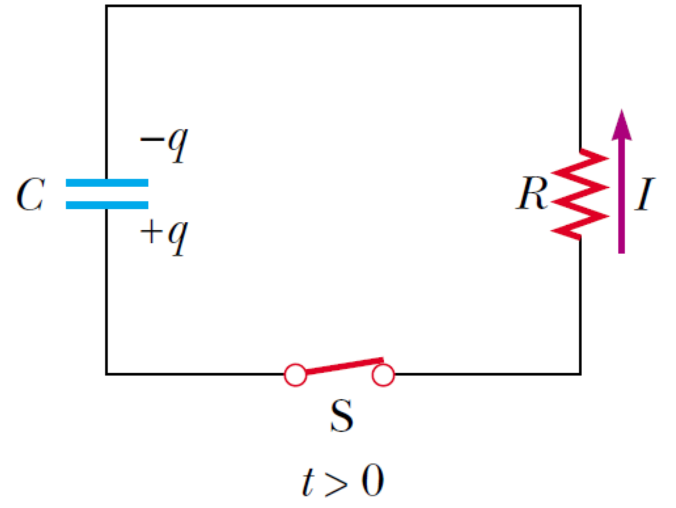

$$-\frac{q}{C} - IR = 0$$

When we substitute *I* = *dq/dt* into this expression, it becomes

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{1}{RC}dt$$

Using the fact that *q* = *Q* at *t* = 0 gives

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

$$q(t) = Qe^{-t/RC}$$

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Differentiating this expression with respect to time gives the instantaneous current as a function of time:

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-t/RC}\right) = -\frac{Q}{RC}e^{-t/RC}$$

where $Q/RC = I_0$ is the initial current.

The negative sign indicates that as the capacitor discharges, the current direction is opposite its direction when the capacitor was being charged. We see that both the charge on the capacitor and the current decay exponentially at a rate characterized by the time constant τ = RC.

Example 28.12 **Charging a Capacitor in an RC Circuit**

An uncharged capacitor and a resistor are connected in series to a battery, as shown in Figure 28.23. If ε=12.0 V, C = 5.00 μF, and R = 8.00 × 10^5 Ω, find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.

Solution The time constant of the circuit is

$$\tau = RC = (8.10^5)(5.10^{-6}) = 4s$$

The maximum charge on the capacitor is

$$Q = C\varepsilon = (5.10^{-6})(12) = 60\mu C$$

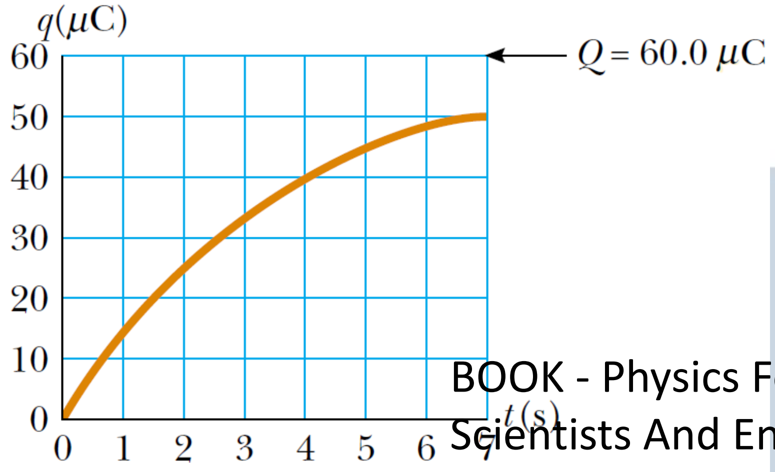
The maximum current in the circuit is

$$I_0 = \frac{\varepsilon}{R} = \frac{12}{8.10^{-5}} = 15\mu A$$

Using these values and Equations 28.14 and 28.15, we find that

$$q(t) = (60\mu C)\left(1 - e^{-t/4}\right)$$

$$I(t) = (15\mu A)e^{-t/4}$$



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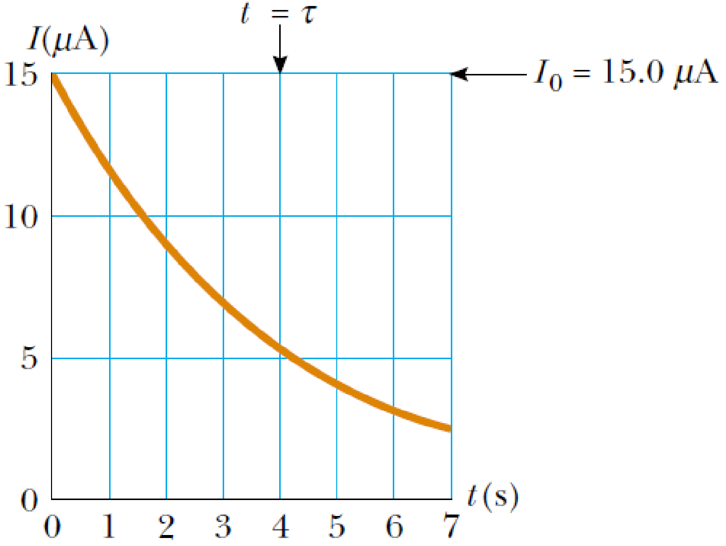

SUMMARY

The emf of a battery is equal to the voltage across its terminals when the current is zero. That is, the emf is equivalent to the open-circuit voltage of the battery.

The equivalent resistance of a set of resistors connected in series is,

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

The equivalent resistance of a set of resistors connected in parallel is found from the relationship

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

Circuits involving more than one loop are conveniently analyzed with the use of Kirchhoff's rules:

1. Junction rule.

$$\sum I_{in} = \sum I_{out}$$

2. Loop rule

$$\sum_{closed\ loop} \Delta V = 0$$

The first rule is a statement of conservation of charge; the second is equivalent to a statement of conservation of energy.

If a capacitor is charged with a battery through a resistor of resistance *R*, the charge on the capacitor and the current in the circuit vary in time according to the expressions:

$$q(t) = Q(1 - e^{-t/RC})$$

$$I(t) = \frac{\varepsilon}{R} e^{-t/RC}$$

If a charged capacitor is discharged through a resistor of resistance R, the charge and current decrease exponentially in time according to the expressions:

$$q(t) = Qe^{-t/RC}$$

$$I(t) = -I_0 e^{-t/RC}$$