Physics 2: Electricity & Magnetism – Sources of the Magnetic Field Part 1

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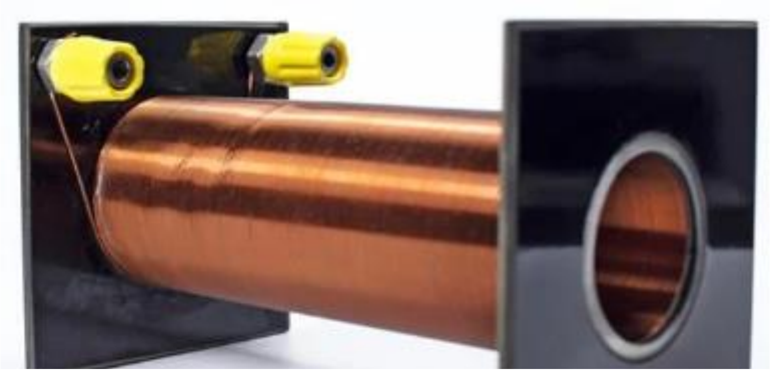


https://www.arborsci.com/products/air-core-solenoid-5a-max

Outline

-The Biot–Savart Law: Able to calculate the magnetic field due to various current distributions

-The Magnetic Force Between Two Parallel Conductors

-Ampère's Law: Useful in calculating the magnetic field of a highly symmetric configuration carrying a steady current

-The Magnetic Field of a Solenoid

-Gauss's Law in Magnetism

-Magnetism in Matter

The Biot–Savart Law

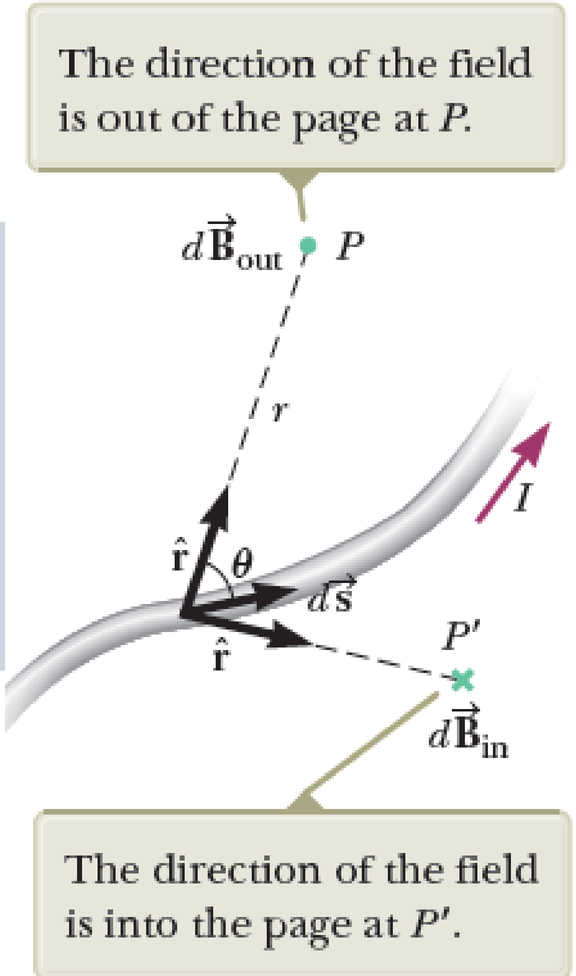
Biot–Savart Law is based on the following experimental observations for the magnetic field $d\mathbf{B}$ at a point *P* associated with a length element $d\mathbf{s}$ of a wire carrying a steady current *I*

- $d\vec{B} \perp d\vec{s}$ (which points in the direction of the current) and $d\vec{B} \perp \hat{r}$ ($\hat{r}$ is the unit vector directed from $d\vec{s}$ toward P)
- $d\vec{B} \propto 1/r^2$, where *r* is the distance from ds to *P*.
- $d\vec{B} \propto I$ and to the magnitude ds of the length element $d\vec{s}$
- $d\vec{B} \propto \sin\theta$

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

- Permeability of free space, $\mu_0 = 4\pi 10^{-7}$ Tm/A

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$



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Example 30.1 Magnetic Field Surrounding a Thin, Straight Conductor

Consider a thin, straight wire carrying a constant current *I* and placed along the *x* axis as shown in Figure 30.3. Determine the magnitude and direction of the magnetic field at point *P* due to this current.

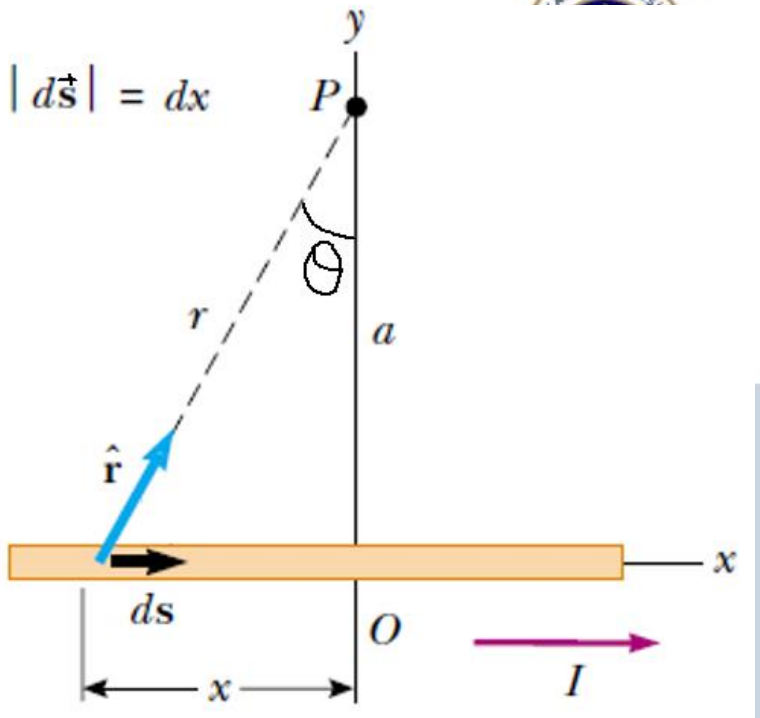

Figure 30.3

Solution: The direction of the magnetic field at point *P* due to the current in this element is out of the page. Taking the origin at *O* and letting point *P* be along the positive *y* axis, with $\hat{k}$ being a unit vector pointing out of the page, we see that

$$d\vec{s} \times \hat{\mathbf{r}} = |d\vec{s} \times \hat{\mathbf{r}}|\hat{\mathbf{k}} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{\mathbf{k}} = (dx \cos\theta)\hat{\mathbf{k}}$$

$d\vec{s}$=dx in magnitude

angle between dx and $\hat{r}$

$$(1) \quad d\vec{\mathbf{B}} = (dB)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{\mathbf{k}}$$

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$$(2) \quad r = \frac{a}{\cos\theta}$$

$$x = -a\tan\theta$$

cosθ=a/r so r=a/cosθ
Notice that tanθ=-*x*/*a* (the negative sign is necessary because *d***s** is located at a negative value of *x*)

$$x = -a\tan\theta$$

$$(3)\quad dx = -a\sec^2\theta\, d\theta = -\frac{a\,d\theta}{\cos^2\theta}$$

$$(1)\quad d\vec{\mathbf{B}} = (dB)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{\mathbf{k}}$$

r=a/cosθ

$$(4)\quad dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\,d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta = -\frac{\mu_0 I}{4\pi a}\cos\theta\, d\theta$$

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

Finalize We can use this result to find the magnitude of the magnetic field of *any* straight current-carrying wire if we know the geometry and hence the angles θ_1 and θ_2. Consider the special case of an infinitely long, straight wire. If the wire in Figure 30.3b becomes infinitely long, we see that

$$\theta_1 = \pi/2,\ \theta_2 = -\pi/2$$

for length elements ranging between positions $x = \infty$ *and* $x = -\infty$. Because

$$\sin(\theta_1 - \theta_2) = [\sin(\pi/2) - \sin(-\pi/2)] = 2$$

Equation becomes

$$B = \frac{\mu_0 I}{2\pi a}$$

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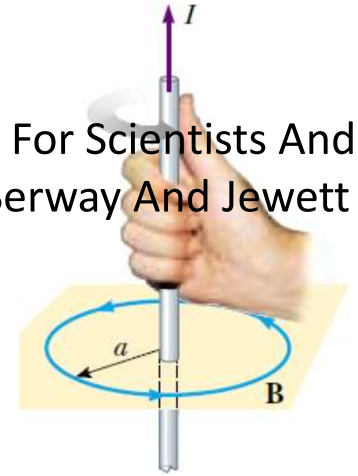

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Example 30.3 **Magnetic Field on the Axis of a Circular Current Loop**

Consider a circular wire loop of radius R located in the yz plane and carrying a steady current I, as in Figure 30.6. Calculate the magnetic field at an axial point P a distance x from the center of the loop.

Solution: In this situation, every length element ds is perpendicular to the vector $\hat{r}$ at the location of the element. Thus, for any element,

$$\left| d\vec{s} \times \hat{r} \right| = ds.1.\sin 90^o = ds$$

$$r^2 = x^2 + R^2$$

Hence, the magnitude of dB due to the current in any length element ds is

$$dB = \frac{\mu_0 I}{4\pi} \frac{\left| d\vec{s} \times \hat{r} \right|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{\left(x^2 + R^2 \right)}$$

The direction of dB is perpendicular to the plane formed by $\hat{r}$ and ds. We can resolve this vector into a component dB_x along the x axis and a component dB_y perpendicular to the x axis. When the components dB_y are summed over all elements around the loop, the resultant component is zero.

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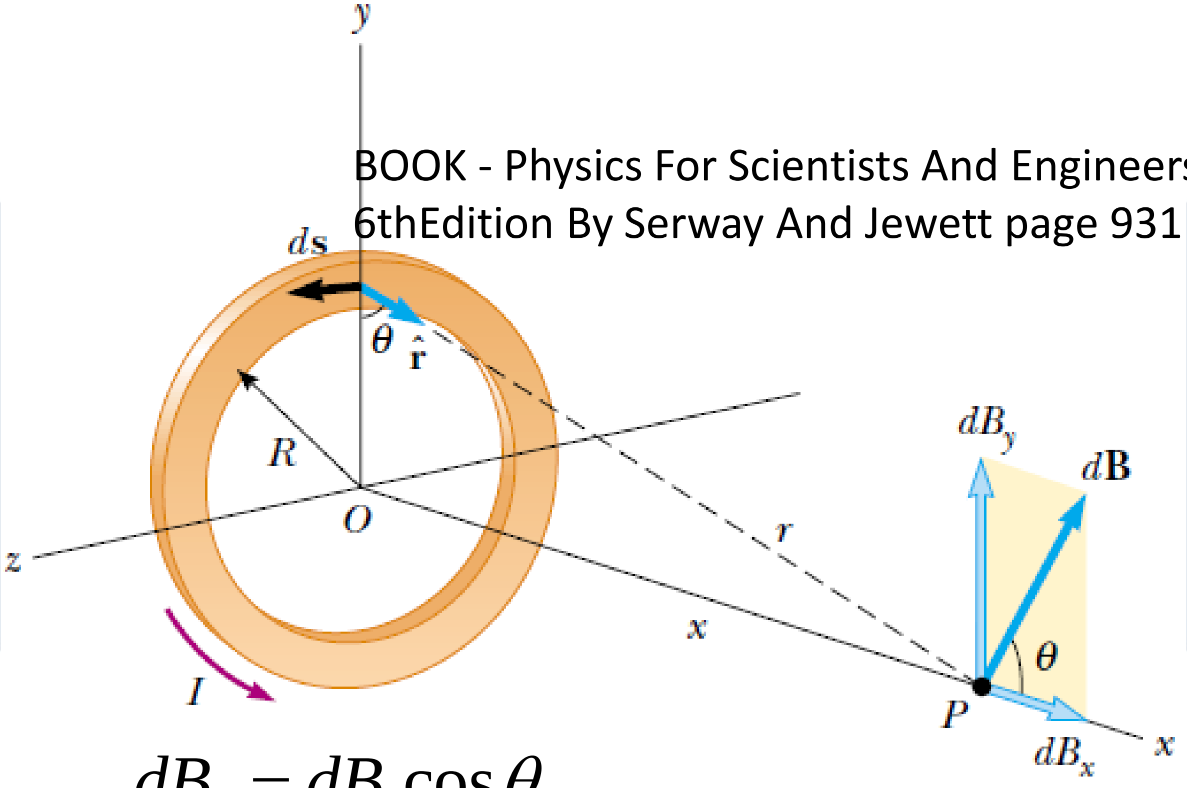
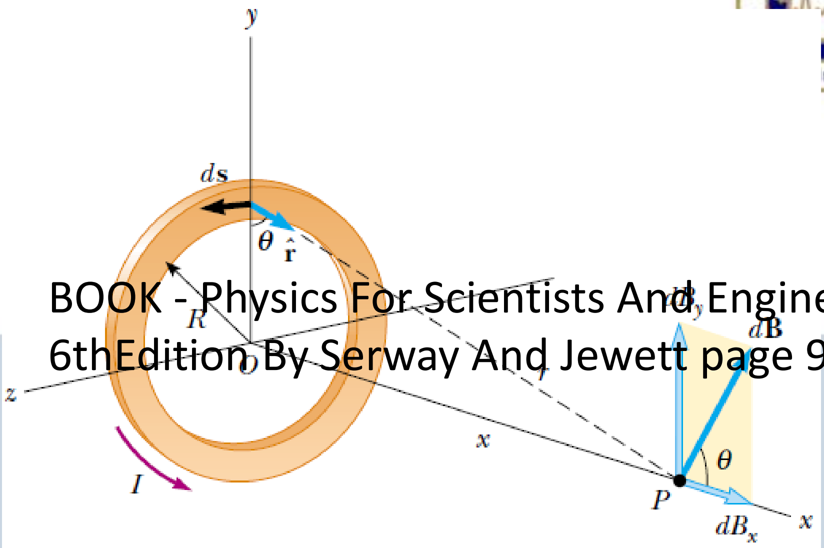


$$dB_x = dB.\cos\theta$$

$$B_x = \oint dB\cos\theta = \frac{\mu_0 I}{4\pi} \oint \frac{ds\cos\theta}{x^2 + R^2}$$

$$B_x = \oint dB\cos\theta = \frac{\mu_0 I}{4\pi}\oint \frac{ds\cos\theta}{x^2+R^2}$$

$$\cos\theta = R/(x^2+R^2)^{1/2}$$



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$$B_x = \oint dB\cos\theta = \frac{\mu_0 I}{4\pi\left(x^2+R^2\right)^{3/2}}\oint ds$$

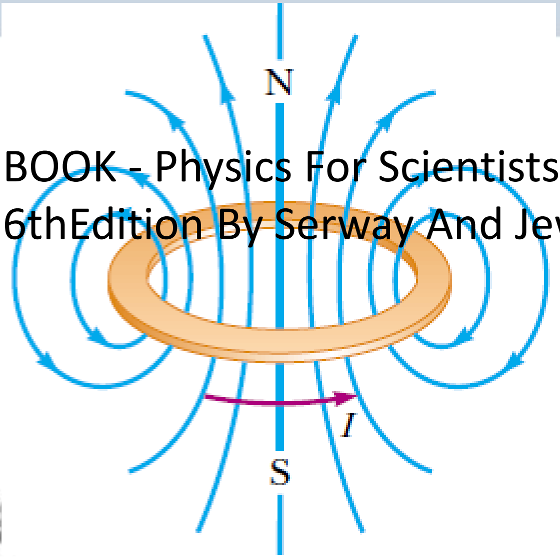
$$\oint ds = 2\pi R$$

$$B_x = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$$

Finalize:

To find the magnetic field at the center of the loop, we set $x = 0$.

$$B = \frac{\mu_0 I}{2R} \quad \text{at } x = 0$$



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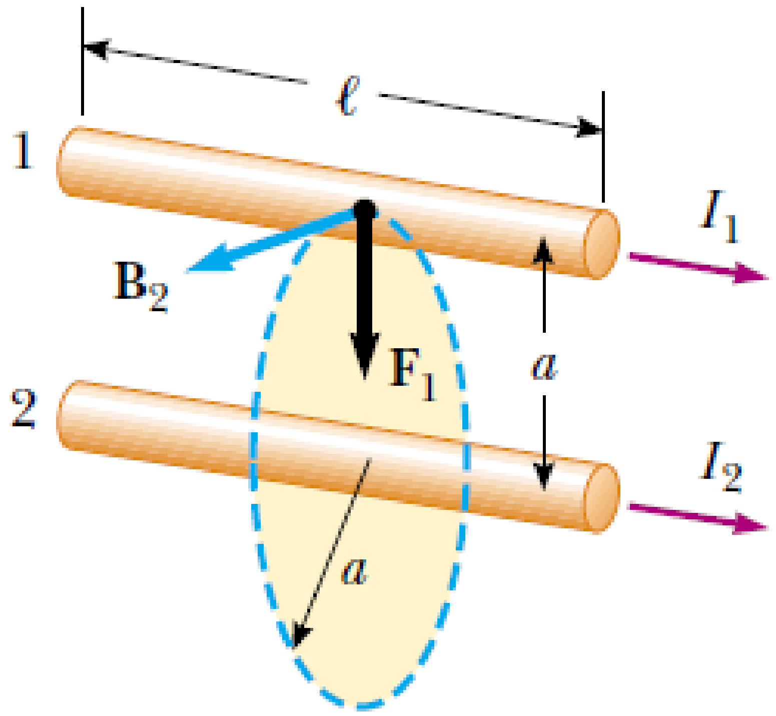
The Magnetic Force Between Two Parallel Conductors

A current in a conductor sets up its own magnetic field. Two current-carrying conductors exert magnetic forces on each other. Wire 2, which carries a current I_2 and is identified arbitrarily as the source wire, creates a magnetic field B_2 at the location of wire 1, the test wire. The magnetic force on a length l of wire 1 is

$$F_1 = I_1 l B_2 = I_1 l \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} l$$



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The direction of F_1 is toward wire 2. If the field set up at wire 2 by wire 1 is calculated, the force F_2 acting on wire 2 is found to be equal in magnitude and opposite in direction to F_1.

When the currents are in opposite directions, the forces are reversed and the wires repel each other.

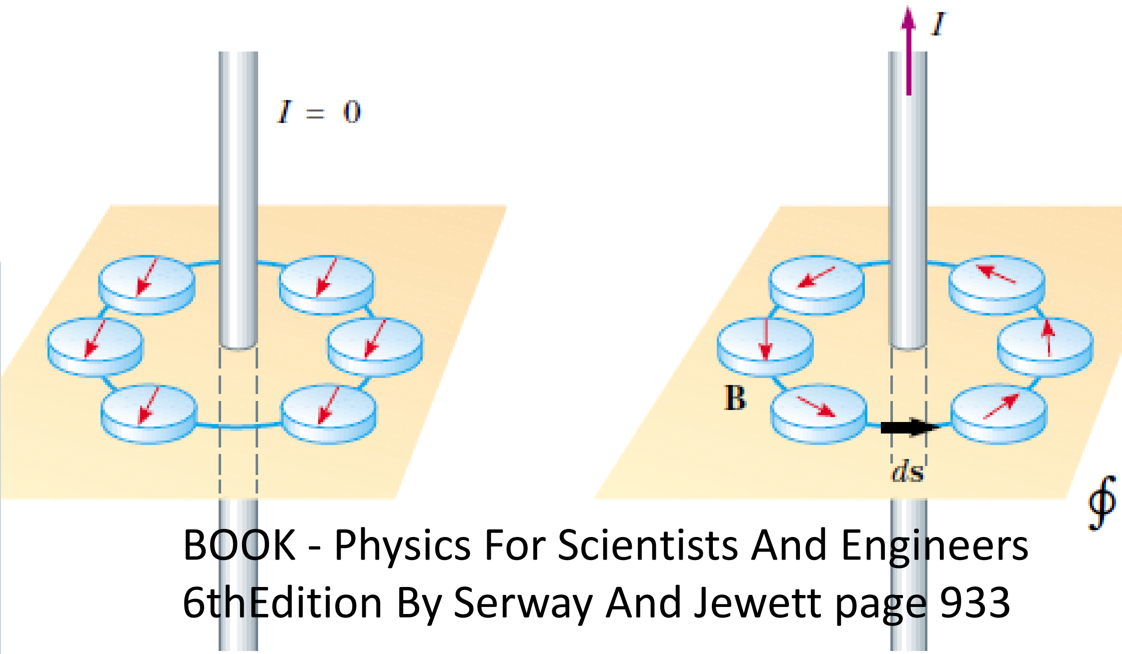
Parallel conductors carrying currents in the same direction attract each other, and parallel conductors carrying currents in opposite directions repel each other. Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply F_B. We can rewrite this magnitude in terms of the force per unit length:

$$\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

The force between two parallel wires is used to define the ampere as follows:

When the magnitude of the force per unit length between two long parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A.

Ampère's Law



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Now let us evaluate the product ***B***.*ds* for a small length element *ds* on the circular path defined by the compass needles.

$$\oint \mathbf{B}\cdot d\mathbf{s} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

$\oint ds = 2\pi r$ is the circumference of the circular path

When the wire carries a strong, steady current, the needles all deflect in a direction tangent to the circle.

Ampère's law

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I$$

The general case, known as Ampère's law, can be stated as follows:

The line integral of B.*ds* around any closed path equals μ_0I, where *I* is the total steady current passing through any surface bounded by the closed path. It is useful only for calculating the magnetic field of current configurations having a high degree of symmetry (similar to Gauss law).

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Example 30.5 **The Magnetic Field Created by a Long Current-Carrying Wire**

A long, straight wire of radius *R* carries a steady current *I* that is uniformly distributed through the cross section of the wire (Fig. 30.12). Calculate the magnetic field a distance *r* from the center of the wire in the regions $r \geq R$ and $r < R$.

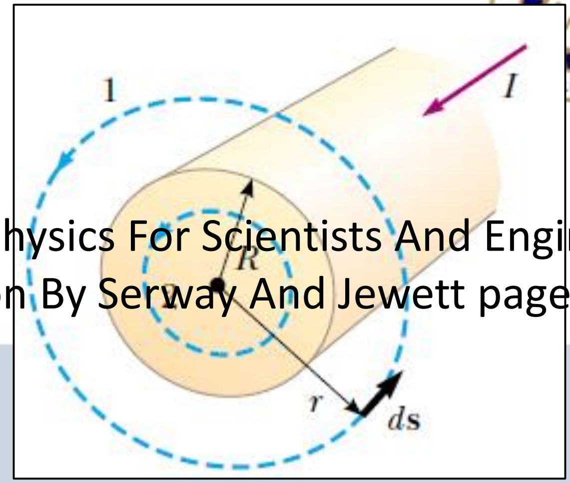

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Fig. 30.12

Solution: From symmetry ***B*** must be constant in magnitude and parallel to *ds* at every point on this circle. Apply Ampere's law and solve for ***B***:

$$\oint \vec{B} d\vec{s} = B(2\pi r) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad \text{for } r \geq R$$

In the interior part I' inside is less than I

$$\frac{I'}{I} = \frac{\pi r^2}{\pi R^2} \longrightarrow I' = \frac{r^2}{R^2} I$$

$$\oint \vec{B} d\vec{s} = B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I \right)$$

$$B = \left(\frac{\mu_0 I}{2\pi R^2} \right) r \qquad \text{for } r < R$$

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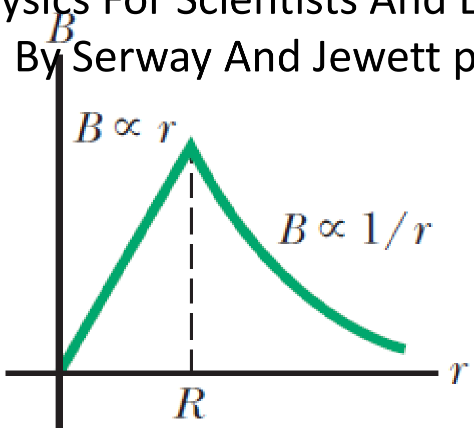

Serway Physics II

Magnetic Field of a Solenoid

A **solenoid** is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire—which we shall call the *interior* of the solenoid—when the solenoid carries a current. If length is much greater than radius, solenoid is ideal.

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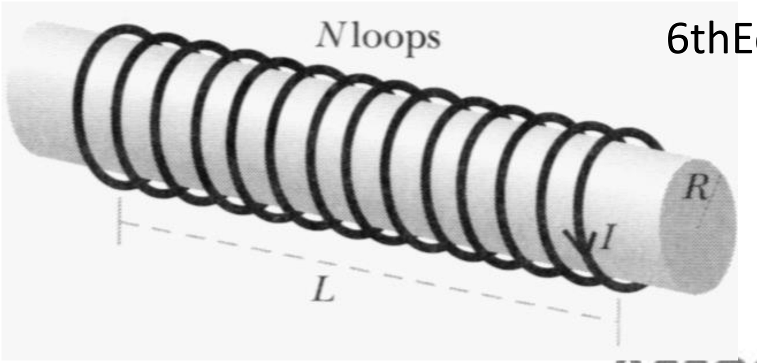

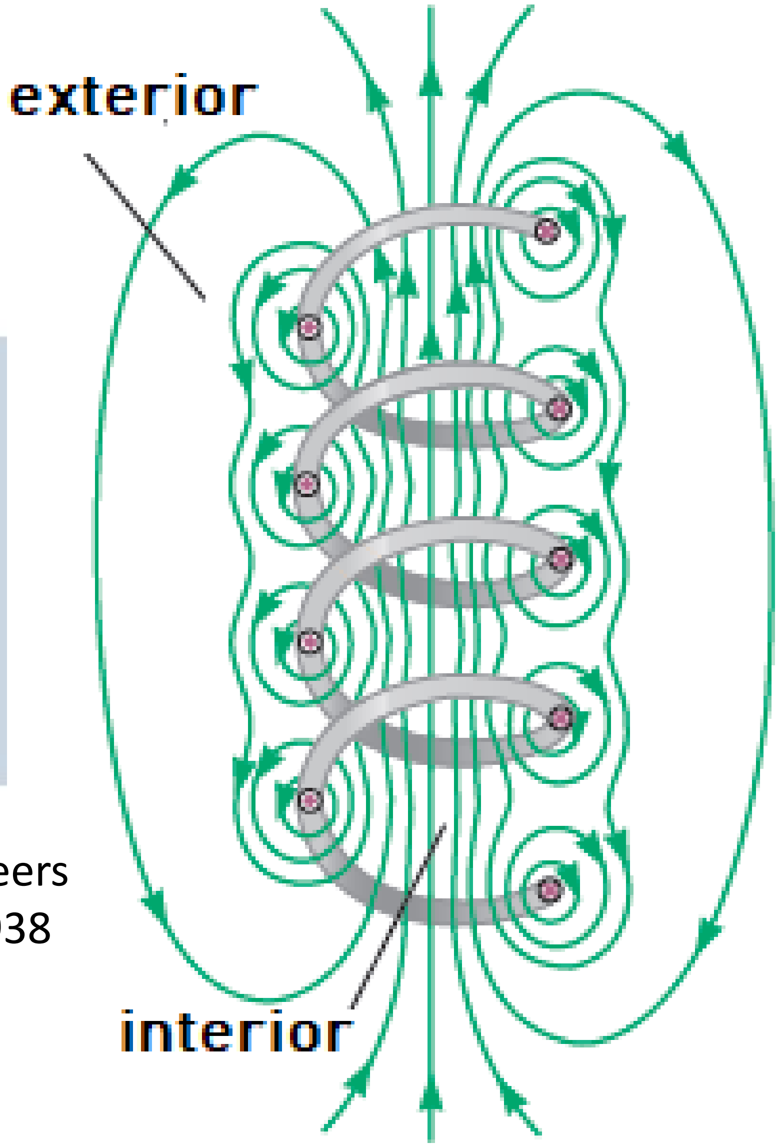

Figure 30.16 The magnetic field lines for a loosely wound solenoid.

For a Ideal Solenoid

Consider the rectangular path of length *l*, and width *w* shown in Figure 30.18. Let's apply Ampère's law to this path by evaluating the integral of **B***ds* over each side of the rectangle. The contribution along side 2,3,4 are zero because the external magnetic field lines are perpendicular to the path.

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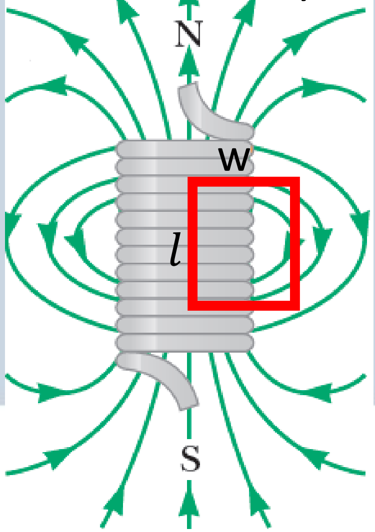
$$\oint \vec{B}d\vec{s} = \int_{path\ 1} \vec{B}d\vec{s} = B\int_{path\ 1} ds = Bl = \mu_0 NI$$

$$B = \mu_0 \frac{N}{l} I = \mu_0 nI$$

n is the number of turns per unit length.

This equation is valid only for points near the center of the solenoid.



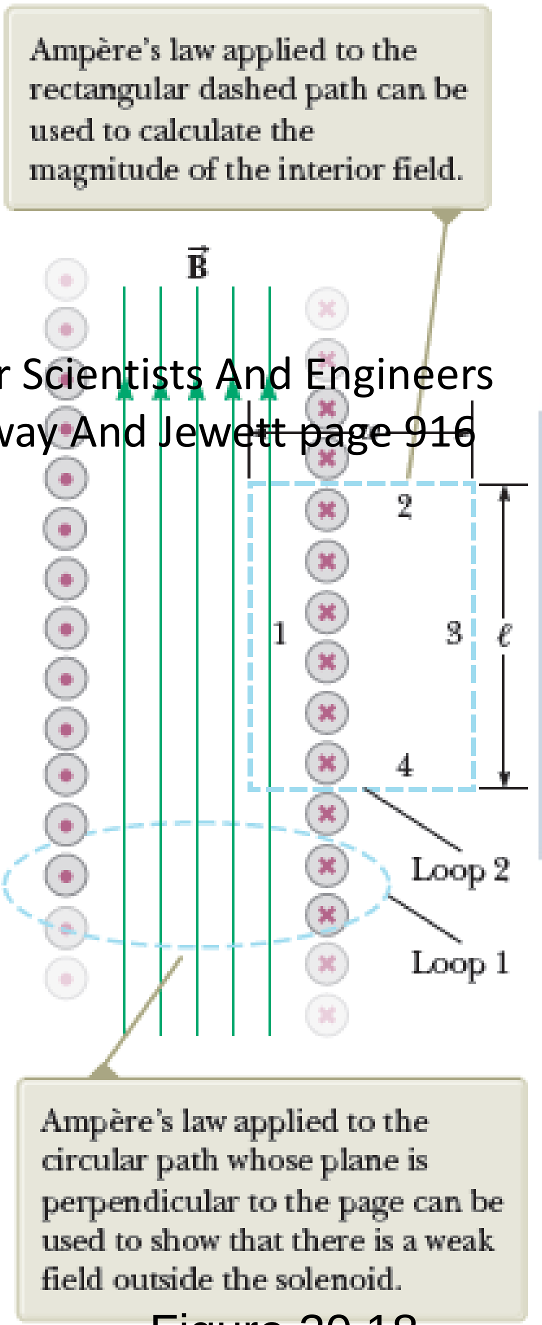


Figure 30.18