ENE 327 – Pumps and Compressors

WEEK 7: SIMILITUDE IN TURBOMACHINES

<u>SIMILITUDE IN TURBOMACHINES</u>

Governing Non-dimensional Parameters [1]

The independent physical variables, which describe the performance of a turbomachine can be given as the:

1. Volumetric flow rate, Q
2. Energy per unit weight, h
3. Dimension of the turbomachine, such as diameter, d
4. Power of the machine, P
5. Rotational speed of the impeller, w
6. Density of the fluid, ρ
7. Viscosity of the fluid, μ

Use energy per unit mass, gh instead of energy per unit weight, h to distinguish between head and diameter of the machine.

The functional relation can be expressed as:

$$f(Q, gh, d, P, \rho, \mu, w) = 0$$

The dimensions of the pyhsical parameters involved can be given as:

$$[Q] = L^3/T\,, [gh] = L^2/T^2\,, [d] = L, [P] = ML^2/T^3\,, [\mu] = M/LT\,, [\rho] = M/L^3\,, [w] = 1/T$$

Since the primary dimensions which are involved in the dimensions of the governing parameters are the length L, time T and mass M the number of dimensions is m=3.

According to the Buckhingham-Pi Theorem, the numbers of non-dimensional parameters is k = n-m = 7-3 = 4. Now, one can select three repeating parameters so that the number of these repeating variables is equal to the number of all parameters involved.

The repeating parameters may be chosen as diameter d, geometric parameter, rotational speed w, kinematic parameter, density μ, dynamic parameter.

The four non-dimensional parametersΠ_Q, Π_P,$\Pi_h$and $\Pi_\mu$can now be set up by multiplying the unselected parameters, Q, gh, P and μ with the repeating parameters d, ρ and w which are raised to unknown exponents.

Hence,

$$\Pi_Q = Q.\rho^a.w^b.d^c$$
$$\Pi_h = gh.\rho^e.w^g.d^h$$
$$\Pi_P = P.\rho^i.w^j.d^k$$
$$\Pi_\mu = \mu.\rho^L.w^P.d^r$$

where a,b,c,e,g,h,i,j,k,L,p,r are the unknown exponents.

$$[\Pi_Q] = [L^3T^{-1}][ML^{-3}]^a[T^{-1}]^b[L]^c = M^0L^0T^0$$

for M: a=0

for T: b=-1

for L: c=-3

$\Pi_Q = Q.\rho^0.w^{-1}.d^{-3} = \frac{Q}{w.d^3}$ Flow coefficient

$$[\Pi_H] = [L^2/T^2][ML^{-3}]^e[T^{-1}]^g[L]^h = M^0L^0T^0$$

for M: e=0

for T: g=-2

for L: h=-2

$\Pi_h = gh.\rho^0.w^{-2}.d^{-2} = \frac{gh}{w^2d^2}$ Head coefficient

$$[\Pi_P] = [ML^2/T^3][ML^{-3}]^i[T^{-1}]^j[L]^k = M^0L^0T^0$$

for M: i=-1

for T: j=-3

for L: k=-5

$\Pi_P = P.\rho^{-1}.w^{-3}.d^{-5} = \frac{P}{\rho w^3 d^5}$ Power coefficient

$$[\Pi_\mu] = [ML^{-4}/T^{-1}][ML^{-3}]^L[T^{-1}]^P[L]^r = M^0L^0T^0$$

for M: L=-1

for T: P=-1

for L: r=-2

$\Pi_\mu = \mu.\rho^{-1}.w^{-1}.d^{-2} = \frac{\mu}{\rho w d^2}$ which is reciprocal of the Reynold's number

Then the functional relationship between the non-dimensional parameters Π_Q, Π_P ,Π_h and Π_μ can be established as:

$$F = F\left(\frac{Q}{w.d^3}, \frac{gh}{w^2d^2}, \frac{P}{\rho w^3 d^5}, \frac{\mu}{\rho w d^2}\right) = 0$$

However, it is not possible to establish the equality of all of these four non-dimensional terms in practice. If one assumes that the efficiencies are the same of two operating conditions, equality of the first three non-dimensional terms, namely Π_Q, Π_P, Π_h and Π_μ can be established.

The first three non-dimensional terms are also referred to as the affinity laws. The last non-dimensional term, Π_μ is the reciprocal of Re number and it is used to correct similarity for viscous effects.

REFERENCES

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