Condensed Matter Physics

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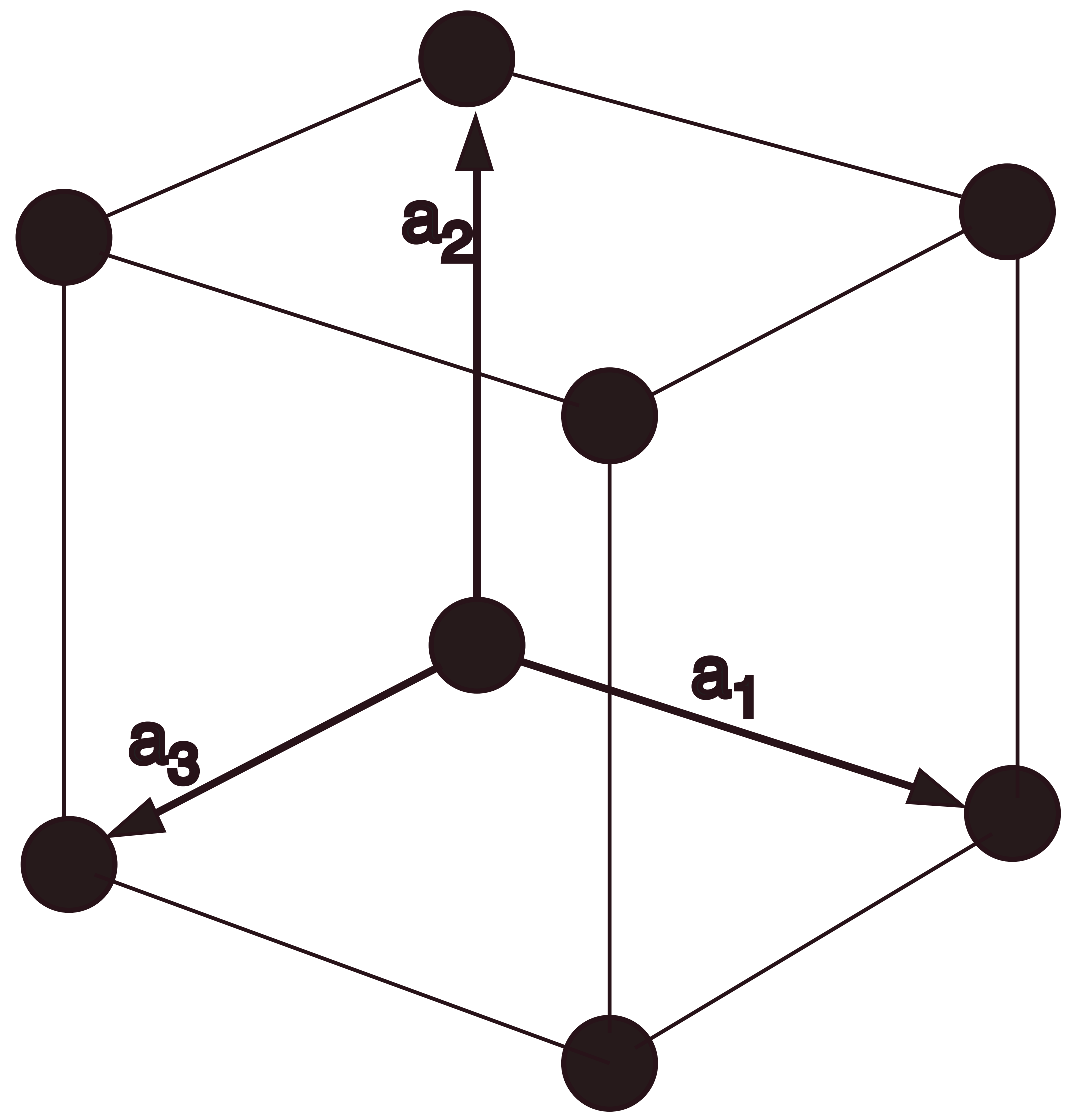
The crystal lattice: Bravais lattice (3D)

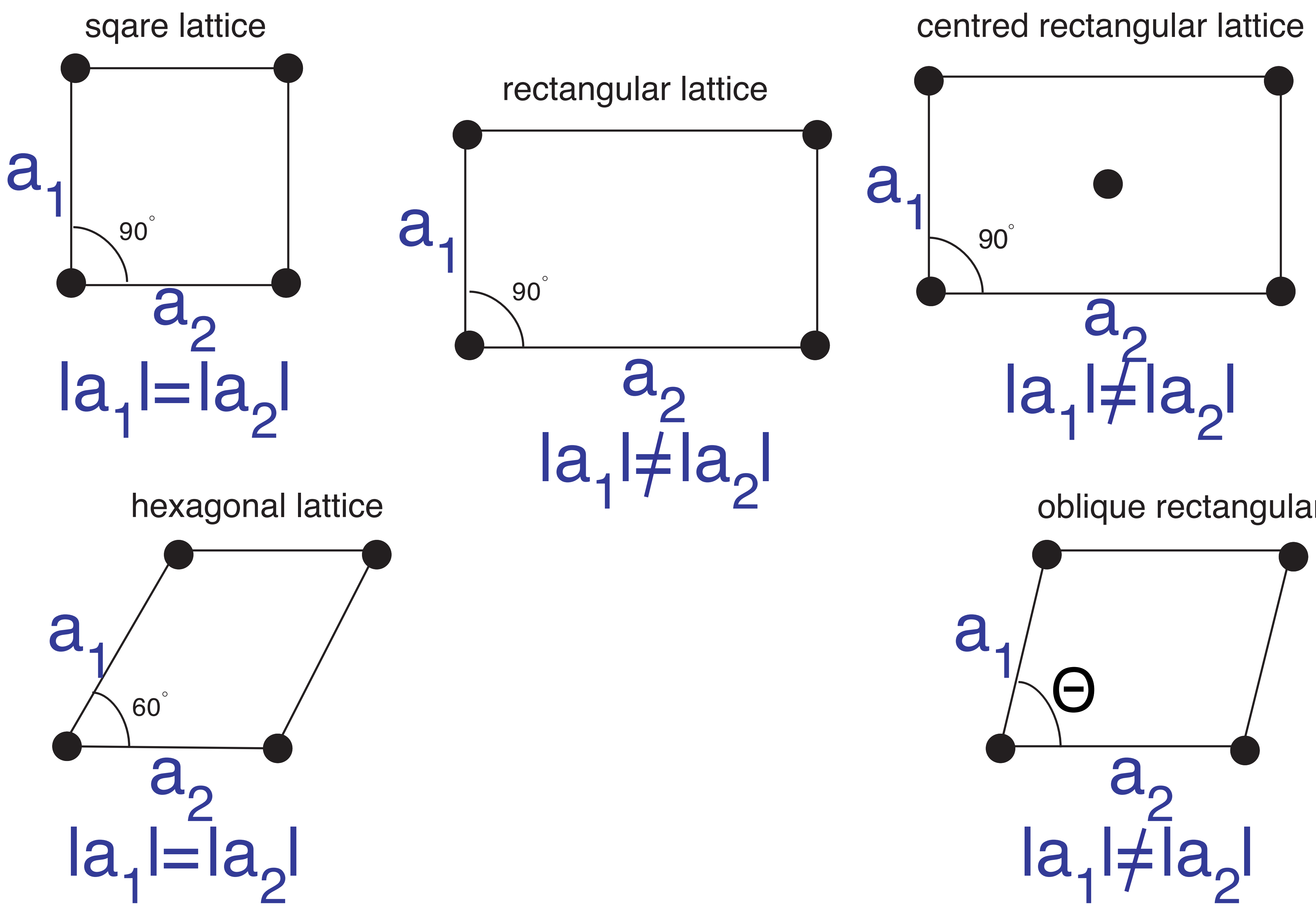
A Bravias lattice is a lattice of points, defined by

$$\mathbf{R} = m\mathbf{a_1} + n\mathbf{a_2} + o\mathbf{a_3}$$



This reflects the translational symmetry of the lattice

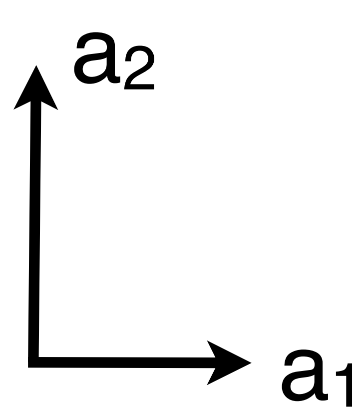
Bravais lattice (2D)



- The number of possible Bravais lattices (of fundamentally different symmetry) is limited to 5 (2D) and 14 (3D).

The crystal lattice: primitive unit cell

Primitive unit cell: any volume of space which, when translated through all the vectors of the Bravais lattice, fills space without overlap and without leaving voids



- Fig 2.2, Solid State Physics: An Introduction, by Philip Hofmann, Wiley-VCH Berlin.

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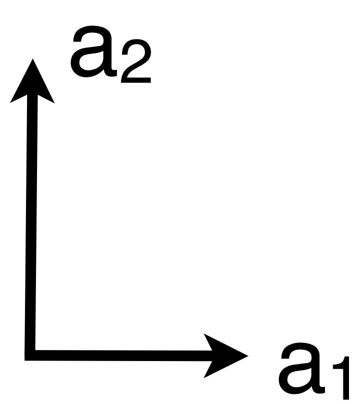

- Fig 2.2, Solid State Physics: An Introduction, by Philip Hofmann, Wiley-VCH Berlin.

The crystal lattice: Wigner-Seitz cell

Wigner-Seitz cell: special choice of primitive unit cell: region of points closer to a given lattice point than to any other.

- Fig 2.2, Solid State Physics: An Introduction, by Philip Hofmann, Wiley-VCH Berlin.

The crystal lattice: basis

- We could think: all that remains to do is to put atoms on the lattice points of the Bravais lattice.
- But: not all crystals can be described by a Bravais lattice (ionic, molecular, not even some crystals containing only one species of atoms.)
- BUT: all crystals can be described by the combination of a Bravais lattice and a basis. This basis is what one "puts on the lattice points".

- Fig 2.3, Solid State Physics: An Introduction, by Philip Hofmann, Wiley-VCH Berlin.

The crystal lattice: one atomic basis

- The basis can also just consist of one atom.

- Fig 2.3, Solid State Physics: An Introduction, by Philip Hofmann, Wiley-VCH Berlin.

The crystal lattice: basis

- Or it can be several atoms.

- Fig 2.3, Solid State Physics: An Introduction, by Philip Hofmann, Wiley-VCH Berlin.

The crystal lattice: basis

- Or it can be molecules, proteins and pretty much anything else.

- Fig 2.3, Solid State Physics: An Introduction, by Philip Hofmann, Wiley-VCH Berlin.

The crystal lattice: one more word about symmetry

- The other symmetry to consider is point symmetry. The Bravais lattice for these two crystals is identical:

- Fig 2.3, Solid State Physics: An Introduction, by Philip Hofmann, Wiley-VCH Berlin.

four mirror lines
4-fold rotational axis
inversion

no additional
point symmetry

The crystal lattice: one more word about symmetry

The Bravais lattice vectors are

$$\mathbf{R}_{mno} = m\mathbf{a_1} + n\mathbf{a_2} + o\mathbf{a_3}$$

We can define a translation operator *T* such that

$$T_{\mathbf{R}_{mno}} F(\mathbf{r}) = F(\mathbf{r} + \mathbf{R}_{mno})$$

This operator commutes with the Hamiltonian of the solid and therefore we can choose the eigenfunctions of the Hamiltonian such that they are also eigenfunctions of the translation operator.

adding the basis keeps translational symmetry but can reduce point symmetry

but it can also add new symmetries (like glide planes here)