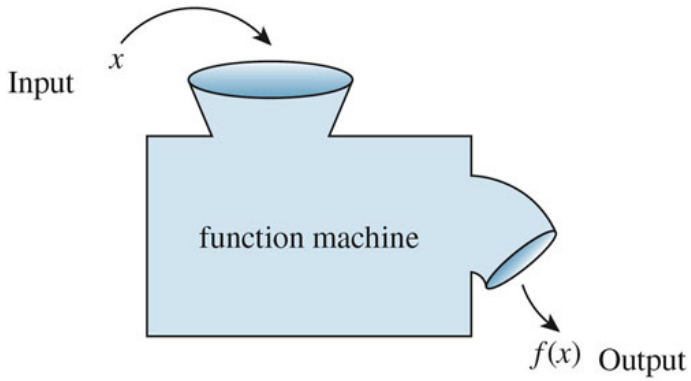
Calculus
Lecture 1

Oktay Ölmez and Serhan Varma

A brief summary of the concept of functions



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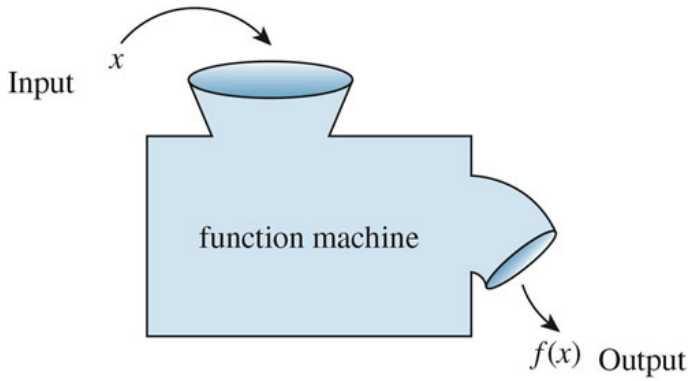

Definition

A function is a rule that assigns to each element in a set A one and only one element in a set B.

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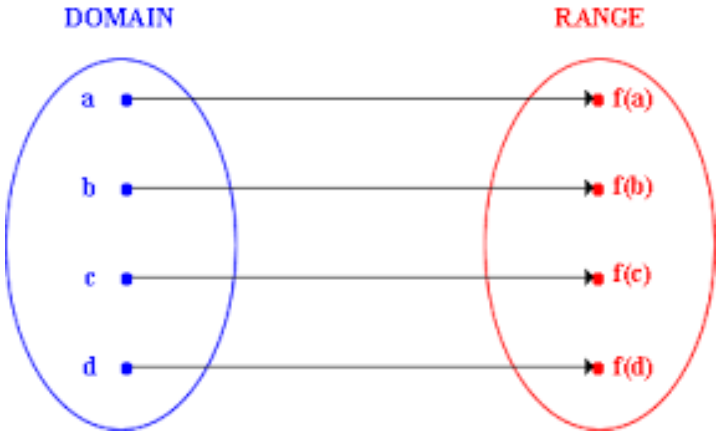

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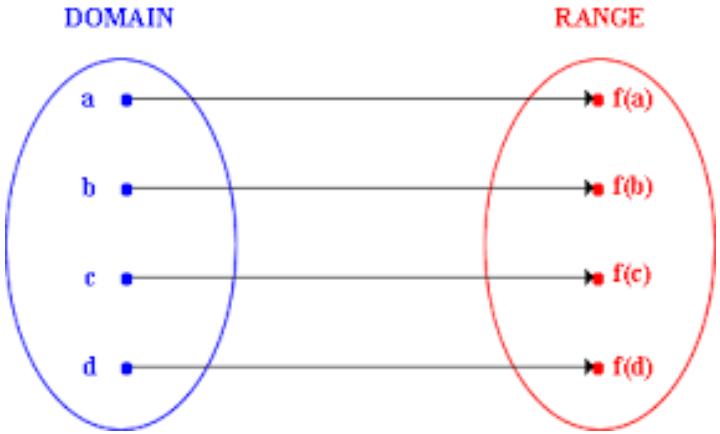

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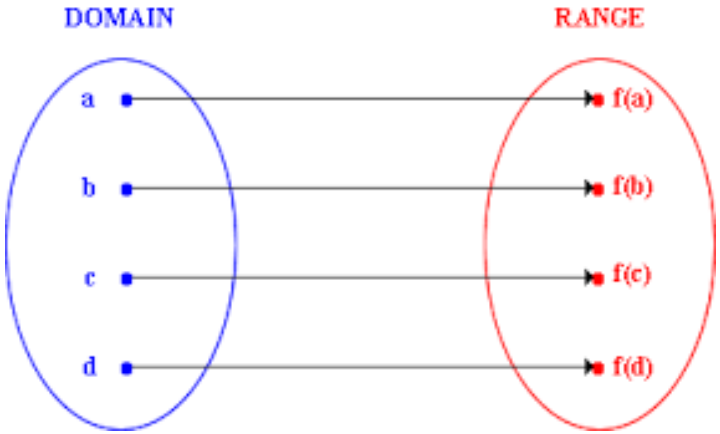

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- If x is an element in the domain of a function f, then the element in B that f associates with x is written $f(x)$ (read f of x) and is called the value of f at x.

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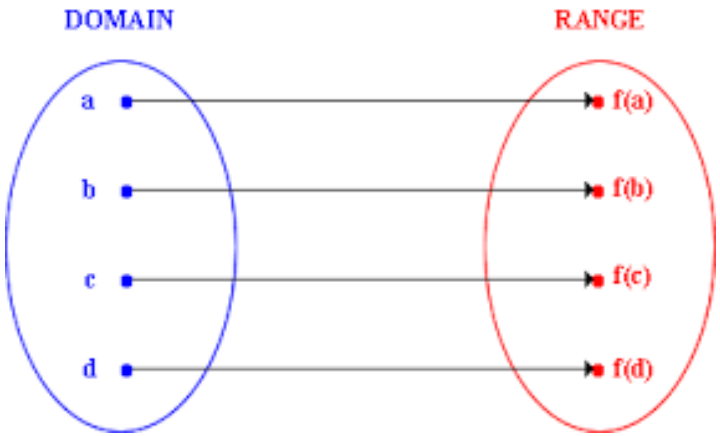

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- If x is an element in the domain of a function f, then the element in B that f associates with x is written $f(x)$ (read f of x) and is called the value of f at x.
- The set comprising all the values assumed by $y = f(x)$ as x takes on all possible values in its domain is called the range of the function f.

Example

An open box is to be made from a rectangular piece of cardboard 16 *inches long and* 10 *inches wide by cutting away identical squares (*x *inches by* x *inches) from each corner and folding up the resulting flaps. Find an expression that gives the volume* V *of the box as a function of* x*. What is the domain of the function?*

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Solution

$$V(x) = (16 - 2x)(10 - 2x)x$$

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Solution

$$V(x) = (16 - 2x)(10 - 2x)x$$

To find the domain, we need to consider the following inequalities:

$$16 - 2x > 0 \quad 10 - 2x > 0 \quad x > 0$$

Thus the domain is $(0, 5)$*.*

Example

Find the domain of the each function:

- *(a)* $\sqrt{x-1}$
- *(b)* $\frac{1}{x^2-4}$
- *(c)* x^2+3

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Answers

- *(a)* $x \geq 1$
- *(b)* $\mathbb{R} \setminus \{\pm 2\}$
- *(c)* $\mathbb{R}$

Graph of a Function of One Variable

Definition

The graph of a function f is the set of all points (x, y) in the xy-plane such that x is in the domain of f and $y = f(x)$.

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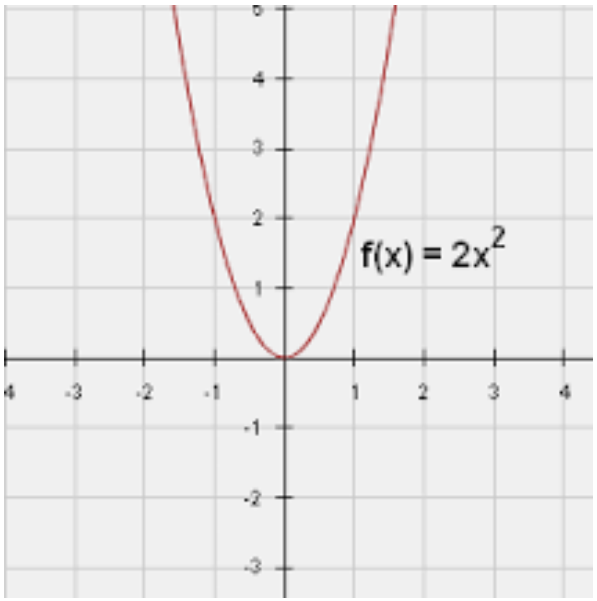


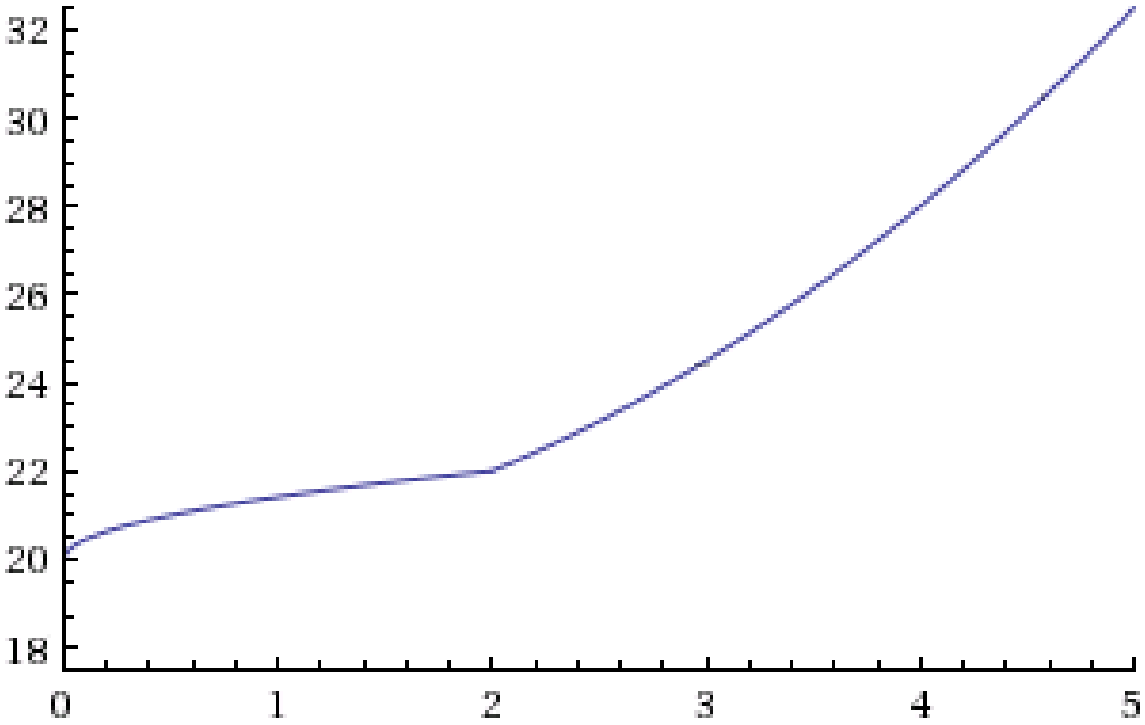
Example

A Finance Company plans to open two branch offices 2 years from now in two separate locations: an industrial complex and a newly developed commercial center in the city. As a result of these expansion plans, The Company's total deposits during the next 5 years are expected to grow in accordance with the rule where

$$f(x) = \begin{cases} \sqrt{2x} + 20 & \text{if } 0 \leq x \leq 2 \\ \frac{x^2}{2} + 20 & \text{if } 2 < x \leq 5 \end{cases}$$

gives the total amount of money (in millions of dollars) on deposit with the company in year x ($x = 0$ corresponds to the present). Sketch the graph of the function f.

Solution



Some special functions: Absolute Value Function

Definition (Absolute value function)

The absolute value function is defined as

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

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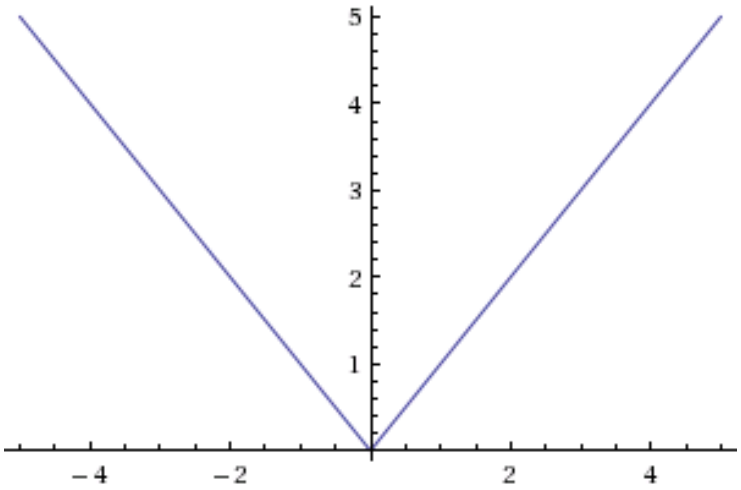

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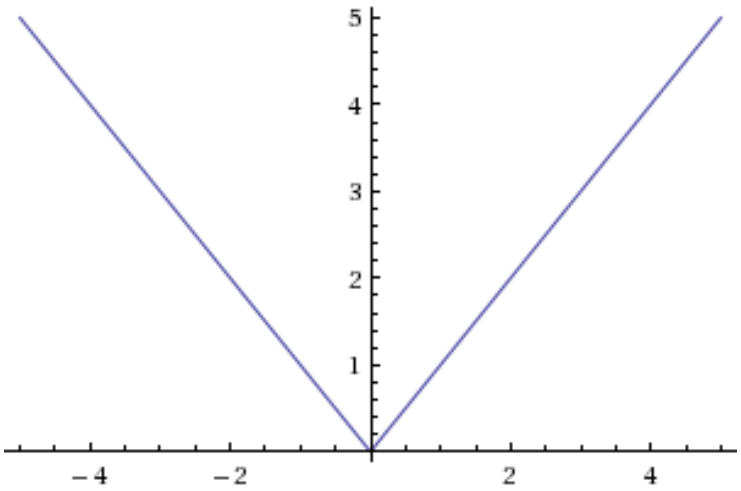

The domain of the absolute value function is the set of all real numbers and the range is the set of all positive real numbers including zero.

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The sign or signum function, sgn, is defined according to

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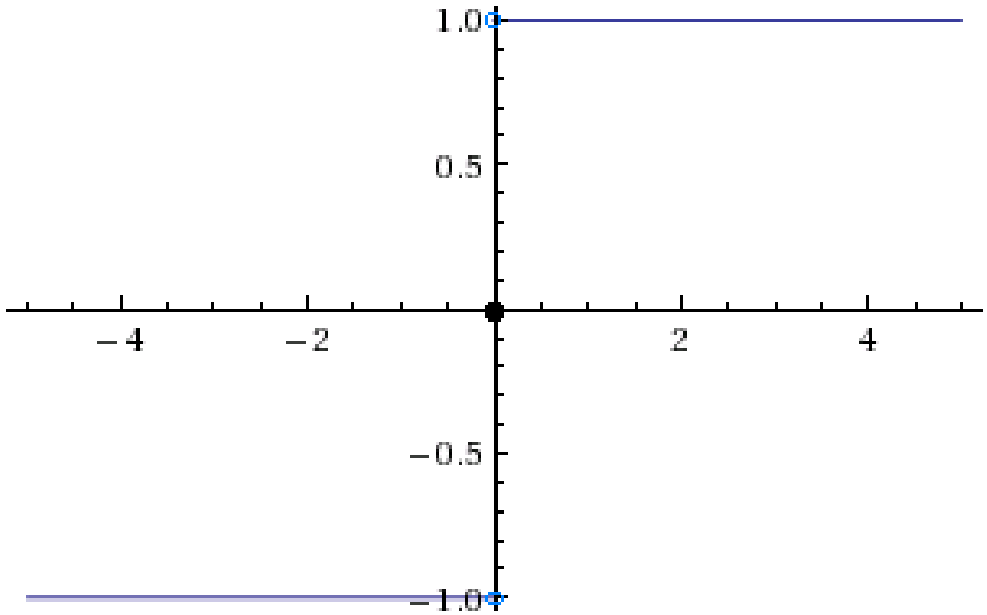

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Floor function, denoted by $\lfloor x \rfloor$*, is defined as the greatest integer less than or equal to any real number. For example,* $\lfloor 2.3 \rfloor = 2$*,* $\lfloor 0.4 \rfloor = 0$ *and* $\lfloor -3.1 \rfloor = -4$*. This function has an infinite number of breaks or steps-one at each integer value in its domain.*

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