Calculus
Lecture 1

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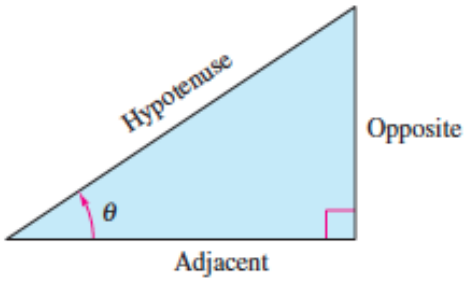
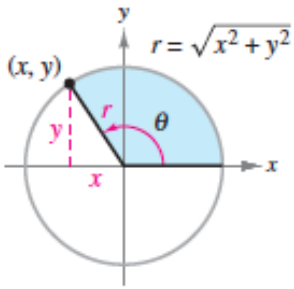
Some special functions: Trigonometric Function





Definitions of the Trigonometric Functions

Right Triangle Definition: $0 < \theta < \frac{\pi}{2}$. (See Figure 8.11.)

$$\sin\theta = \frac{\text{opp.}}{\text{hyp.}} \qquad \csc\theta = \frac{\text{hyp.}}{\text{opp.}}$$

$$\cos\theta = \frac{\text{adj.}}{\text{hyp.}} \qquad \sec\theta = \frac{\text{hyp.}}{\text{adj.}}$$

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} \qquad \cot\theta = \frac{\text{adj.}}{\text{opp.}}$$

Circular Function Definition: θ is any angle in standard position and (x, y) is a point on the terminal ray of the angle. (See Figure 8.12.)

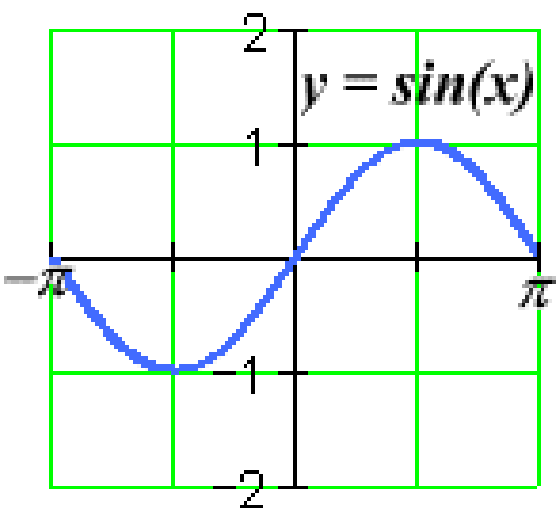
$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

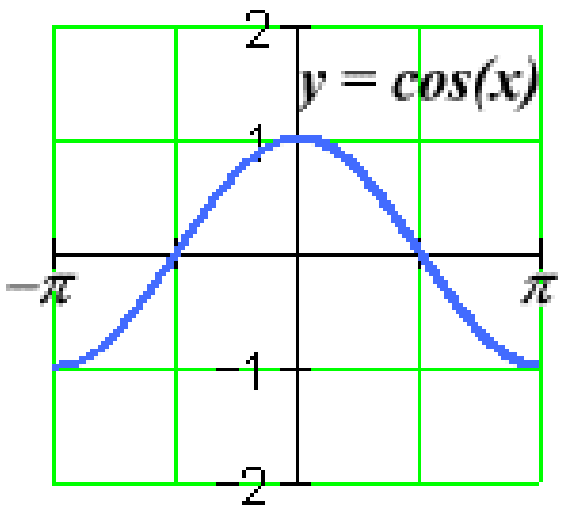
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

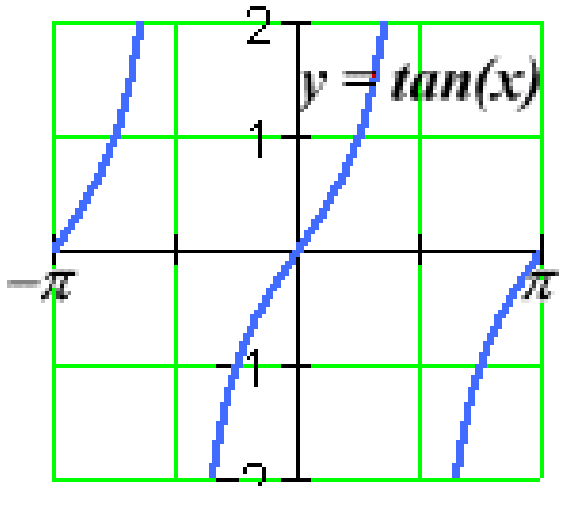
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

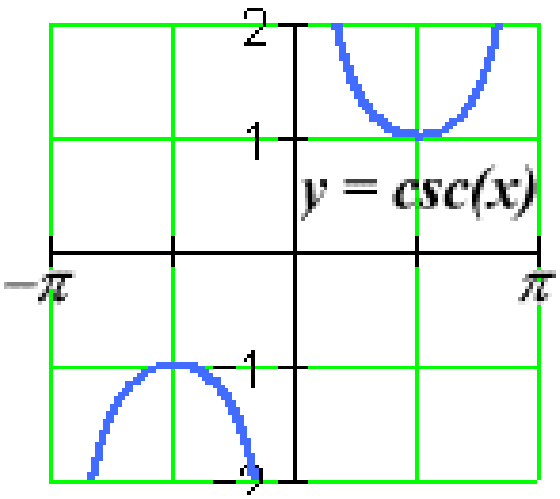
The full names of the trigonometric functions are **sine, cosecant, cosine, secant, tangent,** and **cotangent.**

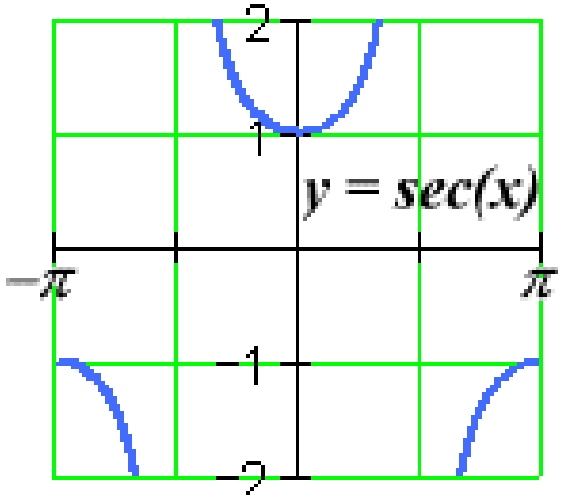
Some special functions: Trigonometric Function

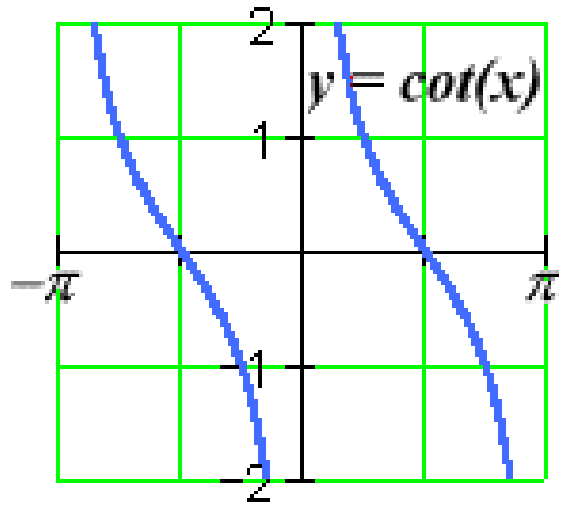






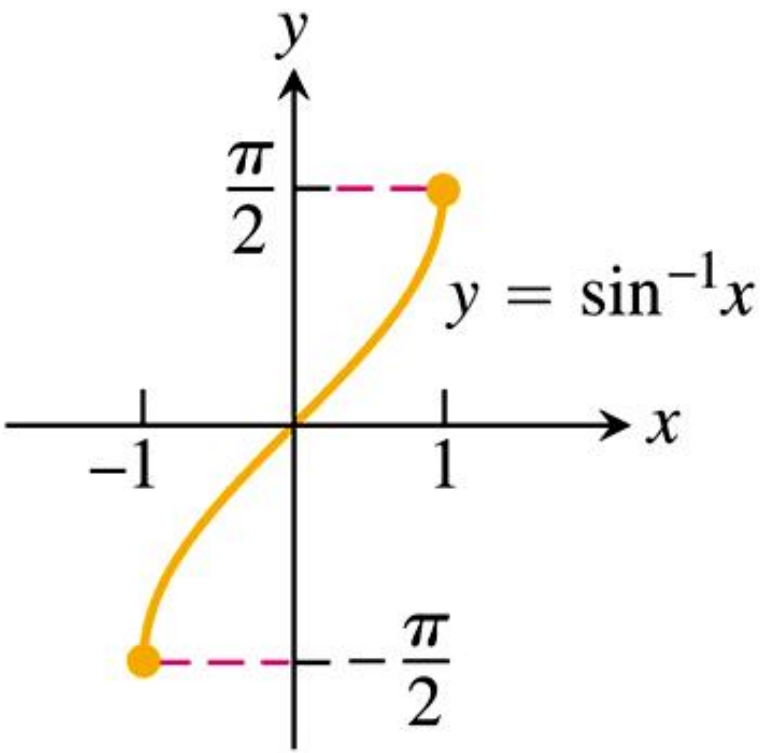






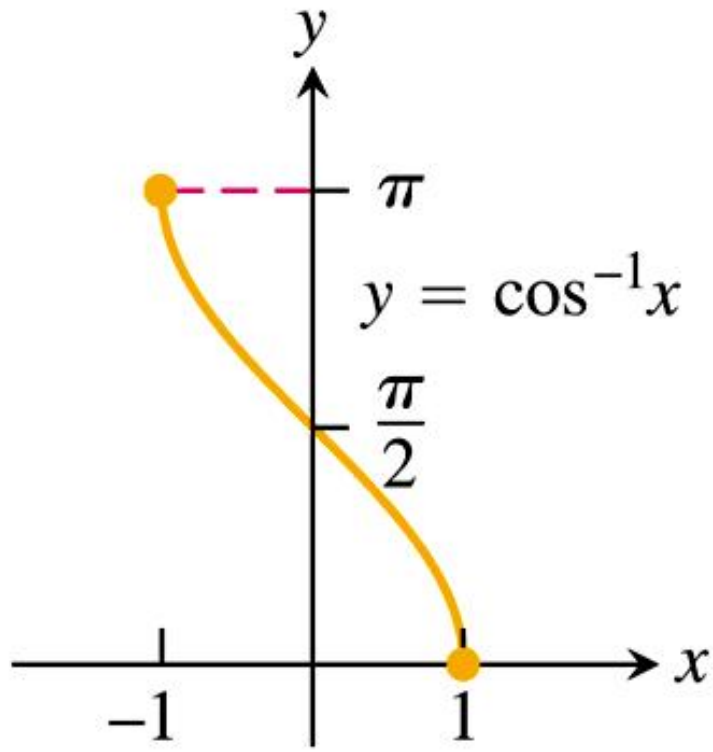
Some special functions

Domain: $-1 \leq x \leq 1$
Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



(a)

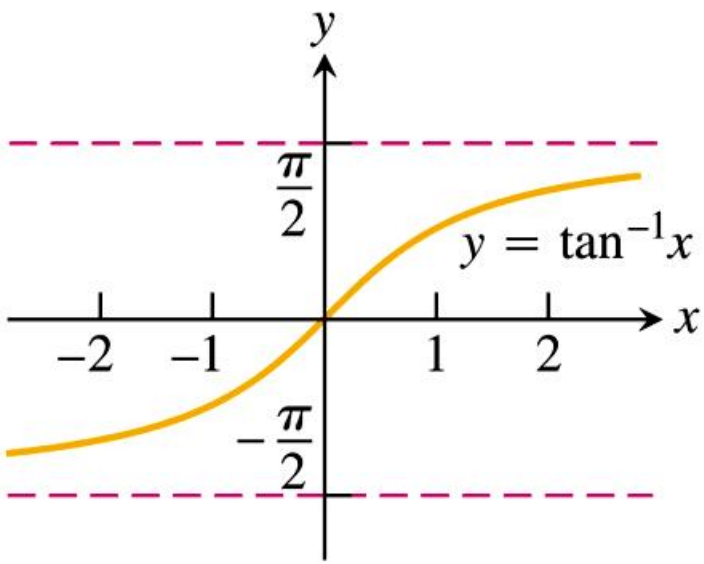
Domain: $-1 \leq x \leq 1$
Range: $0 \leq y \leq \pi$



(b)

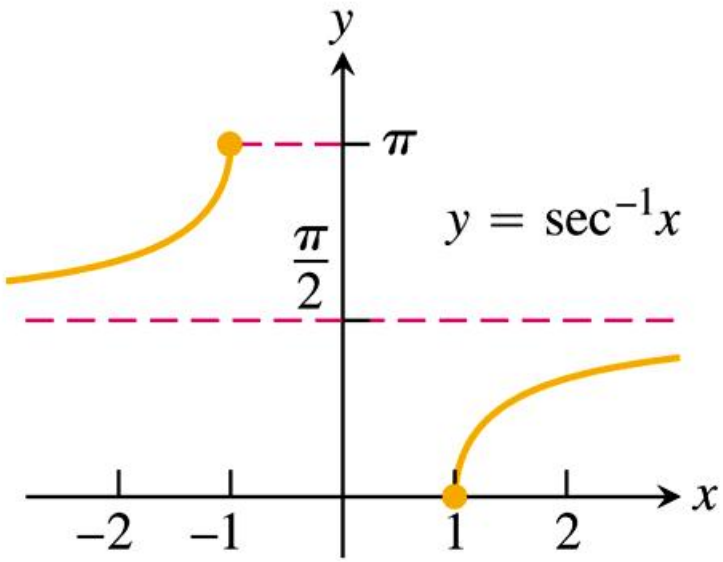
Some special functions

Domain: $-\infty < x < \infty$
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



(c)

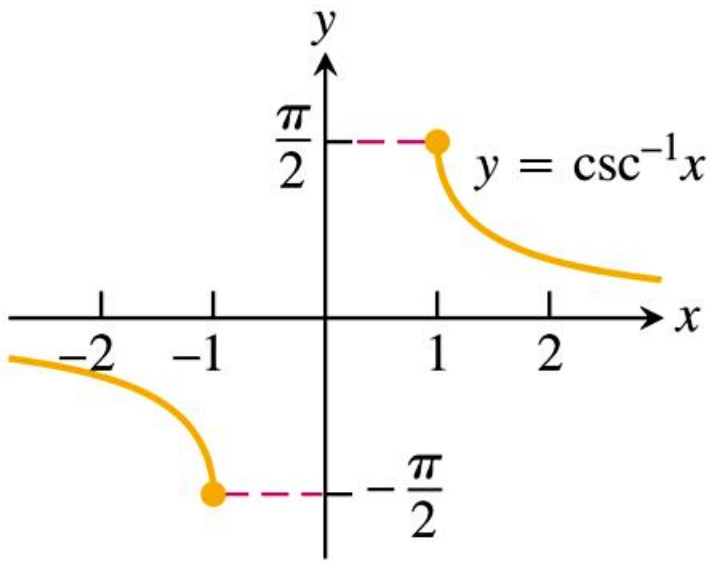
Domain: $x \le -1$ or $x \ge 1$
Range: $0 \le y \le \pi, y \ne \frac{\pi}{2}$



(d)

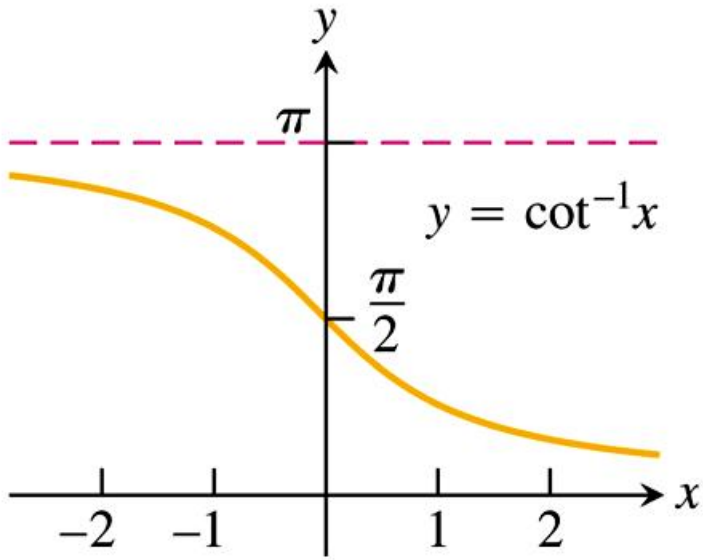
Some special functions

Domain: $x \leq -1$ or $x \geq 1$
Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}, y \neq 0$



(e)

Domain: $-\infty < x < \infty$
Range: $0 < y < \pi$



(f)

Trigonometric identities

θ (degrees)	0	30°	45°	60°	90°	180°	270°
θ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined	0	Undefined

Trigonometric identities

Trigonometric Identities

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

Sum or Difference of Two Angles

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$
$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$
$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

Double Angle

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

Reduction Formulas

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$
$$\sin\theta = -\sin(\theta - \pi)$$
$$\cos\theta = -\cos(\theta - \pi)$$
$$\tan\theta = \tan(\theta - \pi)$$

Half Angle

$$\sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta)$$
$$\cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta)$$

Predator-Prey

Example

The population P of a predator at time t (in months) is modeled by

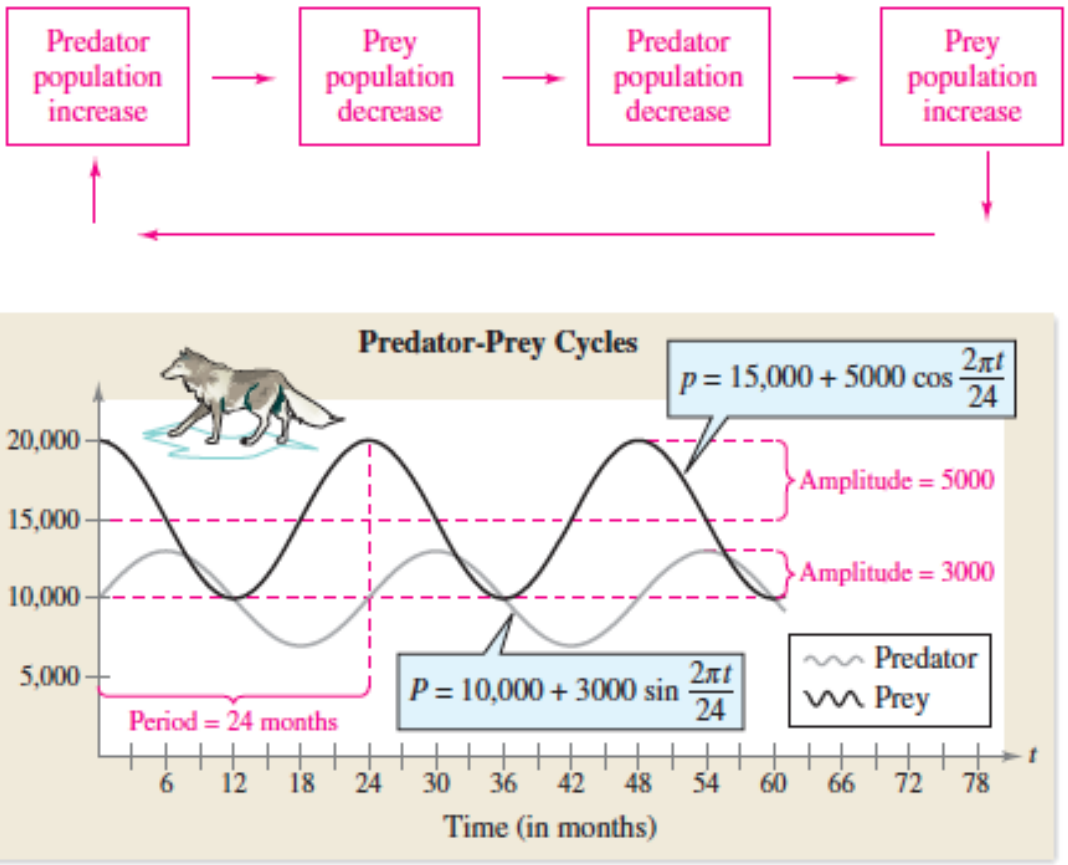
$$P(t) = 10000 + 3000 \sin\left(\frac{2\pi t}{24}\right) \quad t \geq 0$$

and the population p of its primary food source (its prey) is modeled by

$$p(t) = 15000 + 5000 \cos\left(\frac{2\pi t}{24}\right) \quad t \geq 0.$$

Sketch the graph both models on the same set of axes and explain the oscillations in the size of each population.

Model


Predator population increase
Prey population decrease
Predator population decrease
Prey population increase
Predator-Prey Cycles
$p = 15{,}000 + 5000 \cos \frac{2\pi t}{24}$
Amplitude = 5000
Amplitude = 3000
$P = 10{,}000 + 3000 \sin \frac{2\pi t}{24}$
Predator
Prey
Period = 24 months
Population
20,000
15,000
10,000
5,000
6
12
18
24
30
36
42
48
54
60
66
72
78
t
Time (in months)