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- $\lim_{x \to c} \sqrt{f(x)} = \sqrt{\lim_{x \to c} f(x)}$

Useful Limit Theorems

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Theorem

Let f, g and h be functions satisfying for $f(x) \leq g(x) \leq h(x)$ all x in an open interval containing c, except possibly at c. If $\lim_{x \to c} f(x) = L = \lim_{x \to c} h(x)$ then $\lim_{x \to c} g(x) = L$.

Limit Theorems for Trigonometric Functions

Theorem

Let c be a real number. Then,

- $\lim_{x \to c} \cos(x) = \cos(c)$
- $\lim_{x \to c} \sin(x) = \sin(c)$
- $\lim_{x \to c} \csc(x) = \csc(c)$
- $\lim_{x \to c} \sec(x) = \sec(c)$
- $\lim_{x \to c} \cot(x) = \cot(c)$
- $\lim_{x \to c} \tan(x) = \tan(c)$

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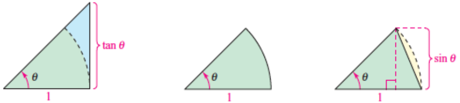

$$\begin{array}{ccccc} \text{Area of triangle} & \geq & \text{Area of sector} & \geq & \text{Area of triangle} \\ \dfrac{\tan\theta}{2} & \geq & \dfrac{\theta}{2} & \geq & \dfrac{\sin\theta}{2} \end{array}$$

Multiplying each expression by $2/\sin\theta$ produces

$$\frac{1}{\cos\theta} \geq \frac{\theta}{\sin\theta} \geq 1$$

and taking reciprocals and reversing the inequalities yields

$$\cos\theta \leq \frac{\sin\theta}{\theta} \leq 1.$$

Because $\cos\theta = \cos(-\theta)$ and $(\sin\theta)/\theta = [\sin(-\theta)]/(-\theta)$, you can conclude that this inequality is valid for *all* nonzero θ in the open interval $(-\pi/2, \pi/2)$. Finally, because $\lim\limits_{\theta\to 0}\cos\theta = 1$ and $\lim\limits_{\theta\to 0} 1 = 1$, you can apply the Squeeze Theorem to conclude that $\lim\limits_{\theta\to 0}(\sin\theta)/\theta = 1$.

What parking space number is the car parked ?