Calculus
Lecture 3

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Intermediate Value Theorem

We say f is continuous on the interval $[a, b]$ if

- f is continuous at every point that belongs to the interval (a, b).
- $\lim_{x \to a^+} f(x) = f(a)$
- $\lim_{x \to b^-} f(x) = f(b)$

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Theorem (IVT)

Let f be a continuous function on $[a, b]$ and W be a number between $f(a)$ and $f(b)$. Then, there is at least one c between a and b such that $f(c) = W$.

IVT Example

Example

Is there any real number c between 0 and $\pi/2$ such that $c = \cos(c)$?

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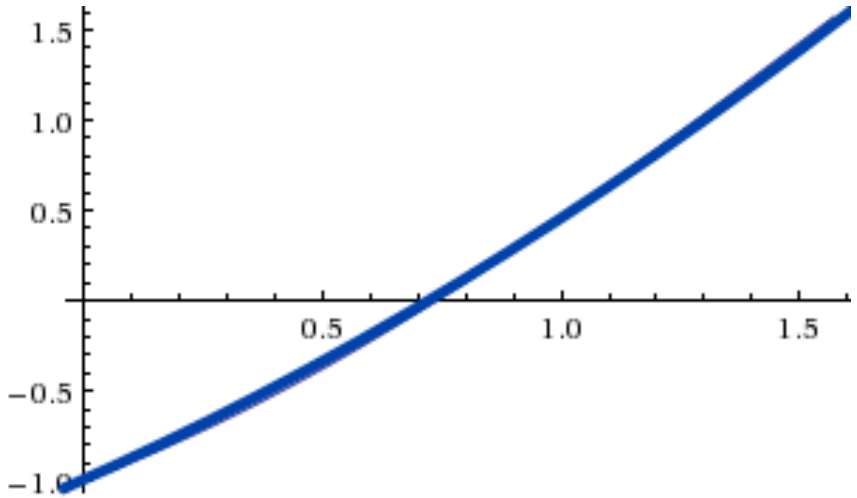
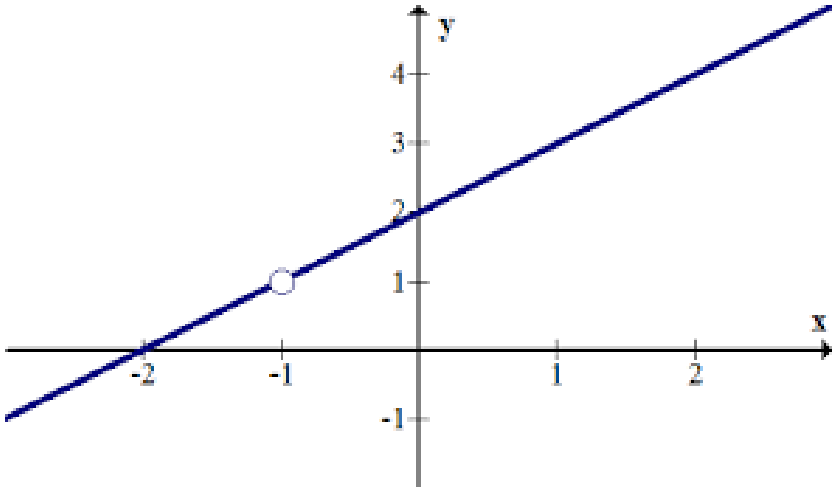


Example

Show that $p(x) = 2x^3 - 5x^2 - 10x + 5$ *has a root somewhere between* -1 *and* 2.

Types of discontunity

- In a **removable discontinuity** $\lim_{x \to c} f(x)$ exists, but $\lim_{x \to c} f(x) \neq f(c)$.
- In a **jump discontinuity**, the right-hand and left-hand limits both exist, but are not equal.
- An **infinite discontinuity** exists when one of the one-sided limits of the function is infinite.
- An **oscillating discontinuity** exists when the values of the function appear to be approaching two or more values simultaneously.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-1
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-1
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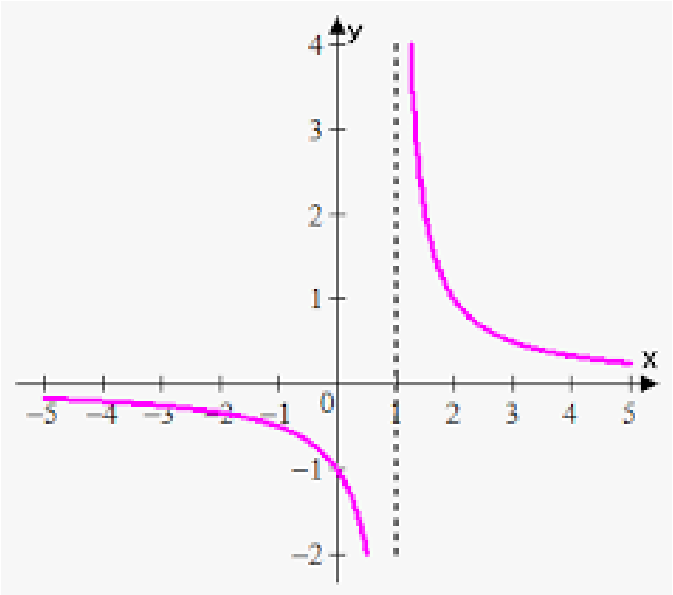
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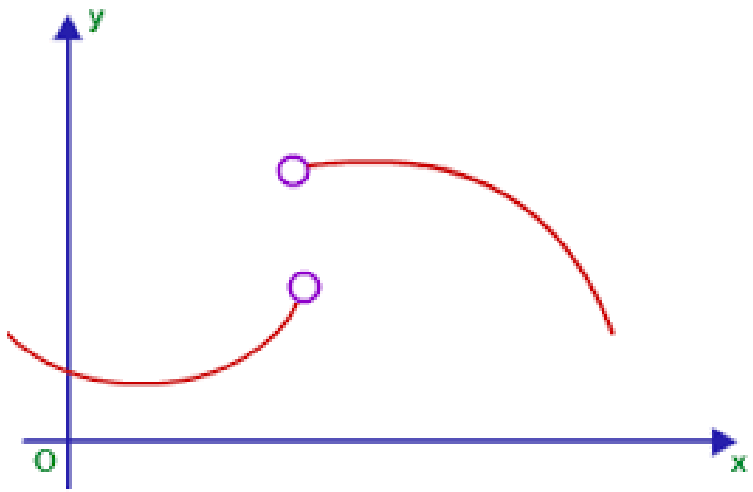
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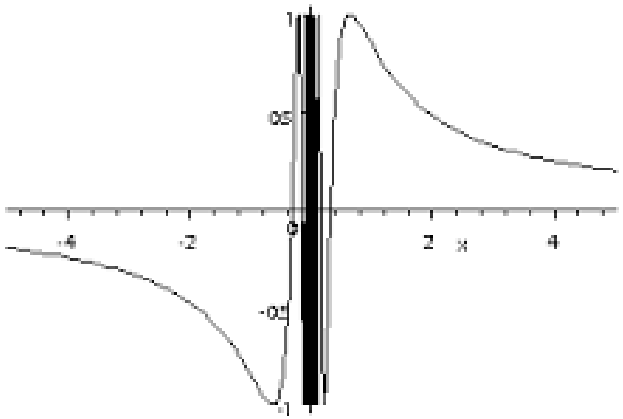
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