Calculus
Lecture 4

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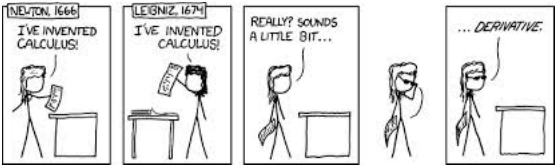
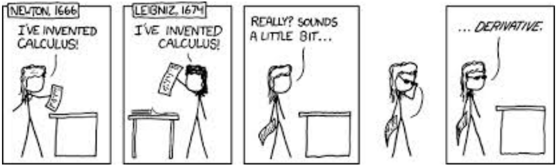
Meaning

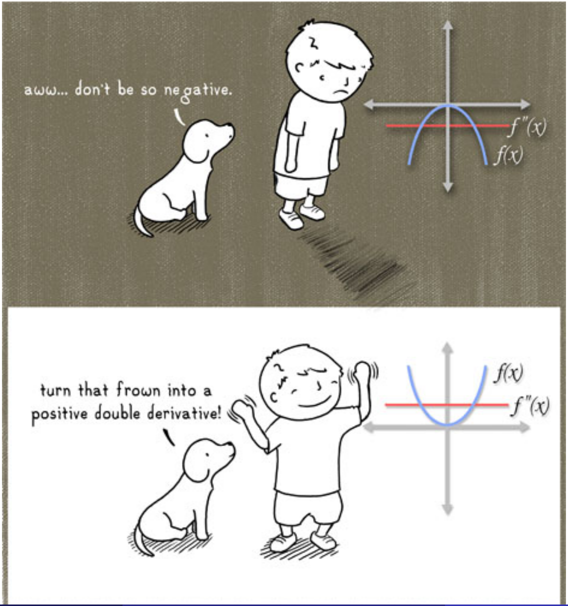


NEWTON, 1666
I'VE INVENTED CALCULUS!
LEIBNIZ, 1674
I'VE INVENTED CALCULUS!
REALLY? SOUNDS A LITTLE BIT...
... DERIVATIVE.

Meaning



- imitative of the work of another artist, writer, etc., and usually disapproved of for that reason.
- (of a product) having a value deriving from an underlying variable asset.
- something which is based on another source.


aww... don't be so negative.
f''(x)
f(x)
turn that frown into a positive double derivative!
f(x)
f''(x)

Slope of tangent line

Consider the graph of $y = f(x)$.

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- $m_{sec} = \dfrac{f(x+h) - f(x)}{h}$.
- $m_{tan} = \lim_{h \to 0} m_{sec} = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ (provided that this limit exists and not equals to either ∞ or $-\infty$).

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Suppose $f(x)$ is the position function of a moving object.

- $v_{avg} = \dfrac{f(x+h) - f(x)}{h}$.
- $v = \lim\limits_{h \to 0} v_{avg} = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$. ($v$ is the instantaneous velocity)

Secant to Tangent

Derivative

Definition

Derivative of a function f is another function f' whose value at number x is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example

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Apply the definition to find the derivative of the function $f(x) = x^2$.

Solution

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h\to 0} \frac{h(2x+h)}{h} \\ &= \lim_{h\to 0} 2x + h = 2x. \end{aligned}$$

Differentiability implies continuity

Theorem

If $f'(c)$ exists then f is continuous at c.

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But the converse may not be true !

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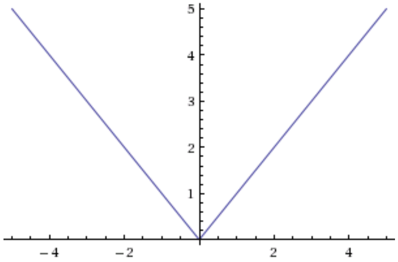

$f(x) = |x|$ is not differentiable at $x = 0$

By definition, we need to show the following limit exists at $x = 0$.

$$f'(x) = \lim_{h \to 0} \frac{|x+h| - |x|}{h}$$

Thus we just need to consider,

$$\lim_{h \to 0} \frac{|0+h| - |0|}{h} = \lim_{h \to 0} \frac{|h|}{h}.$$

But, it is evident that this limit DNE.

Example

The rate of change of electric charge with respect to time is called current. Suppose that $\frac{t^3}{3} + t$ *coulombs of charge flow through a wire in* t *seconds. Find the current in amperes after 3 seconds. When will a 26-amperes fuse in the line?*

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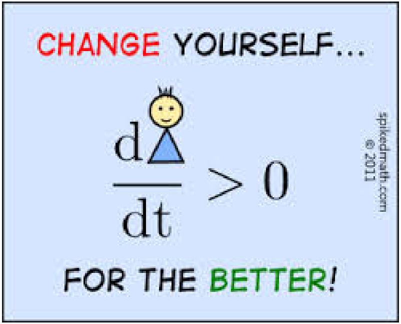
- Δy means change in y.
- If $x_1 = 2$ and $x_2 = 4$, Then $\Delta y = f(4) - f(2)$.
- $m_{sec} = \dfrac{\Delta y}{\Delta x}$.
- $m_{tan} = \lim_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \dfrac{dy}{dx}$

Change yourself!

Change yourself!



Equation of a Tangent Line

Example

Find the equation of the tangent line to the graph of $y = \frac{1}{x}$ *at* $x = 1$.