

Calculus

Lecture 4

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Rules for Derivative

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or

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Higher order derivatives

Good day Displacement!



Good day
to you too!

 x

Hi there Velocity!



Hi!

 $\frac{d}{dx}$

How are you doing
Acceleration?



I'm doing quite well!
and yourself?

 $\frac{d^2}{dx^2}$

It sure is a glorious
day isn't it?



pff... whatever.

jerk.

 $\frac{d^3}{dx^3}$

Example

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An object moves along a horizontal coordinate line in a such a way that its position at time t is specified by

$$s(t) = t^3 - 12t^2 + 36t - 30.$$

- *When is the velocity is 0?*
- *When is the velocity is positive?*
- *When is the object moving left?*
- *When is the acceleration is positive?*

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An object thrown directly upward is at the height of $s(t) = -16t^2 + 48t + 256$ feet after t seconds.

- *When is its initial velocity?*
- *When does it reach its maximum height ?*
- *With what speed does it hit the ground?*