

Calculus

Lecture 5

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Implicit Differentiation

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- The equation $y^3 + 7y = x^3$ defines y implicitly.

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- $$\frac{dy}{dx} = \frac{3x^2}{3y^2 + 7}.$$
- The method of finding $\frac{dy}{dx}$ without solving the given equation explicitly for y in terms of x is called implicit differentiation.

Implicit Differentiation

Example

Find the equation of the tangent line to

$$x^2 + y^2 = 9$$

at the point $(2, \sqrt{5})$.

Implicit Differentiation

Example

Consider the equation

$$x^2 + xy + y^2 = 1.$$

Find equations for y' and y'' in terms of x and y only.

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- In this case, we may still find dy/dt since dy/dt and dx/dt are related rates.

Example

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A small balloon is released at a point 150 feet away from an observer, who is on level ground. If the balloon goes straight up at a rate of 8 feet per second, how fast is the distance from the observer to the balloon increasing when the balloon is 50 feet high?

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Water is pouring into a conical tank at the rate of 8 cubic feet per minute. If the height of the tank is 12 feet and the radius of its circular opening is 6 feet, how fast is the water level rising when the water is 4 feet deep?

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- Differentiate the equation implicitly.
- Solve for the desired derivative

Example

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A woman standing on a cliff is watching a motorboat through a telescope as the boat approaches the shore lie directly below her. If the telescope is 250 feet above the water level and if the boat is approaching at 20 feet per second, at what rate is the angle of the telescope changing when the boat is 250 feet from the shore?