PEN205
MODERN PHYSICS

Relativity - I

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Introduction

In our daily life, we mostly deal with objects whose speed is much more less than speed of light.

Motion of these objects are described by Newtonian mechanics → this formalism is very successful in describing a wide range of phenomena that occur at low speeds

However, it fails to describe properly the motion of objects whose speeds approach that of light.

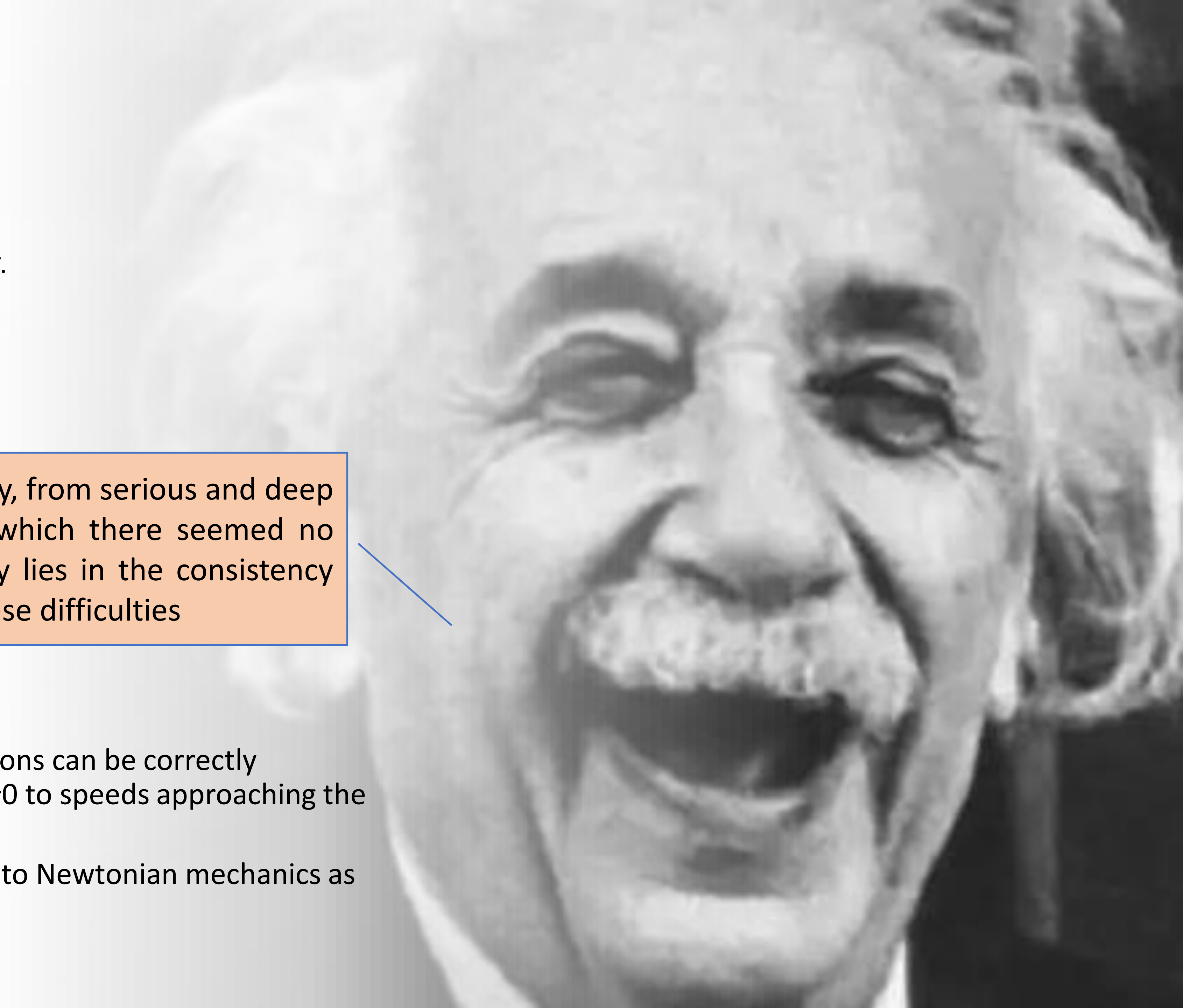
If we accelerate electrons or other charged particles through a large electric potential difference.

Newton→ if the potential difference is increased by a factor of 4, the electron's kinetic energy is four times greater and its speed should 1.98*c*

Exp. Results → speed of the electron—as well as the speed of any other object in the Universe—always remains less than the speed of light, regardless of the size of the accelerating voltage (0.99c)

Newtonian mechanics is contrary to modern experimental results and is clearly a limited theory.

In 1905, at the age of only 26, Einstein published his special theory of relativity.



The relativity theory arose from necessity, from serious and deep contradictions in the old theory from which there seemed no escape. The strength of the new theory lies in the consistency and simplicity with which it solves all these difficulties

- With this theory, experimental observations can be correctly predicted over the range of speeds from v=0 to speeds approaching the speed of light.
- At low speeds, Einstein's theory reduces to Newtonian mechanics as a limiting situation.

Einstein was working on electromagnetism when he developed the special theory of relativity. He was convinced that Maxwell's equations were correct, and in order to reconcile them with one of his postulates, he was forced into the revolutionary notion of assuming that space and time are not absolute.

Einstein's Relativity

Special Theory of Relativity (1905)

Deals with non-accelerating reference frames

Applies to all physical phenomena in the absence of gravity

The laws of physics are the same in all inertial frames, and the speed of light is the same for all observers.

Why "Special" ?—> Theory restricts itself to observers in uniform or constant relative motion

General Theory of Relativity (1916)

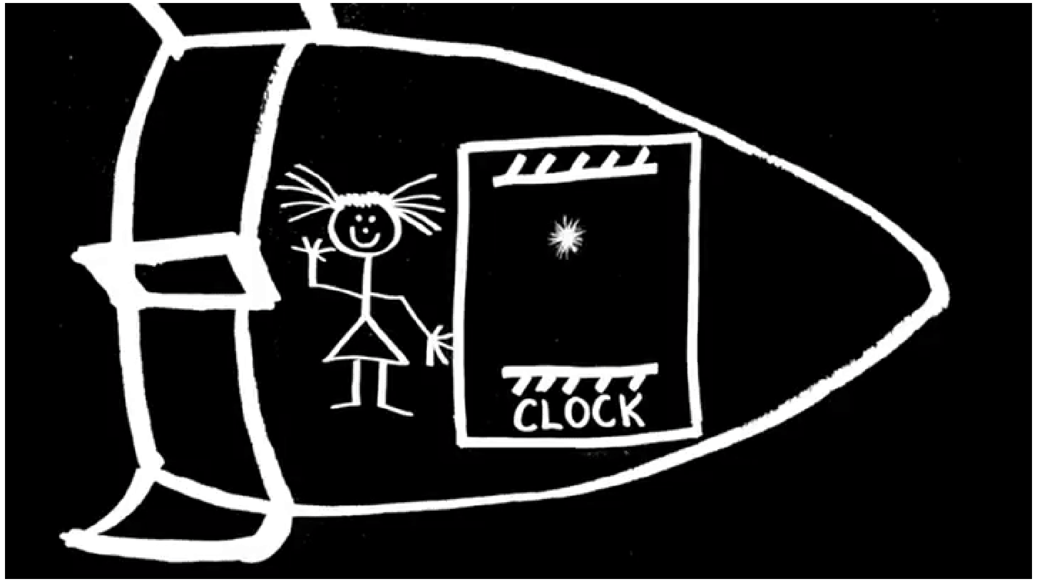
Deals with accelerating reference frames

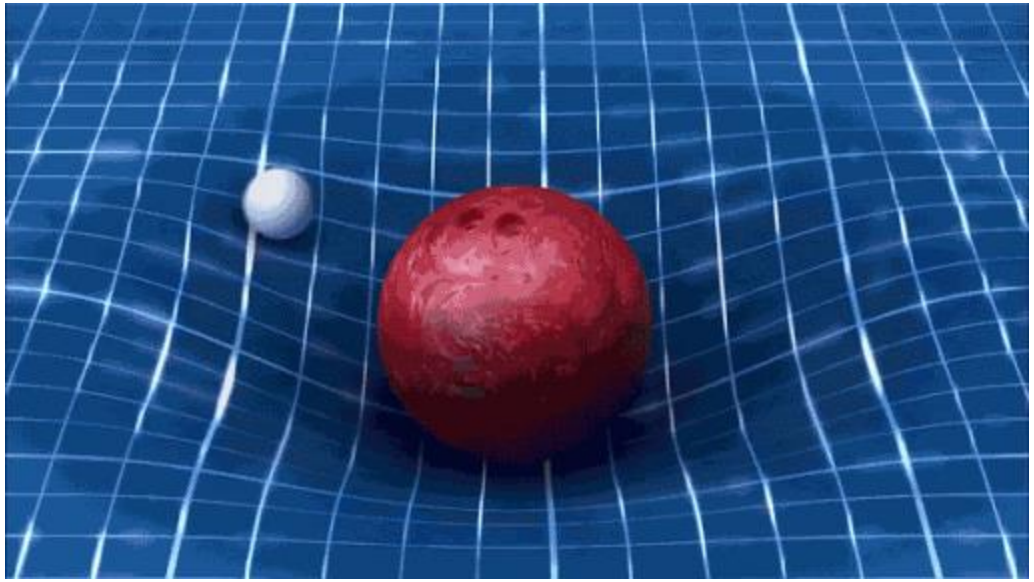
explains the law of gravitation and its relation to other forces of nature

Includes Special Relativity

Acceleration and gravity are indistinguishable

Gravity is the geometrical property of space-time





Special Relativity

Space and Time

General Relativity

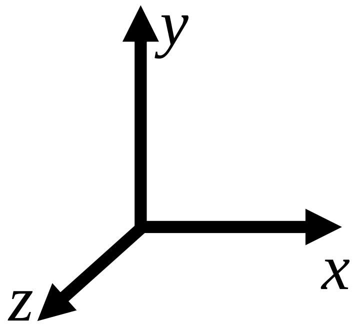
Gravitiy

In this issue;

- ✓ The Principle of Galilean Relativity
- ✓ The Michelson Morley Experiment
- ✓ Einstein’s Principle of Relativity
- ✓ Consequences of the Special Theory of Relativity
- ✓ The Lorentz Transformation Equations
- ✓ The Lorentz Velocity Transformation Equations
- ✓ Relativistic Linear Momentum and the Relativistic Form of Newton’s Laws
- ✓ Relativistic Energy
- ✓ Mass and Energy

The Principle of Galilean Relativity

To describe a physical event, we must establish a frame of reference.



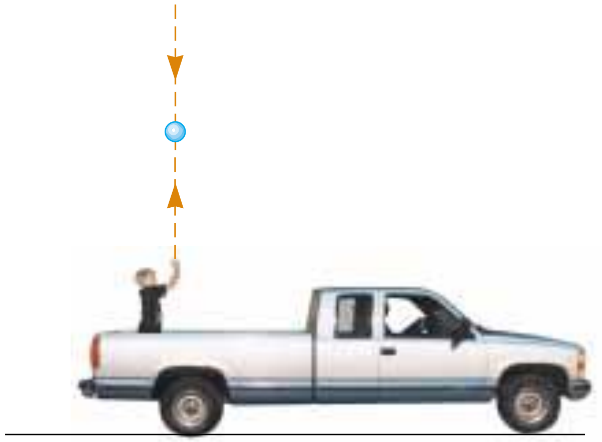
An inertial frame of reference is one in which an object is observed to have no acceleration when no forces act on it.

Furthermore, any system moving with constant velocity with respect to an inertial frame must also be in an inertial frame.

the results of an experiment performed in a moving car with uniform velocity $\overset{\text{identical}}{=}$ results of the same experiment performed in a stationary car

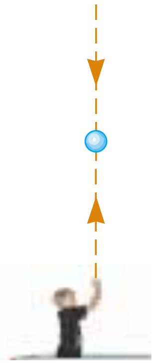
Principle of Galilean Relativity: The laws of mechanics must be the same in all inertial frames of reference.

Consider a passenger in the truck moving with a constant velocity throws a ball straight up.



(air effects are neglected)

The passenger observes that the ball moves in a vertical path.

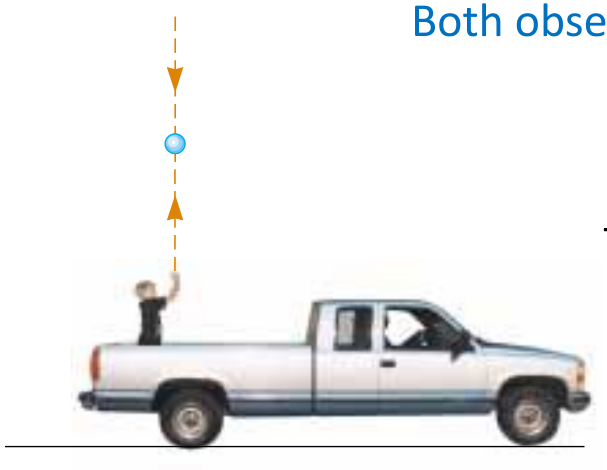


The motion of the ball appears to be precisely the same as if the ball were thrown by a person at rest on the Earth.

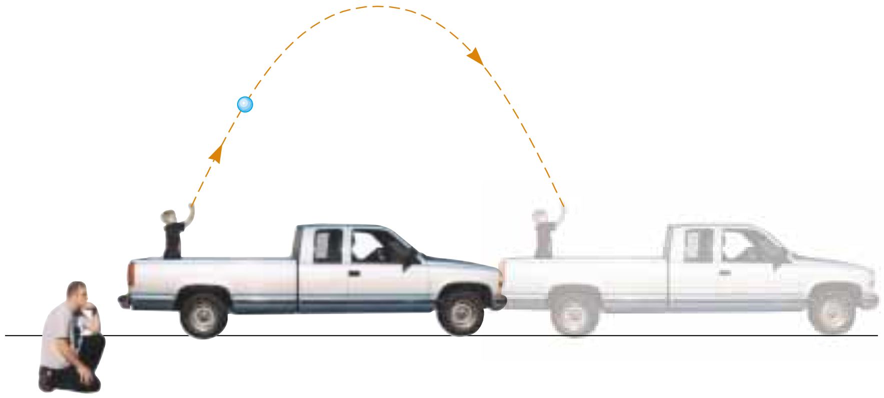
Both observers agree on the laws of physics—they each throw a ball straight up and it rises and falls back into their hand.

The law of universal gravitation and the equations of motion under constant acceleration are obeyed whether the truck is at rest or in uniform motion.

Both observers agree on the laws of physics. But do they agree on the path of ball?



The observer in the truck sees the ball move in a vertical path.



The observer on the ground sees the path of the ball as a parabola

According to the observer on the ground, the ball has a horizontal component of velocity equal to the velocity of the truck and a vertical component due to gravity.

Although the two observers disagree on certain aspects of the situation, they agree on the validity of Newton's laws.

Effect of gravity is the same whether truck is moving with constant speed or at rest.

No mechanical experiment can detect any difference between the two inertial frames. The only thing that can be detected is the relative motion of one frame with respect to the other.

The laws of mechanics must be the same in all inertial frames of reference!!!

Consider two inertial frames S and S’

The frame S’ moves with a <u>constant velocity</u> **v** along the common x and x' axes,

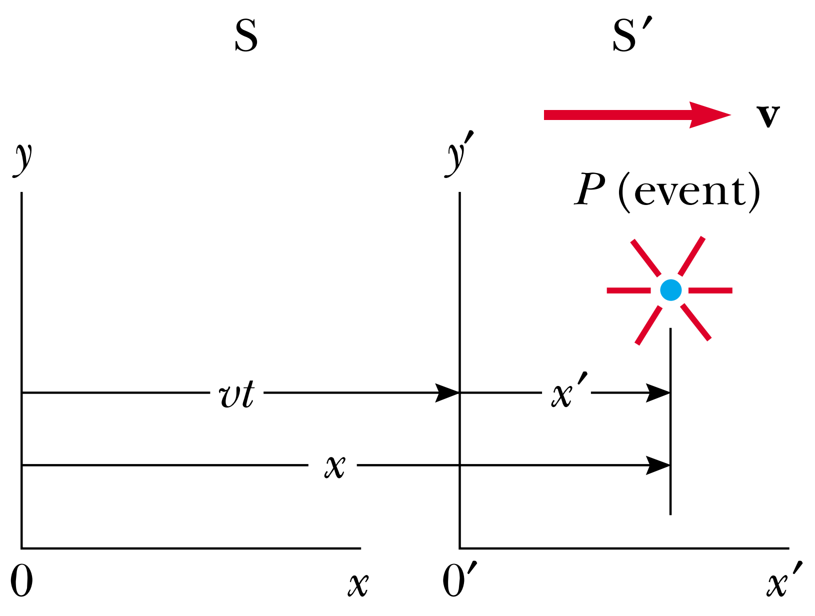
v is measured relative to S

Assume that the origins of S and S’ coincide at t=0

An event occurs at point P in space at some instant of time

An observer in S describes the event with space–time coordinates (x, y, z, t),

An observer in S’ uses the coordinates (x , y , z , t) to describe the same event

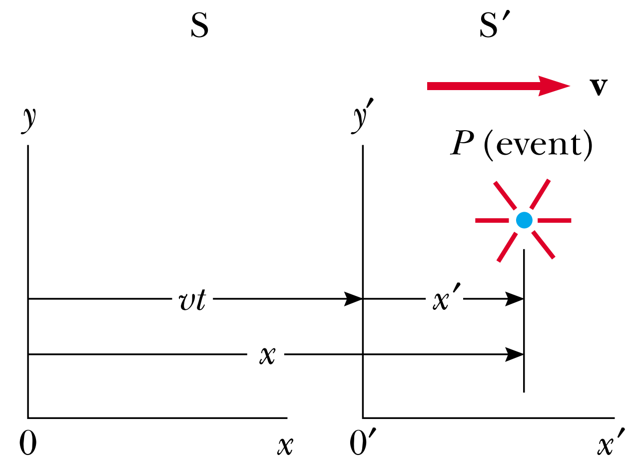


$$\boxed{\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}}$$

Galileo space-time transform equations

Equal in both!!

Lets suppose that a particle moves through a displacement of magnitude *dx* along the *x* axis in a time interval *dt* as measured by an observer in S.



$x' = x - vt$ $\longrightarrow$ $$\boxed{\begin{aligned}\frac{dx'}{dt} &= \frac{dx}{dt} - v \\ u_x' &= u_x - v\end{aligned}}$$

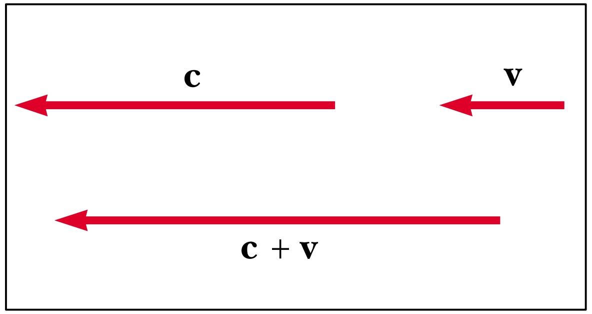
Galileo velocity transform equations

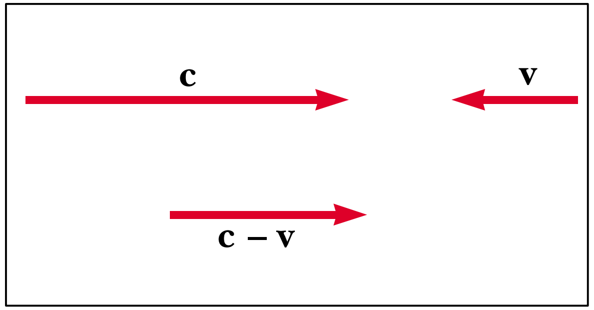
What happens when v approaches to c ?

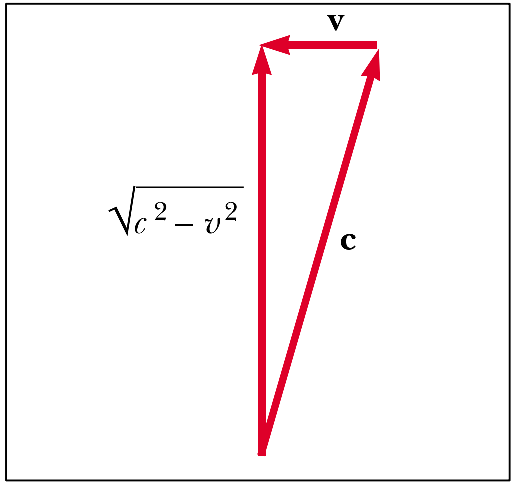
$t' = t$ → turns out to be incorrect in situations where *v* is comparable to the speed of light

Does Principle of Galilean relativity also applies to electricity, magnetism, and optics?

Remember: 1800s → Physicists thought that light waves moved through *ether* and that the speed of light was *c* only in a special, absolute frame at rest with respect to the ether







v is the speed of the ether relative to the Earth

- If the Galilean velocity transformation equations are valid speed of light must be different relative to ether.

- If Maxwell's electromagnetic laws are correct speed of light must be the same for all inertial frame of references.

CONTRADICTION!!!!

- To resolve this contradiction in theories, we must conclude that either (1) the laws of electricity and magnetism are not the same in all inertial frames or (2) the Galilean velocity transformation equation is incorrect.

- Michelson and Morley solved this contradiction.

The Michelson Morley Experiment

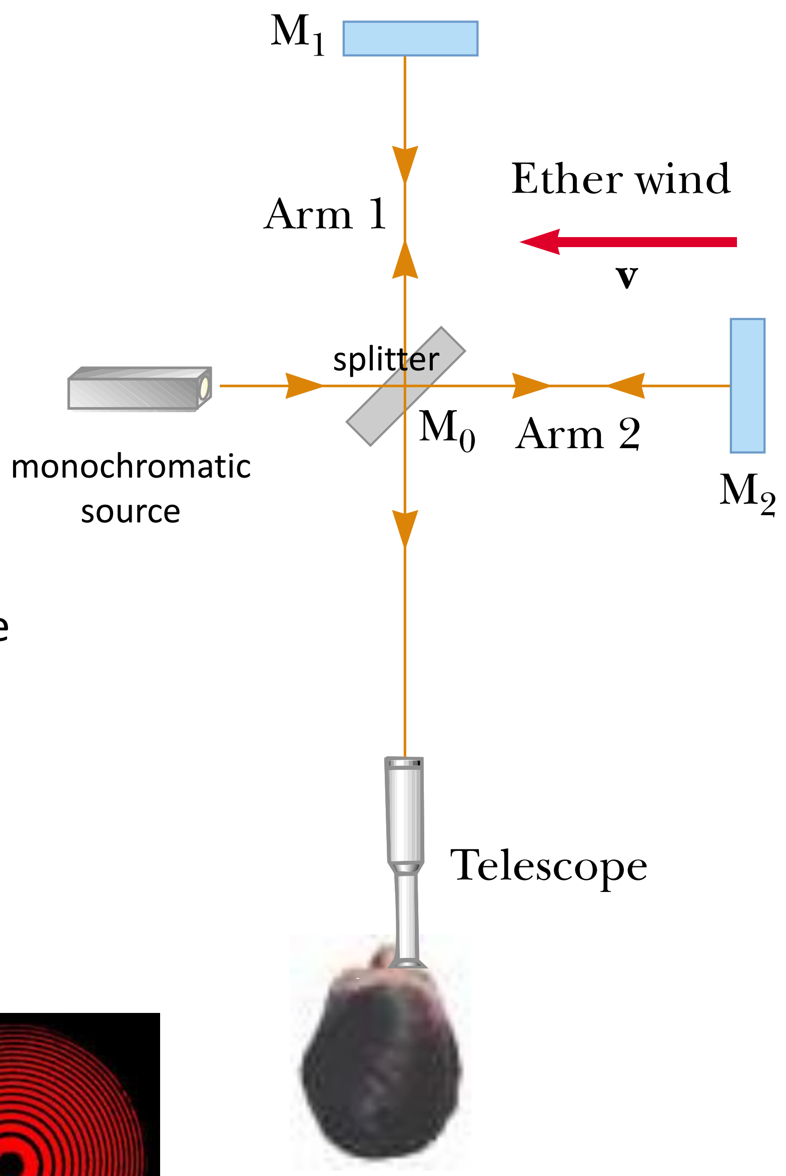
The most famous experiment designed to detect small changes in the speed of light was first performed in 1881.

The experiment was designed to determine the velocity of the Earth relative to that of the hypothetical ether.
The experimental tool used was the Michelson interferometer.

Arm 2 is aligned along the direction of the Earth's motion through space.

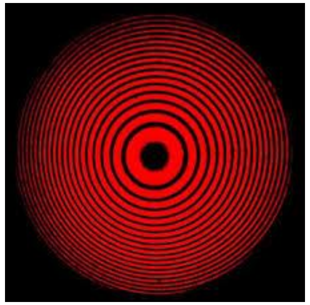
The Earth moving through the ether at speed *v* is equivalent to the ether flowing past the Earth in the opposite direction with speed *v*.

This ether wind blowing in the direction opposite the direction of Earth's motion should cause the speed of light measured in the Earth frame to be $c - v$ as the light approaches mirror M_2 and $c + v$ after reflection.



no fringe shift was ever observed ←

Interference pattern
bright and dark circular fringes



Results of Experiment:

•Light is now understood to be an electromagnetic wave, which requires no medium for its propagation.

•The idea of an ether became unnecessary.

• Velocity of light is constant.

•Galileo velocity transformations are not valid at velocity of light.

Albert Michelson
(1852-1931)

Edward Morley
(1838-1923)

Einstein's Principle of Relativity

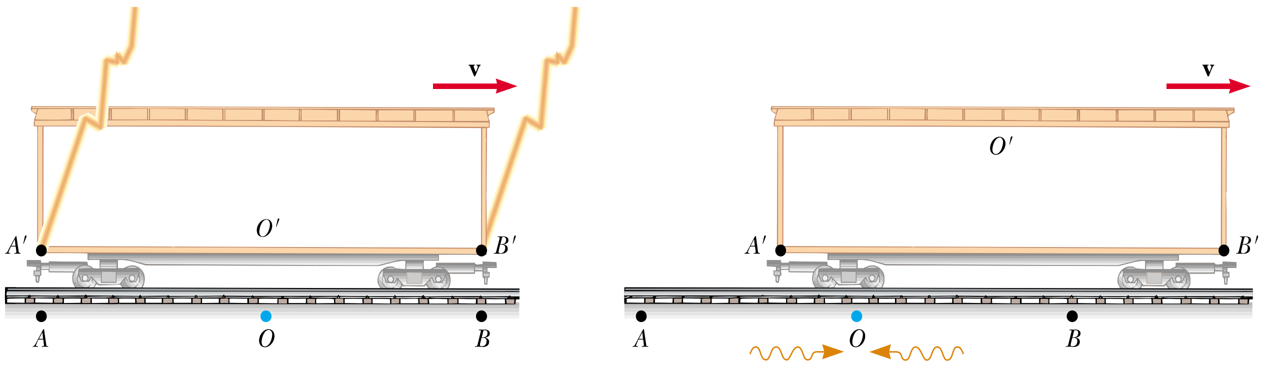
With the failure of the Galilean velocity transformation equation in the case of light, Einstein proposed a theory that boldly removed these difficulties and at the same time completely altered our notion of space and time.

He based his special theory of relativity on two postulates

1. The principle of relativity: The laws of physics must be the same in all inertial reference frames.

2. The constancy of the speed of light: The speed of light in vacuum has the same value, $c=3x10^8$ m/s, in all inertial frames.

Simultaneity and the Relativity of Time



A boxcar moves with uniform velocity, and two lightning bolts strike its ends leaving marks on the boxcar and on the ground

An observer *O'* moving with the boxcar is midway between *A'* and *B'* , and a ground observer *O* is midway between *A* and *B*

→ Lets assume that light signals reach the observer O at the same time, so this observer thinks that light signals have traveled at the same speed over equal distances, and so rightly concludes that the events at *A* and *B* occurred simultaneously

Now consider the same events as viewed by observer *O'…*

When the light reaches to O →O' has already moved, so;

→light signal *from B'* has already swept past *O'*

→ but the signal from *A* has not yet reached *O'*

According to Einstein, *the two observers must find that light travels at the same speed.*
Therefore, observer *O'* concludes that the lightning strikes the front of the boxcar before it strikes the back!!!

This thought experiment clearly demonstrates that the two events that appear to be simultaneous to observer *O* do not appear to be simultaneous to observer *O'.*

two events that are simultaneous in one reference frame are in general not simultaneous in a second frame moving relative to the first.

Which observer is right?

That is, simultaneity is not an absolute concept but rather one that depends on the state of motion of the observer.

Results of Special Relativity;

1- Time Dilation
2- Length Contraction

Results of Special Relativity

1. Time Dilation

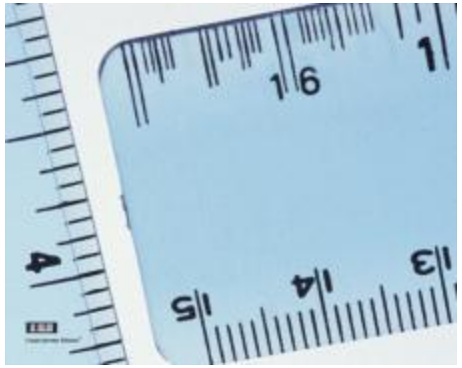
•Time was constant for all inertial frame of references according to Newtonian mechanics:

$$t' = t$$

•In relativistic mechanics the time interval between two events depend on the frame of reference in which they are measured.

$$t' \neq t$$

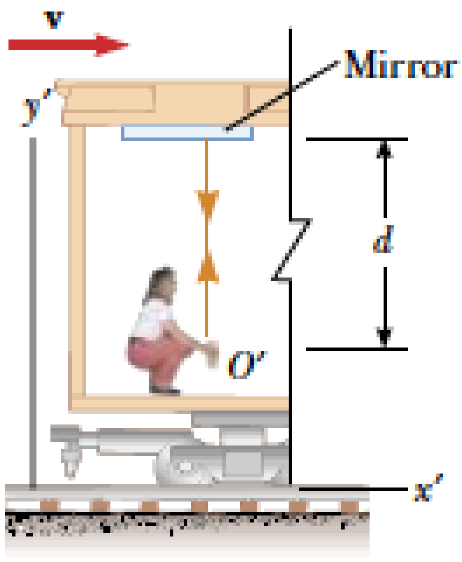
2. Length Contraction:



In relativistic mechanics the distance between two points between two events depend on the frame of reference in which they are measured.

1. Time Dilation

Observers in different inertial frames can measure different time intervals between a pair of events



(a)

Consider a vehicle moving to the right with a speed v.

A mirror is fixed to the ceiling of the vehicle, and observer O' at rest in the frame attached to the vehicle holds a flashlight a distance d below the mirror.

At some instant, the flashlight emits a pulse of light directed toward the mirror (event 1), and at some later time after reflecting from the mirror, the pulse arrives back at the flashlight (event 2).

Observer O' carries a clock and uses it to measure the time interval Δt_p between these two events.

proper time

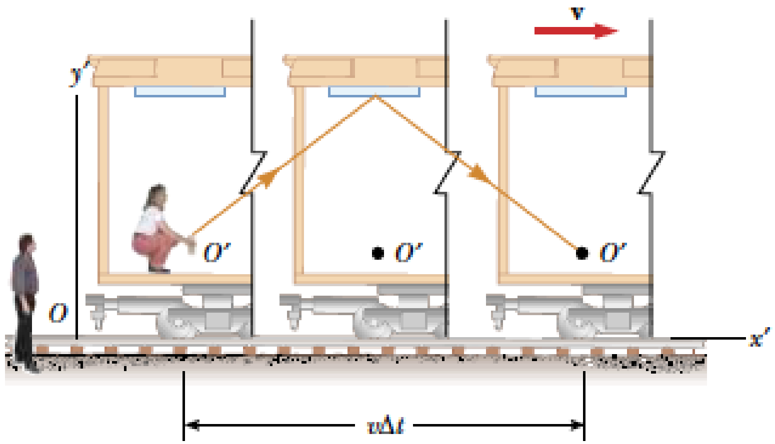
Δt_p can be calculated for the O' observers as; $$\Delta t_p = \frac{\text{distance traveled}}{\text{speed}} = \frac{2d}{c}$$

Let's consider the same pair of events as viewed by observer O in a second frame.

According to this observer, the mirror and flashlight are moving to the right with a speed v

By the time the light from the flashlight reaches the mirror, the mirror has moved to the right a distance $v\,\Delta t/2$

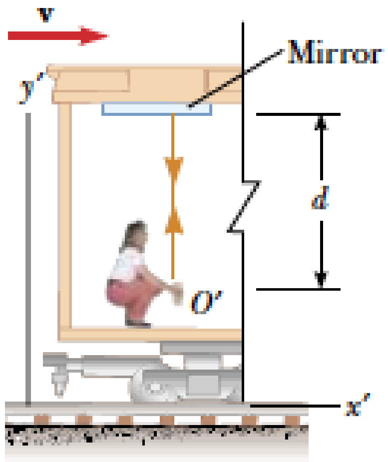
Δt is the time interval required for the light to travel from O' to the mirror and back to O' as measured by O.



(b)

O concludes that, because of the motion of the vehicle, if the light is to hit the mirror, it must leave the flashlight at an angle with respect to the vertical direction.

So, we see that the light must travel farther in (b) than in (a)

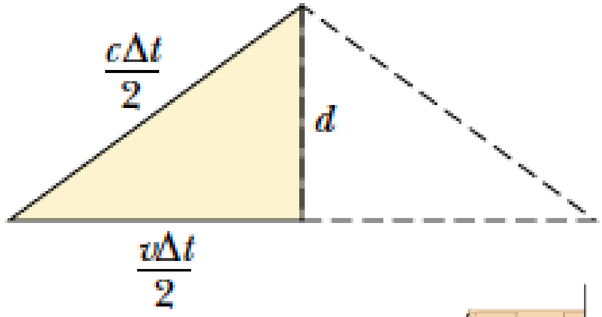


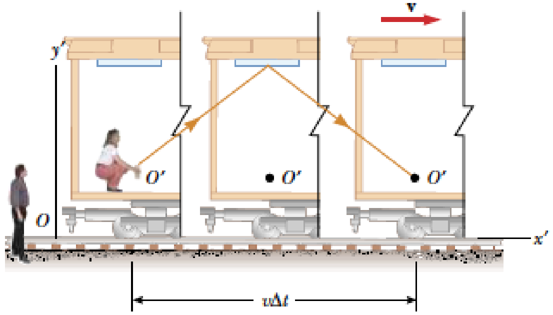
(a)

➔ According to the second postulate of the special theory of relativity, both observers must measure *c* for the speed of light

Because the light travels farther according to O, it follows that the time interval Δt measured by O is longer than the time interval Δt_p measured by O'.

$$\left(\frac{c\,\Delta t}{2}\right)^2 = \left(\frac{v\,\Delta t}{2}\right)^2 + d^2$$





Solving for Δt →

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$\Delta t_p = 2d/c$ → $\boxed{2d = \Delta t_p\, c}$

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\, \Delta t_p$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

γ is always greater than unity → $\Delta t > \Delta t_p$

Time Dilation

the proper time interval is the time interval between two events measured by an observer who sees the events occur at the same point in space.