PEN205
MODERN PHYSICS

Relativity - II

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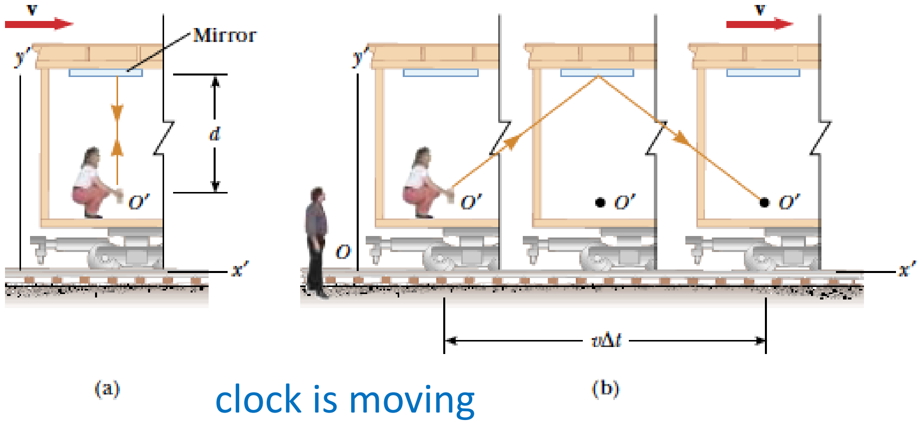
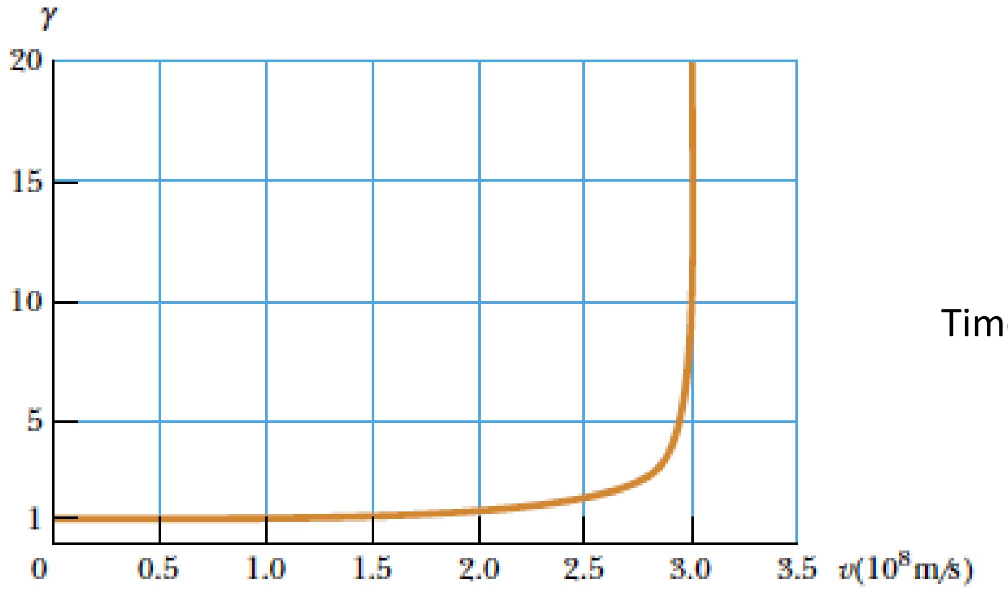
Time Dilation → $\Delta t > \Delta t_p$

The time interval Δt measured by an observer moving with respect to a clock is longer than the time interval Δt_p measured by an observer at rest with respect to the clock.



clock is moving

Approximate Values for γ at Various Speeds	
v/c	γ
0.001 0	1.000 000 5
0.010	1.000 05
0.10	1.005
0.20	1.021
0.30	1.048
0.40	1.091
0.50	1.155
0.60	1.250
0.70	1.400
0.80	1.667
0.90	2.294
0.92	2.552
0.94	2.931
0.96	3.571
0.98	5.025
0.99	7.089
0.995	10.01
0.999	22.37



Time dilation is not an everyday issue

$v/c = 0.1 \rightarrow \quad \gamma = 0.5\ \%$

• If a clock is moving with respect to you, the time interval between ticks of the moving clock is observed to be longer than the time interval between ticks of an identical clock in your reference frame.

➔ Thus, it is often said that a moving clock is measured to run more slowly than a clock in your reference frame by a factor .

•We can generalize this result by stating that all physical processes, including chemical and biological ones, are measured to slow down when those processes occur in a frame moving with respect to the observer.

➔ For example, the heartbeat of an astronaut moving through space would keep time with a clock inside the spacecraft. Both the astronaut's clock and heartbeat would be measured to slow down according to an observer on Earth comparing time intervals with his own clock.

What happens if v → c

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t_p$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \longrightarrow \infty$$

$$\Delta\tau \rightarrow \infty$$

Time stops for a light wave !!! ???

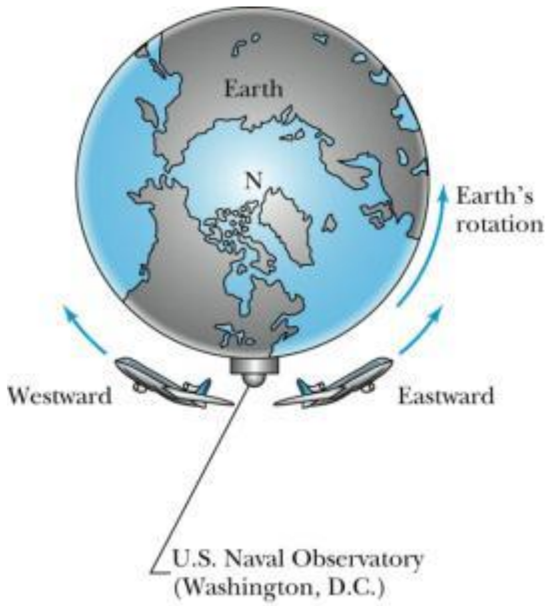
Experimental Verification of Time Dilation

An experiment reported by Hafele and Keating provided direct evidence of time dilation (1971).

Time intervals measured with four cesium atomic clocks in jet flight were compared with time intervals measured by Earth-based reference atomic clocks.

Considered factors;

- periods of speeding up and slowing down relative to the Earth
- variations in direction of travel,
- gravitational field experienced by the flying clocks and the Earth-based clock



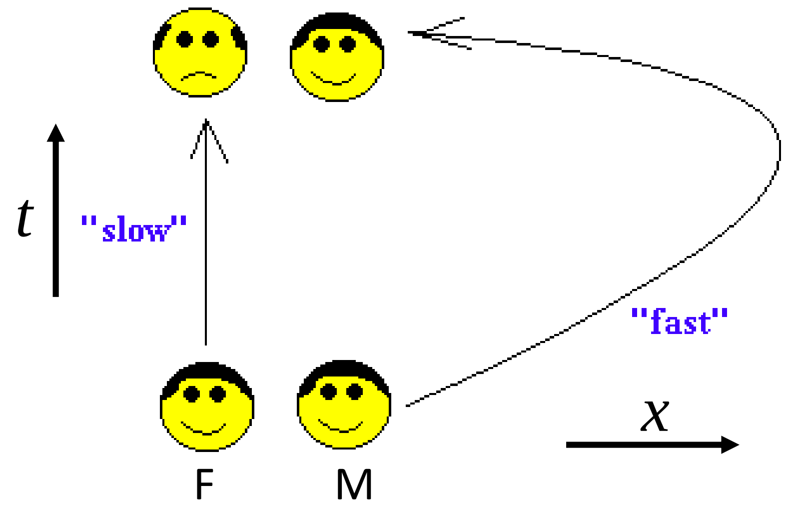
Result → Relative to the atomic time scale of the U.S. Naval Observatory, the flying clocks lost 59 ± 10 ns during the eastward trip and gained 273 ± 7 ns during the westward trip. . . .

The Twin Paradox

The Set-up

•Mary and Frank are twins.

•Mary, an astronaut, leaves on a trip many lightyears (ly) from the Earth at great speed and returns; Frank decides to remain safely on Earth.



The Problem

•Frank knows that Mary's clocks measuring her age must run slow, so she will return younger than he.

•However, Mary (who also knows about time dilation) claims that Frank is also moving relative to her, and so his clocks must run slow.

The Paradox

•Who, in fact, is younger upon Mary's return?

The Twin-Paradox Solution

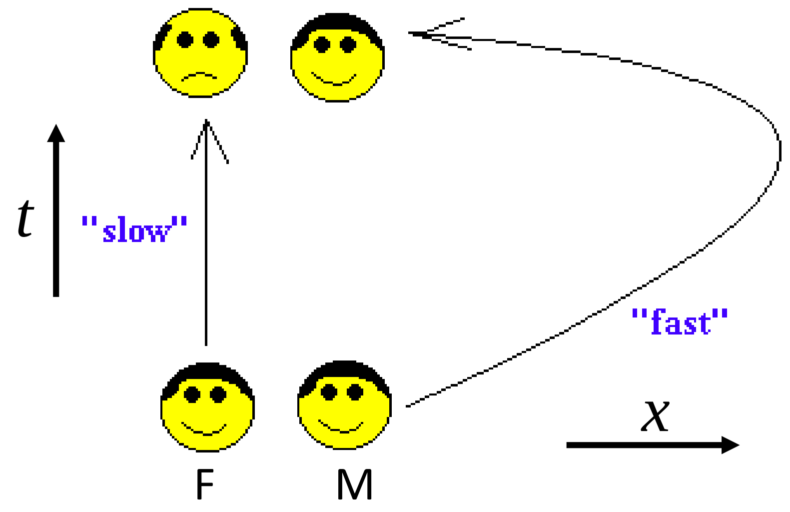
•Frank's clock is in an inertial system during the entire trip.

•But Mary's clock is not.

•As long as Mary is traveling at constant speed away from Frank, both of them can argue that the other twin is aging less rapidly.

•But when Mary slows down to turn around, she is not in an inertial frame anymore. She returns in a completely different inertial frame.

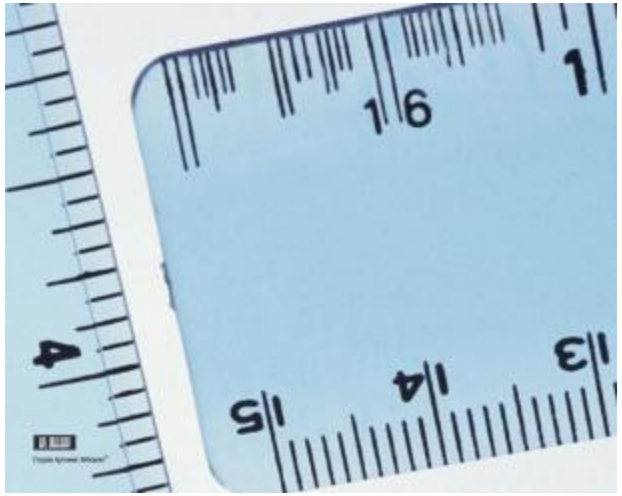
•Mary's claim is no longer valid, because she doesn't remain in the same inertial system. Frank does, however, and Mary ages less than Frank.



2. Length Contraction:

The measured distance between two points also depends on the frame of reference

When both endpoints of an object (at rest in a given frame) are measured in that frame, the resulting length is called the Proper Length (Lp).



We’ll find that the proper length is the largest length observed.

Observers in motion will see a contracted object. This is called length contraction.

Lets consider a spacecraft traveling with a speed *v* from one star to another.

There are two observers:
- on the Earth
- in the spacecraft.

Note: length contraction takes place only along the direction of motion.

1. star

2. star

x

Observer in a spacecraft measures time interval as Δt_p

$$\Delta t_p = \Delta t/\gamma$$

distance between stars as L:

The observer at rest on the Earth measures the distance between the stars to be the proper length L_p.

$$\Delta t = L_p/v.$$

$$L_p = v\,\Delta t$$

$$L = v\,\Delta t_p = v\,\frac{\Delta t}{\gamma}$$

$$L = \frac{L_p}{\gamma} = L_p\sqrt{1-\frac{v^2}{c^2}}$$

$$L = \frac{L_p}{\gamma} = L_p \sqrt{1 - \frac{v^2}{c^2}}$$

→ less than unity

Remember that;

L → measured by the observer in spacecraft

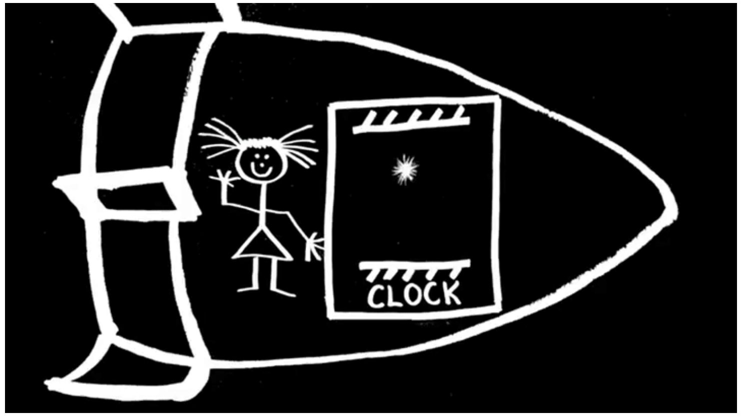
L_p → measured by the observer on Earth

If an object has a proper length L_p when it is measured by an observer at rest with respect to the object, then when it moves with speed v in a direction parallel to its length, its length L is measured to be shorter

The proper length → measured by an observer for whom the end points of the length remain fixed in space.

The proper time interval → measured by someone for whom the two events take place at the same position in space.

on Earth	on spacecraft
L_p	L
Δt	Δt_p



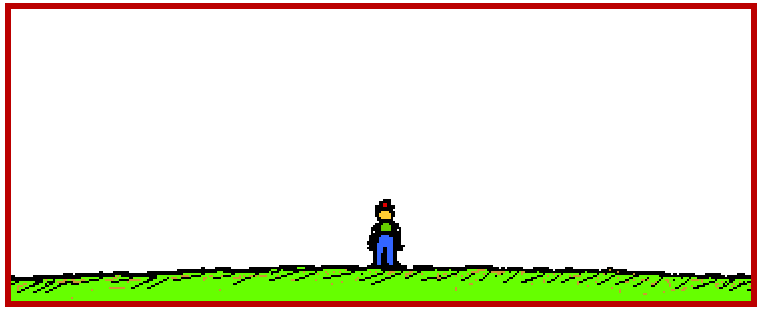
$L < L_p$

$\Delta t > \Delta t_p$

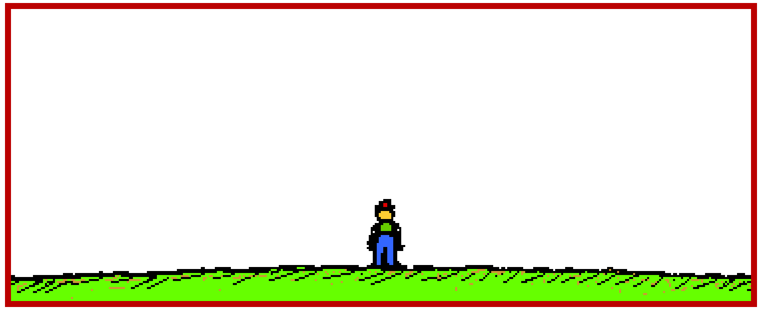
Length Contraction

A fast-moving plane at different speeds.

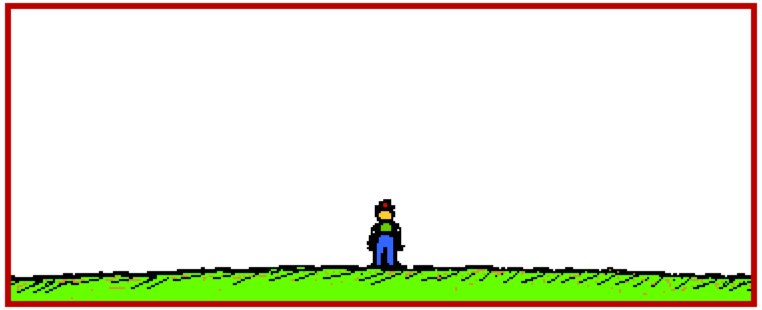
v = 10% c



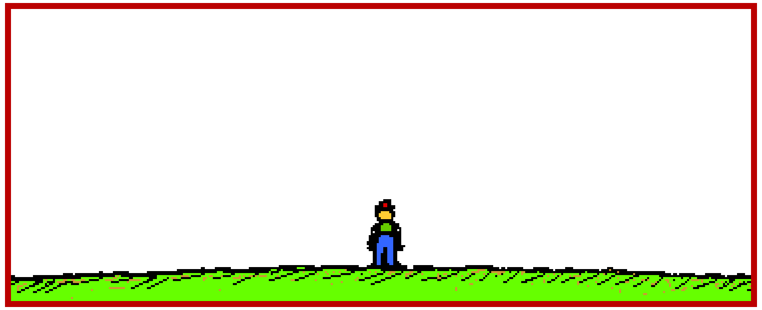
v = 80% c

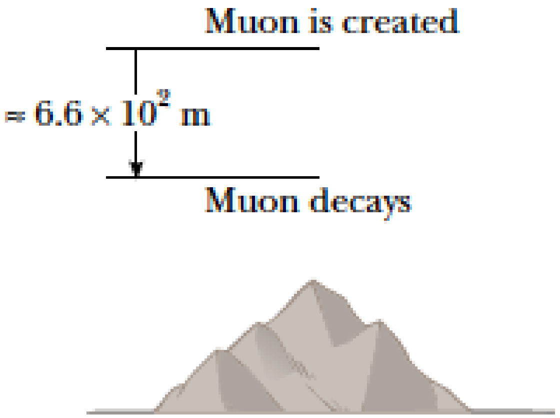


v = 99% c



v = 99.9% c



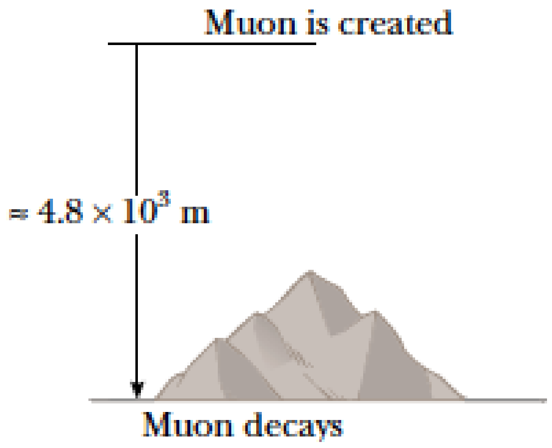


Without relativistic considerations, muons created in the atmosphere and traveling downward with a speed of 0.99c travel only about 6.6×10^2 m before decaying with an average lifetime of 2.2 μs. Thus, very few muons reach the surface of the Earth.

For observer travelling with muons:

- Measures proper time
- Measures the length of mountain shorter than its proper length

Length contraction
No time dilation



With relativistic considerations, the muon's lifetime is dilated according to an observer on Earth. As a result, according to this observer, the muon can travel about 4.8×10^3 m before decaying. This results in many of them arriving at the surface.

For observer at rest on earth:

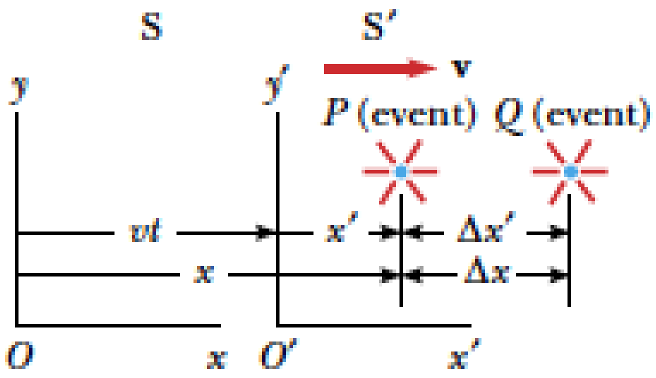
- Measures longer time interval
- Measures the length of mountain as proper length

No length contraction
Time dilation

The Lorentz Transformation Equations

The Galilean transformation is not valid when *v* approaches the speed of light.

Lorentz transformation equations → $v \le 0 < c$



Events occur at points P and Q and are observed by an observer at rest in the S frame and another in the S" frame, which is moving to the right with a speed v.

The equations that are valid for all speeds and enable us to transform coordinates from S to S’ are the **Lorentz transformation equations**:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

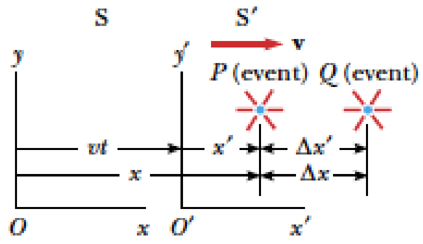
$$t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \gamma(x - vt) \qquad y' = y \qquad z' = z \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$ S → S' equations

$$x = \gamma(x' + vt') \qquad y = y' \qquad z = z' \qquad t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$ S' → S equations

If $v \ll c$ → $\gamma = 1$ so, Lorentz equations transform to Galilean equations;



$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t$$

difference in coordinates between two events or the time interval between two events as seen by observers O and O' ;

$$\left.\begin{aligned} \Delta x' &= \gamma(\Delta x - v\,\Delta t) \\ \Delta t' &= \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) \end{aligned}\right\} S \rightarrow S'$$

$$\left.\begin{aligned} \Delta x &= \gamma(\Delta x' + v\,\Delta t') \\ \Delta t &= \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right) \end{aligned}\right\} S' \rightarrow S$$

The Lorentz Velocity Transformation Equations

Suppose two observers in relative motion with respect to each other are both observing the motion of an object.

Previously, we defined an event as occurring at an instant of time.
Now, we wish to interpret the "event" as the motion of the object.

an object has a velocity component u'_x measured in the S' frame → $u'_x = \dfrac{dx'}{dt'}$

$$dx' = \gamma(dx - v\,dt)$$

$$dt' = \gamma\left(dt - \frac{v}{c^2}\,dx\right)$$

→ We have these equations from Lorentz transformation eq.

Observed velocity in S

$u_x = dx/dt$

$$u'_x = \frac{dx'}{dt'} = \frac{dx - v\,dt}{dt - \dfrac{v}{c^2}\,dx} = \frac{\dfrac{dx}{dt} - v}{1 - \dfrac{v}{c^2}\dfrac{dx}{dt}}$$

$$\boxed{u'_x = \frac{u_x - v}{1 - \dfrac{u_x v}{c^2}}}$$

$$u'_y = \frac{u_y}{\gamma\left(1 - \dfrac{u_x v}{c^2}\right)} \quad \text{and} \quad u'_z = \frac{u_z}{\gamma\left(1 - \dfrac{u_x v}{c^2}\right)}$$

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$v << c$ → $u'_x \approx u_x - v$ → Galilean transformation equations

$u_x = c$ → $$u'_x = \frac{c - v}{1 - \frac{cv}{c^2}} = \frac{c\left(1 - \frac{v}{c}\right)}{1 - \frac{v}{c}} = c$$

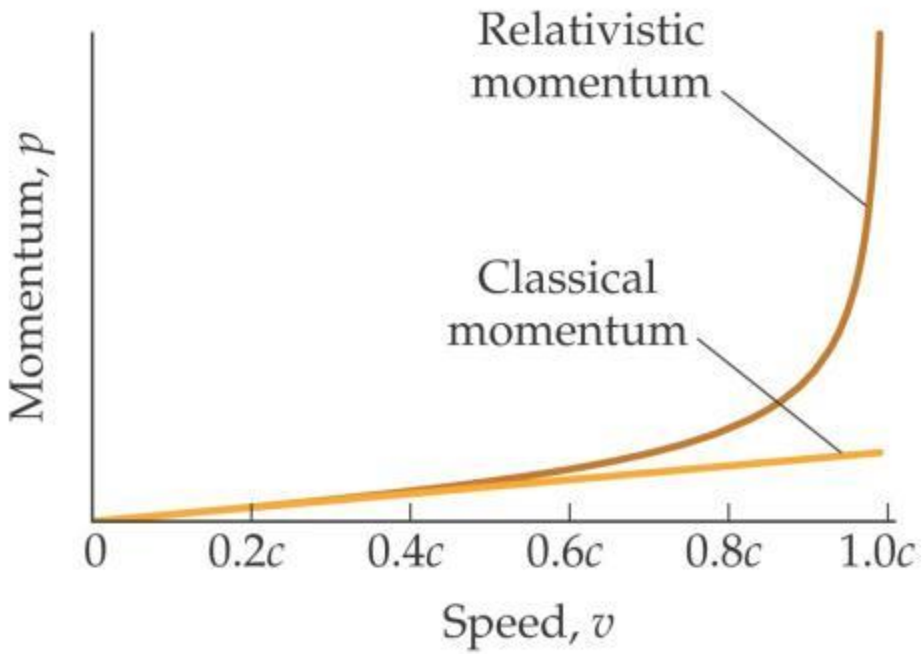
A speed measured as *c* by an observer in S is also measured as *c* by an observer in S'.
It is independent of the relative motion of S and S'

Einstein → the speed of light must be c relative to all inertial reference frames

Relativistic Linear Momentum and the Relativistic Form of Newton's Laws

Because the laws of physics must remain unchanged under the Lorentz transformation, we must generalize Newton's laws and the definitions of linear momentum and energy to conform to the Lorentz transformation equations and the principle of relativity.

$$\vec{p} = \frac{m_0 \vec{u}}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma m_0 \vec{u}$$



u: the velocity of the particle

m: the mass of the particle

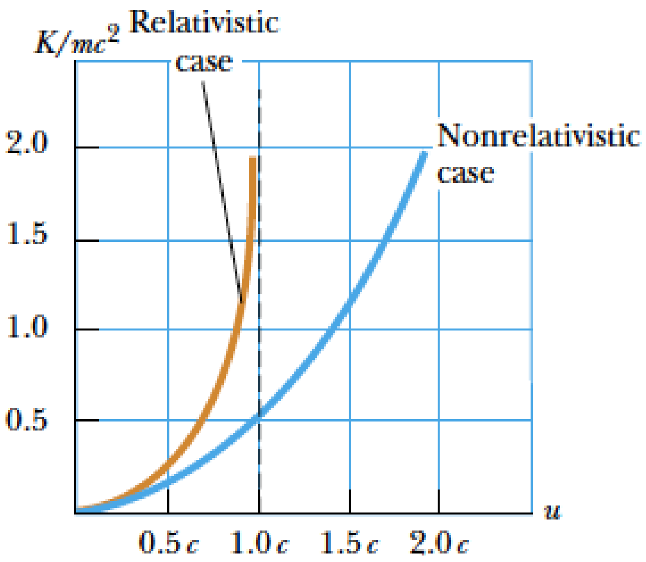
The relativistic force F acting on a particle whose linear momentum is **p** → $\mathbf{F} \equiv \frac{d\mathbf{p}}{dt}$

Relativistic Energy

$$K = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} - mc^2 = \gamma mc^2 - mc^2 = (\gamma - 1)\,mc^2$$

$$\boxed{K = mc^2\left(\gamma - 1\right)}$$

$$u/c \ll 1 \longrightarrow K = \frac{1}{2}mu^2$$



$$\boxed{K = mc^2(\gamma - 1)}$$

The constant term mc^2 which is independent of the speed of the particle, is called the rest energy E_R of the particle:

$$E_R = mc^2$$

The term γmc^2 depends on the particle speed, and defined as the total energy E

$$E = \frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma mc^2$$

Total energy = kinetic energy + rest energy

$$E = K + mc^2$$

→ mass is a form of energy

In many situations, the linear momentum or energy of a particle is measured rather than its speed.

$$E = \gamma mc^2 \qquad p = \gamma mu$$

$$E^2 = p^2c^2 + (mc^2)^2$$

If particle at rest, p = 0 → $\boxed{E_R = mc^2}$

If m = 0 → $E = pc$

$$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$$

the mass of an electron is 9.11×10^{-31} kg. Hence, the rest energy of the electron is

$$\begin{aligned} m_e c^2 &= (9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^{8}\ \text{m/s})^2 = 8.20 \times 10^{-14}\ \text{J} \\ &= (8.20 \times 10^{-14}\ \text{J})(1\ \text{eV}/1.60 \times 10^{-19}\ \text{J}) = 0.511\ \text{MeV} \end{aligned}$$

The Electron Volt (eV)

The work done in accelerating a charge through a potential difference is given by $W = qV$.

For a proton, with the charge $e = 1.602 \times 10^{-19}$ C and a potential difference of 1 V, the work done is:

$$W = (1.602 \times 10^{-19}\ \text{C})(1\ \text{V}) = 1.602 \times 10^{-19}\ \text{J}$$

The work done to accelerate the proton across a potential difference of 1 V could also be written as:

$$W = (1\ e)(1\ \text{V}) = 1\ \text{eV}$$

Thus eV, pronounced "electron volt," is also a unit of energy. It's related to the SI (*Système International*) unit joule by:

$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$