Calculus
Lecture 6

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Maxima and minima

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- $f(c)$ is an extreme value of f on S if it is either the maximum value or the minimum value.
- the function we want to maximize or minimize is called objective function.

Does f have a maximum or a minimum value on S?

If f is continuous on a closed interval $[a, b]$, then f attains both a maximum value and minimum value there.

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Any point of one of these three types is called critical point of f.

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Let f be a continuous function defined on an interval $I = [a, b]$.

- Find the critical points of f on I
- Evaluate f at each of these critical points. The largest of these values is the maximum value, the smallest is the minimum value.

Let f be defined on an interval I.

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- f is strictly monotone on I if it is either increasing or decreasing on I

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Concave up and Concave down

Let f be differentiable on an open interval I. We say that f is concave up on I if f' is increasing on I, and we say that f is concave down on I if f' is decreasing on I.

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Let f be differentiable on an open interval I. We say that f is concave up on I if f' is increasing on I, and we say that f is concave down on I if f' is decreasing on I. We call $(c, f(c))$ an inflection point of the graph of f, if f is concave up on one side of c and and concave down on the other side.

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- If $f'(x) < 0$ for all x in (a, c) and $f'(x) > 0$ for all x in (c, b) then $f(c)$ is a local minimum.

Second Derivative Theorem

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Let f' and f'' exist at every point in an open interval $I = (a, b)$ containing c and suppose $f'(c) = 0$.

- If $f''(c) < 0$ then $f(c)$ is a local maximum of f.
- If $f''(c) > 0$ then $f(c)$ is a local minimum of f.