SAMPLE OF EXAM

1. Determine whether $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & -3 & 1 \end{bmatrix}$ is invertible or not. If it is, find it.

2. Consider the linear system

$$\begin{aligned} x_1 + \mathbf{m}x_2 - 3x_3 &= 5 \\ 2x_1 + 3\mathbf{m}x_2 - 6x_3 &= 8 \\ -x_1 + \mathbf{m}x_2 + \mathbf{n}x_3 &= -9. \end{aligned}$$

By using the Gauss elimination method**,** find all values of **m** and **n** for which

a) the system is inconsistent,

b) the system has exactly one solution,

c) the system has infinitely many solutions.

3. Some of the entries of the 3×3 matrix A are known as $A = \begin{bmatrix} . & 1 & 2 \\ 2 & . & . \\ . & -1 & 1 \end{bmatrix}$.

Suppose that $det\,(A) = -6$ and that $A^{-1} = \begin{bmatrix} . & x & . \\ . & . & . \\ . & . & . \end{bmatrix}$. Find x.

4. Answer each of the following questions **True (T)** or **False (F)**.

a) If a system of linear equations has two different solutions, then it must have infinitely many solutions .....

b) Every system of linear equations has at least one solution .....

c) If A and B are $n \times n$ invertible matrices, then the equality $(A+B)(A-B) = A^2 - B^2$

is always true .....

d) If A is an $n \times n$ matrix such that $A^3 = I_n$, then A is invertible .....

e) If A is 3×3 matrix such that $A^2 = A$, then $\det(A^5) = \det A$ .....

Good luck!

SAMPLE OF EXAM

1. Let W be the set of all 2×2 matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $\det A = 0$. Is W a subspace of M_{22}? Justify your answer.

2. Consider the vector space $\mathbb{R}^3$ with the standard operations.

a) For what values of x are the vectors $v_1 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix}$ linearly independent?

b) For what values of y the vector $u = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$ can not be spanned by the vectors $u_1 = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 1 \\ 3 \\ y \end{bmatrix}$?

3. Let $S = \{v_1, v_2, v_3\}$ and $T = \left\{ \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \right\}$ be ordered basis for $\mathbb{R}^3$. The transition matrix from the basis T to the basis S is $P_{S \leftarrow T} = \begin{bmatrix} -2 & 1 & -1 \\ 7 & 0 & 2 \\ 3 & 0 & 1 \end{bmatrix}$. Determine the basis S. $\left(\text{Note that } P_{S \leftarrow T}^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 3 \\ 0 & 3 & 7 \end{bmatrix} \right)$

4. Find a basis for the subspace $U = \left\{ \begin{bmatrix} a+c & a-b \\ b+c & b-a \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$ of M_{22}. What is the dimension of U?

Good luck!