ENE 206 – Fluid Mechanics

WEEK 7

- ## Introduction

- Bernoulli equation is obtained by applying the Newton's second law of motion to an infinitesimal fluid element, which is moving along an instantaneous streamline without considering the viscous effects and non-conservative body forces.
- When non-conservative body forces are introduced, then extended Bernoulli equation which is valid for viscous fluids is obtained.

- ## The Bernoulli equation

- The equation of motion for inviscid fluid will be integrated along a streamline. To do so, the equation of motion is first derived in the streamline coordinates and then integrated along a streamline which is shown in Figure 5.1.

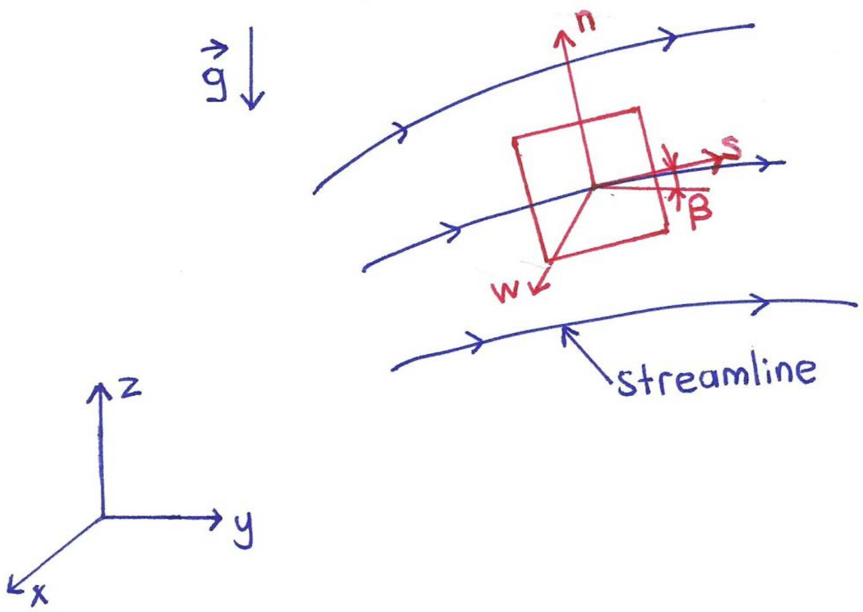

Figure 5.1. An infinitesimal fluid element moving along an instantaneous streamline

The Newton's second law of motion is applied in the streamline direction (*s*-direction) to the infinitesimal fluid element of mass, *dm* with sides, *ds*, *dn* and *dw.*

The summary of the derivation of the Bernoulli equation is given as below:

$$dF_s = a_s\, dm \qquad (5.1)$$

where dF_s denotes the sum of external surface and body forces acting on the defined infinitesimal fluid element, a_s is the acceleration of this element in the *s*-direction.

$$dF_s = dF_{bs} + dF_{ps} \tag{5.2}$$

where dF_{bs} is referred to as the body force acting in the *s*-direction and dF_{ps} is the pressure force in the *s*-direction. dF_s is also expressed as:

$$dF_s = \rho \frac{DV}{dt} ds dn dw \tag{5.3}$$

where V is the velocity of the infinitesimal fluid element, and $dm = \rho ds dn dw$

The body force can be expressed as below:

$$dF_{bs} = -\rho g \frac{\partial z}{\partial s} ds dn dw \tag{5.4}$$

In an inviscid flow there is no shear stress.

The sum of the pressure forces acting on the infinitesimal fluid element in the *s*-direction can be given as:

$$dF_{ps} = -\frac{\partial p}{\partial s} ds dn dw \tag{5.5}$$

Then, the total force acting on the infinitesimal fluid element in the *s*-direction by using eqn. 5.2 can be rearranged as below:

$$dF_s = dF_{bs} + dF_{ps} = -\rho g \frac{\partial z}{\partial s} ds dn dw - \frac{\partial p}{\partial s} ds dn dw \tag{5.6}$$

Finally, eqn. 5.2 becomes

$$\rho \frac{DV}{dt} = -\rho g \frac{dz}{ds} - \frac{\partial p}{\partial s} \tag{5.7}$$

Eqn. 5.7 can take the following form by considering the fact that the velocity is always tangent to the streamline:

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial s} = -g \frac{\partial z}{\partial s} - \frac{1}{\rho} \frac{\partial p}{\partial s} \tag{5.8}$$

Now, integrating eqn. 5.8 partially with respect to the distance along an instantaneous streamline

$$\int \frac{\partial V}{\partial t} ds + \int V \frac{\partial V}{\partial s} ds + \int \frac{1}{\rho} \frac{\partial p}{\partial s} ds + g \int \frac{\partial z}{\partial s} ds = \beta(t) \tag{5.9}$$

where $\beta(t)$ is the Bernoulli function.

Rearranging eqn. 5.9 one can obtain the eqn. as follows:

$$\int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + \int \frac{\partial p}{\rho} + gz = \beta(t) \tag{5.10}$$

The right hand side of the eqn. 5.10 is constant at any instant of time. The eqn. is known as the Bernoulli equation, which is named in the honor of Bernoulli (1700-1782).

- Bernoulli equation for the steady flow of an incompressible fluid in the absence of gravitational acceleration:

$\frac{\partial}{\partial t}=0$; $\rho=$ constant, g = 0 no gravity. Then eqn. 5.10 becomes

$$\frac{V^2}{2}+\frac{p}{\rho}=\text{constant} \qquad (5.11)$$

- Bernoulli equation for the steady flow of an incompressible fluid:

  $\frac{\partial}{\partial t}=0$; $\rho=$ constant. Then eqn. 5.10 becomes

$$\frac{V^2}{2}+\frac{p}{\rho}+gz=\text{constant} \qquad (5.12)$$

- Bernoulli equation for the steady of a barotropic fluid:

  For a steady flow $\partial/\partial t=0$ Then eqn. 5.10 becomes:

$$\frac{V^2}{2}+\int\frac{\partial p}{\rho}+gz=\beta(t) \qquad (5.13)$$

- ## The Extended Bernoulli equation

- This is a more general case of the Bernoulli equation to include the viscous terms in eqn. 5.10. Nonconservative body forces and viscous stresses are also present in the equation. The first law of thermodynamics for a one-dimensional and steady flow of an incompressible fluid through a streamtube is combined with the enthalpy to obtain the flowing equation:

$$\frac{p_1}{\rho}+\frac{V_1^2}{2}+gz_1=\frac{p_2}{\rho}+\frac{V_2^2}{2}+gz_2-w_s+w_f \qquad (5.14)$$

  In the above equation, the term w_s represents the work done on the fluid per unit mass by the surroundings and the term w_f can be defined as the frictional work per unit mass.

  According to the chosen convention, the flow conditions at the downstream or after the flow process are designated by subscript 2, while the flow conditions at the upstream or before the flow process are designated by subscript 1.

- ## Velocity measurements

- Pitot tube: A glass tube which is used for the measurement of the flow velocity. This tube directly measures the static pressure in the flow channel which is used to determine the velocity using the Bernoulli equation.

- ## Flow rate measurements in closed conduits

- **Orifice meter**: A thin plate with an opening, which is circular.

- **Nozzle flow meter**: It is placed in a pipe, has a well rounder entrance.

- **Venturi meter**: It is composed of a conical contraction, a straight throat and a conical expansion.

- **References**

1. Aksel, M.H., 2016, “Notes on Fluids Mechanics”, Vol. 1, METU Publications
2. Anderson, J.D., 1995, “Computational Fluid Dynamics”, McGraw Hill Book Company, New York
3. Daugherty, R.L., Franzini, J.B., and Finnemore, E.J., 1985, “Fluid Mechanics with Engineering applications”, 8th Edition, McGraw Hill Book Company, New York
4. Fox, R.W. and McDonald, A.T., 1994, “Introduction to Fluid Mechanics”, 4th Edition, John Wiley and Sons, Inc., New York
5. Prandtl, L., and Tietjens, O.G., 1957, “Fundamental of Hydro and Aero-mechanics”, Dover Publications, Inc., New York
6. Roberson, J.A. and Crowe, T.C., 1985, “Engineering Fluid Mechanics”, 3rd Edition, Houghton Mifflin Company, Boston
7. Sabersky, R.H., Acosta, A.J. and Hauptmann, E.G., 1971, “Fluid Flow”, Macmillan Publishing C., Inc. New York
8. Shames, I.H., 1982, “Fluid Mechanics”, 2nd Edition, McGraw Hill Book Company, New York.
9. Vennard J.K. and Street R.L., 1982, “Elementary Fluid Mechanics”, 7th Edition, John Wiley and Sons, Inc., New York.