Chapter 24 Section 1:
Gauss’s Law

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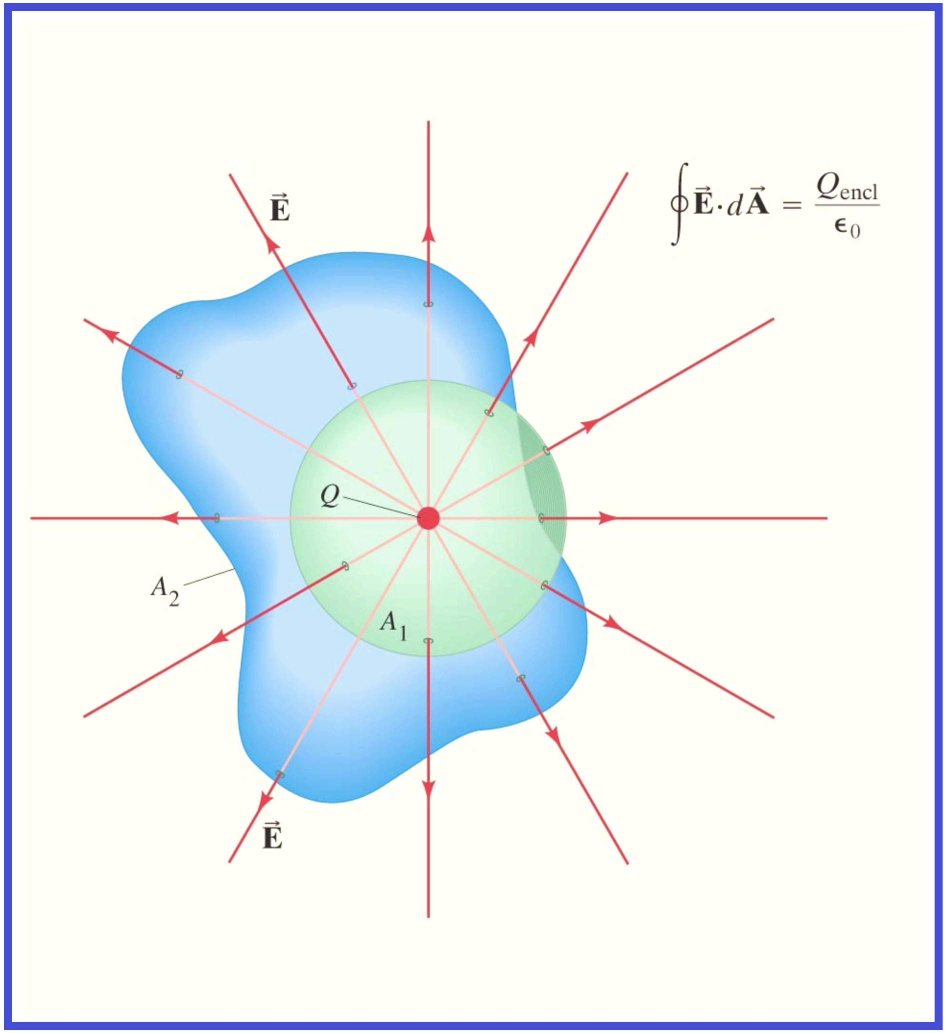
Reference Book: “Physics for Scientists and Engineers” by R. A. Serway & J. W. Hewett

Similar Book: “Physics for Scientists&Engineers” by

D.C.Giancoli

Chapter 24: Gauss's Law



Outline of Chapter 24

- **Electric Flux**
- ***Gauss's Law***
- **Applications of *Gauss's Law***
- **Experimental Basis of *Gauss's* & *Coulomb's Laws***

Gauss' s Law

- **Gauss' s Law** can be used as ***an alternative procedure for calculating electric fields***.
- It is based on **the inverse-square behavior of the electric force** between point charges.
- It is convenient in calculations of **the electric field of *highly symmetric charge distributions***.
- ***Gauss' s Law*** is important in understanding and verifying the properties of conductors in electrostatic equilibrium.

Johann Carl Friedrich Gauss

(1736–1806, Germany)

- Mathematician, Astronomer & Physicist.
- Sometimes called the

  "Prince of Mathematics" (?)
- A child prodigy in math and science

- **Age 3:** He informed his father of a mistake in a payroll calculation & gave the correct answer!!
- **Age 7:** His teacher gave the problem of summing all integers 1to 100 to his class to keep them busy. Gauss quickly wrote the correct answer **5050** on his slate!!
- Whether or not you believe all of this, it is 100% true that he

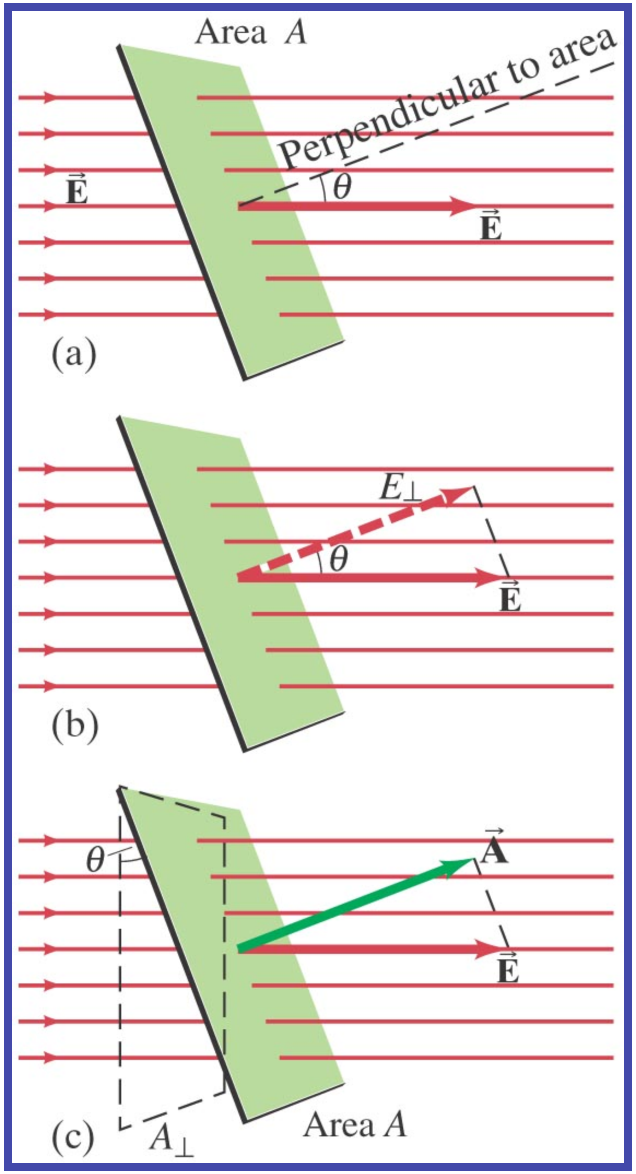
Made a HUGE number of contributions to Mathematics, Physics, & Astronomy!!

Johann Carl Friedrich Gauss

Genius! **He made a *HUGE* number of contributions to Mathematics, Physics, & Astronomy** Some are:

1. Proved **The Fundamental Theorem of Algebra**, that every polynomial has a root of the form **a+bi.**
2. Proved **The fundamental Theorem of Arithmetic**, that every natural number can be represented as a product of primes in only one way.
3. Proved that every number is the sum of at most 3 triangular numbers.
4. Developed the method of least squares fitting & many other methods in statistics & probability.
5. Proved many theorems of integral calculus, including the divergence theorem (when applied to the **E** field, it is what is called ***Gauss's Law***).
6. Proved many theorems of number theory.
7. Made many contributions to the orbital mechanics of the solar system.
8. Made many contributions to Non-Euclidean geometry
9. One of the first to rigorously study the Earth's magnetic field

Section 24.1: Electric Flux



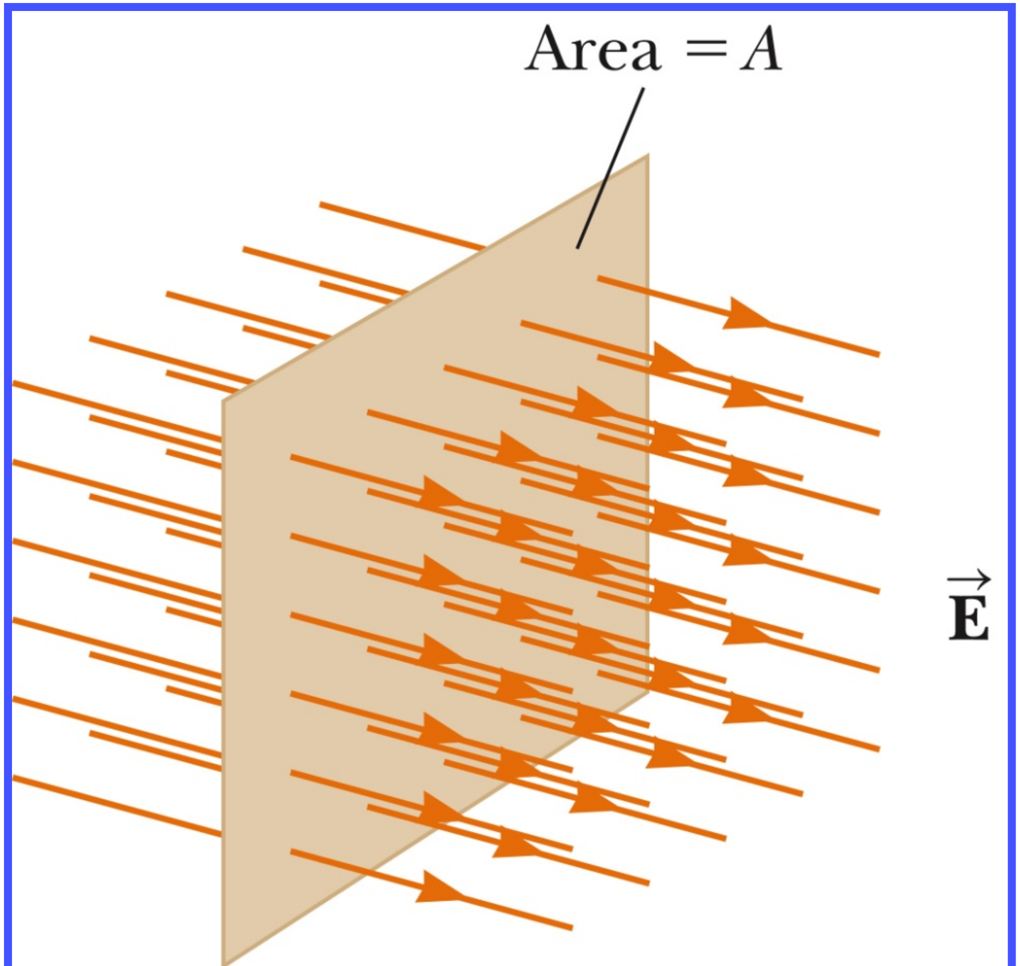
- The **Electric Flux** Φ_E through a cross sectional area **A** is proportional to the total number of field lines crossing the area & is ***defined as*** (***constant* E *only!***):

$$\Phi_E = \vec{E} \cdot \vec{A}.$$

$$\Phi_E = E_\perp A = E A_\perp = E A \cos\theta,$$

- The ***Electric Flux*** is defined as the product of the magnitude of the electric field **E** & the surface area, **A**, perpendicular to the field. $\mathbf{\Phi_E = EA}$

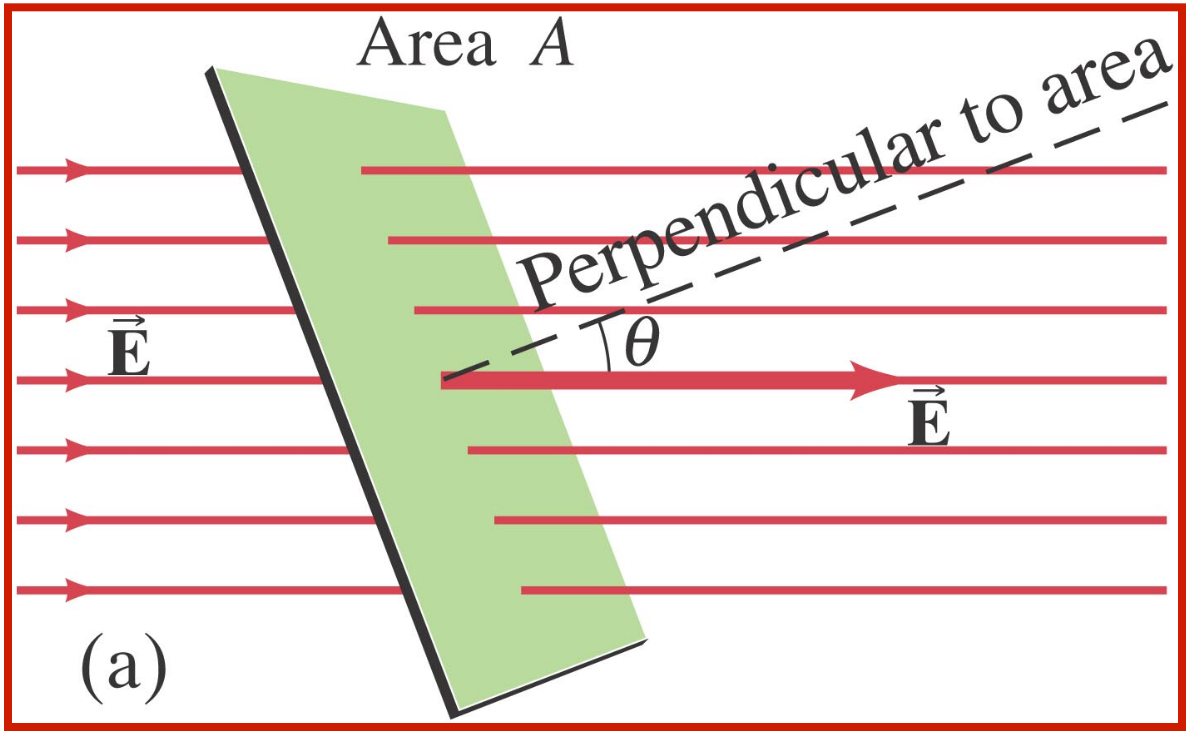
Flux Units:
$N \cdot m^2/C$



Example: Electric flux.

•Calculate the electric flux through the rectangle shown.

The rectangle is **10 cm** by **20 cm**. **E = 200 N/C**, & **θ = 30°.**



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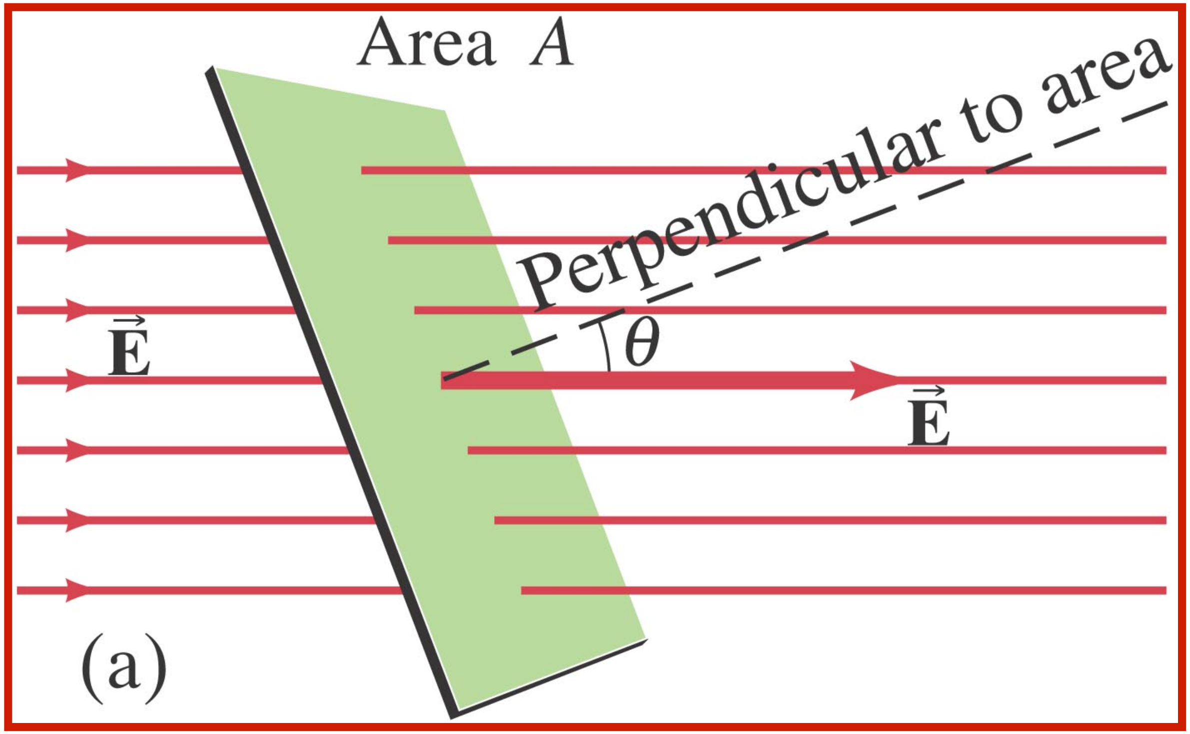


Solution

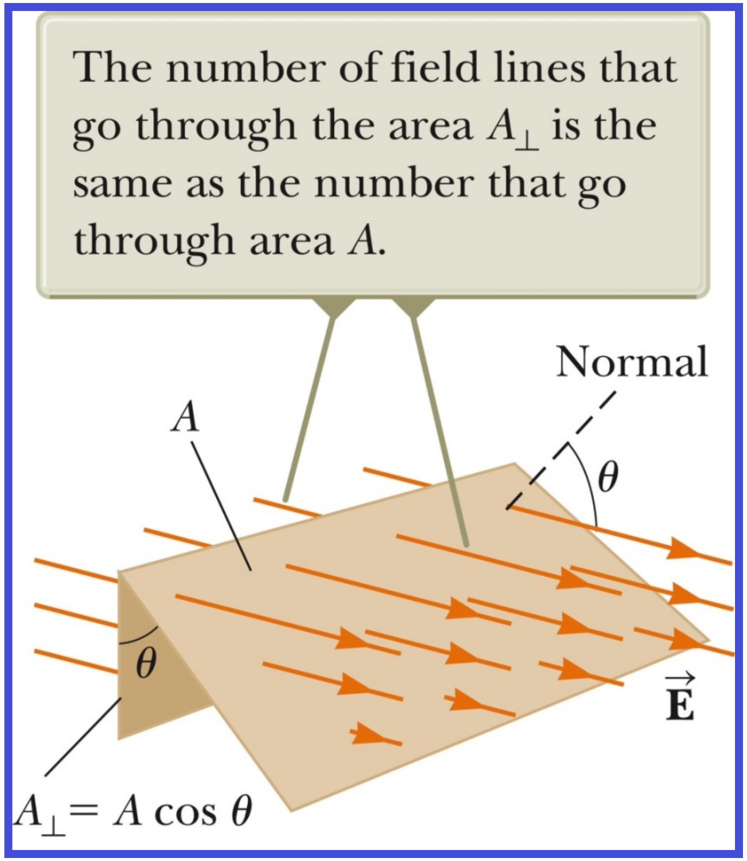
$$\Phi_E = EA\cos(30^0), \quad A = (0.02)m^2$$

$$\Phi_E = (200)(0.02)\cos(30^0) = 3.46 \text{ N m}^2/\text{C}$$

Electric Flux, General Area

- The electric flux is proportional to the number of electric field lines penetrating some surface.
- The field lines may make some angle **θ** with the perpendicular to the surface.
- Then

$$\Phi_E = EA\cos\theta$$



Electric Flux: *Interpreting Its Meaning*

$$\Phi_E = EA\cos\theta$$

- Φ_E is a maximum when the surface is perpendicular to the field: $\theta = 0°$
- Φ_E is zero when the surface is parallel to the field: $\theta = 90°$
- If the field varies over the surface, $\Phi_E = EA\cos\theta$ is valid for only a small element of the area.

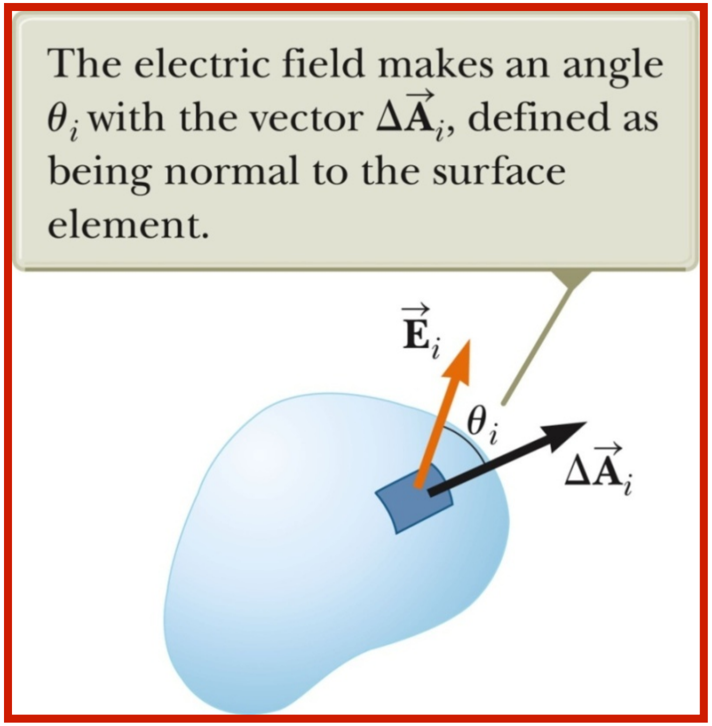
Electric Flux, General

- In the more general case, look at a small area element.
- In general, $\Delta\Phi_E$ becomes

$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i = \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i$$

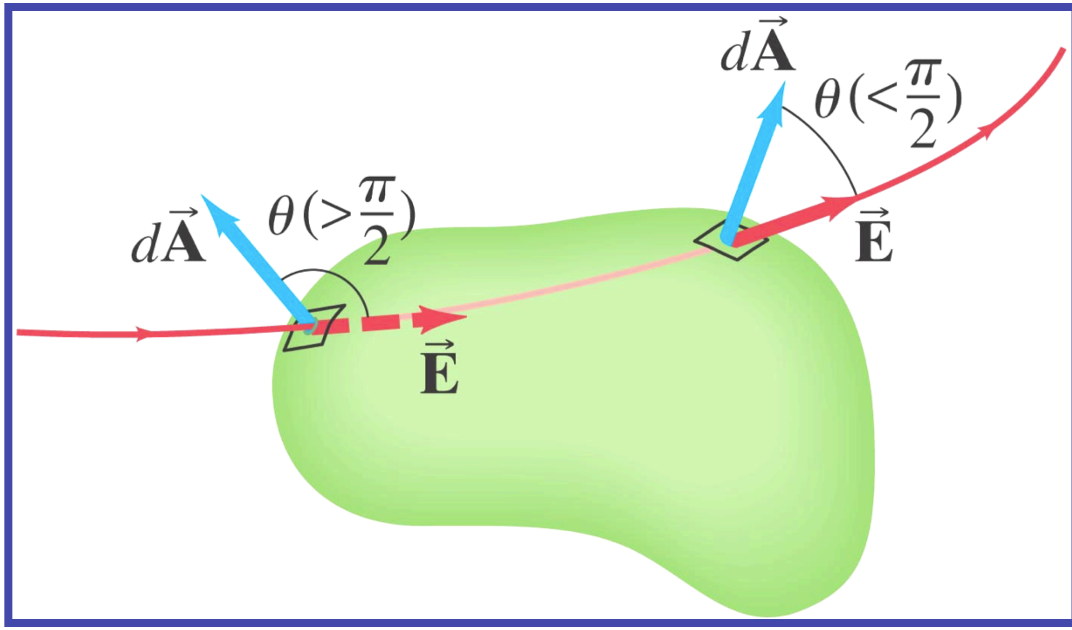
$$\Phi_E = \lim_{\Delta A_i \to 0} \sum E_i \cdot \Delta A_i$$

$$\Phi_E = \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$



- The surface integral means that the integral must be evaluated over the surface in question. In general, the value of the flux will depend both on the field pattern & on the surface.

- The **Electric Flux Φ_E** through a **closed surface** is ***defined*** as the closed surface integral of the scalar (dot) product of



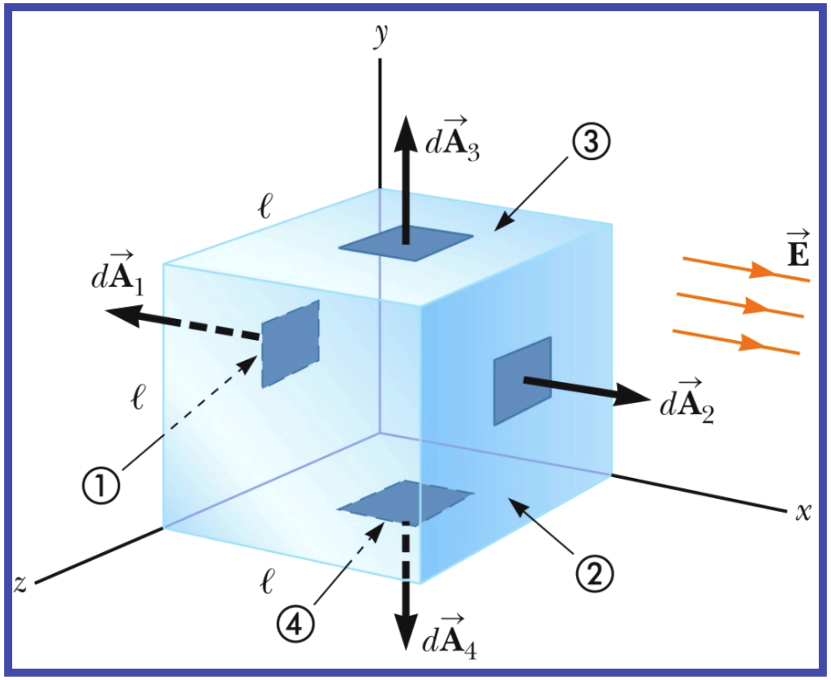
al surface area **dA**.

$$\Phi_E \approx \sum_{i=1}^{n} \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i ,$$

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}},$$

Flux Through a Cube, Example 24.1

Consider a uniform electric field $\vec{\mathbf{E}}$ oriented in the x direction in empty space. A cube of edge length ℓ is placed in the field, oriented as shown in Figure 24.5. Find the net electric flux through the surface of the cube.



- The field lines pass through 2 surfaces perpendicularly & are parallel to the other 4 surfaces.
- For face **1**, $\Phi_E = -E\ell^2$
- For face **2**, $\Phi_E = E\ell^2$
- For the other sides, **E = 0**. Therefore,

$$\Phi_E \text{ (total)} = 0$$

Section 24-2: Gauss’ s Law

- The net number of **E** field lines through a closed surface ***is proportional to the charge enclosed***, & to the flux, which gives

Gauss’ s Law:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\epsilon_0},$$

- This is a ***VERY POWERFUL*** method, which can be used to find the **E** field especially in situations where there is a ***high degree of symmetry.*** It can be shown that, of course, the **E** field calculated this way is identical to that obtained by **Coulomb’ s Law**. Often, however, in such situations, it is often ***MUCH EASIER*** to use **Gauss’ s Law** than to use **Coulomb’ s Law.**

For a **Point Charge:**

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\, dA = E \oint dA = E(4\pi r^2).$$

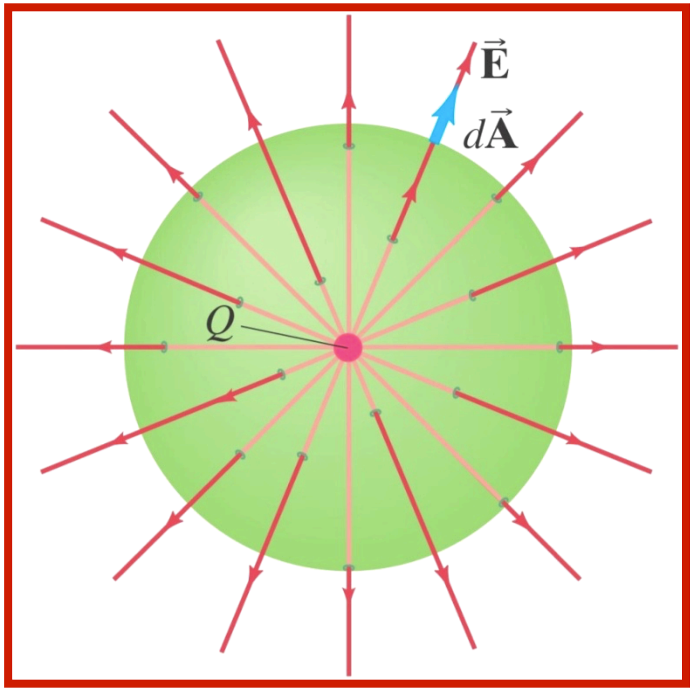
Therefore,

$$\frac{Q}{\epsilon_0} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(4\pi r^2).$$

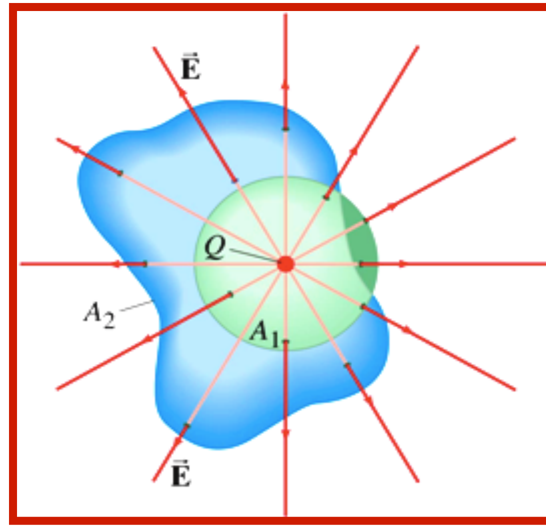
Of course, solving for **E** gives the same result as

Coulomb's Law:

$$E = \frac{Q}{4\pi\epsilon_0 r^2}.$$



- Using **<u>Coulomb's Law</u>** to evaluate the integral of the field of a point charge over the Surface of a sphere of surface area **A_1** surrounding the charge gives:



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,dA = \frac{Q}{4\pi\epsilon_0 r^2}\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}.$$

- Now, consider a point charge surrounded by an
- ***<u>Arbitrarily Shaped</u>*** closed surface of area **A_2**. It can be
- seen that ***the same flux passes through*** **A_2** as passes
- through the spherical surface **A_1**. So, ***<u>This Result is</u>***
- ***<u>Valid for any Arbitrarily Shaped Closed Surface.</u>***

- The power of this is that you (the problem solver) can choose the closed surface

(**called a *Gaussian Surface***)

at your convenience!

- In cases where there is a large amount of symmetry in the problem,

this will simplify the calculation considerably,

as we' ll see.

Now, consider a **Gaussian Surface** enclosing several point charges. We can use the superposition principle to show that:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \left(\Sigma \vec{\mathbf{E}}_i\right) \cdot d\vec{\mathbf{A}} = \Sigma \frac{Q_i}{\epsilon_0} = \frac{Q_{\text{encl}}}{\epsilon_0}.$$

So

Gauss's Law is valid for ANY Charge Distribution.

Note, though, that it only refers to the field due to charges ***within the Gaussian surface*** charges outside the surface will also create fields.

Conceptual Example: Flux from Gauss' s law.

Consider the 2 Gaussian surfaces, $\mathbf{A_1}$ & $\mathbf{A_2}$, as shown. The only charge present is the charge Q at the center of surface $\mathbf{A_1}$. Calculate the net flux through each surface, $\mathbf{A_1}$ & $\mathbf{A_2}$.