Applications of Gauss's Law

Example

Spherical Conductor

A thin spherical shell of radius $\mathbf{r_0}$ possesses a total net charge **Q** that is uniformly distributed on it. **Calculate** the electric field at points.

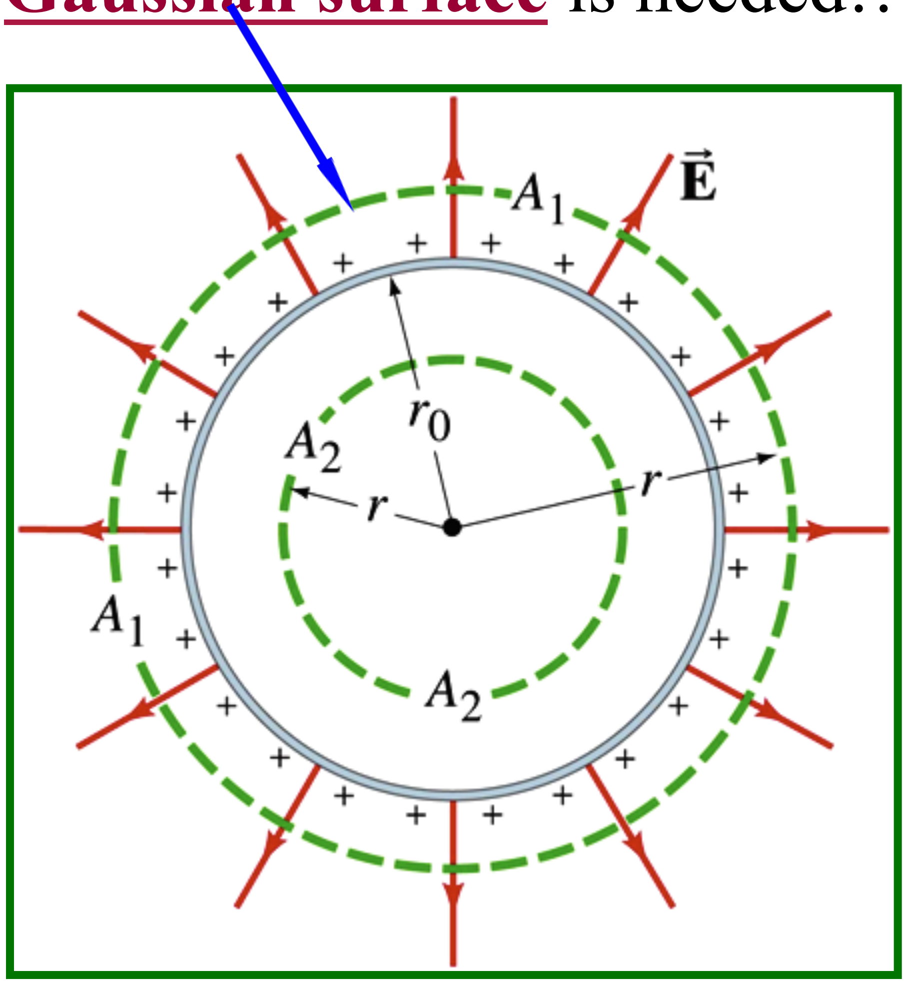
(a) Outside the shell ($\mathbf{r > r_0}$)

and

(b) Inside the shell ($\mathbf{r < r_0}$)

(c) What if the conductor were a solid sphere?

By ***symmetry***, clearly a ***SPHERICAL*** **Gaussian surface** is needed!!



Example

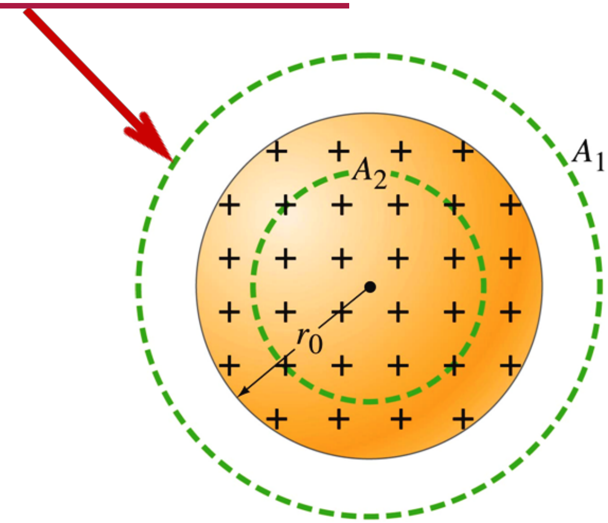
Solid Sphere of Charge

An electric charge **Q** is distributed uniformly throughout a nonconducting sphere, radius r_0. Calculate the electric field

(a) Outside the sphere ($r > r_0$)

& **(b)** Inside the sphere ($r < r_0$).

By *symmetry*, clearly a ***SPHERICAL*** **Gaussian surface** is needed!!



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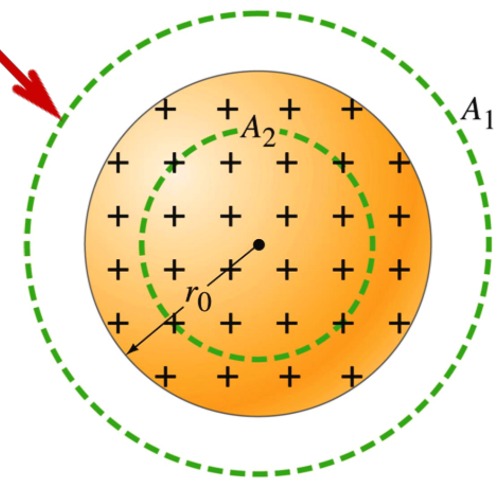

Results

Outside ($\mathbf{r > r_0}$):

$$\mathbf{E = Q/(4\pi\varepsilon_0 r^2)}$$

Inside ($\mathbf{r < r_0}$):

$$\mathbf{E = (Qr)/(4\pi\varepsilon_0 r_0^3)}$$

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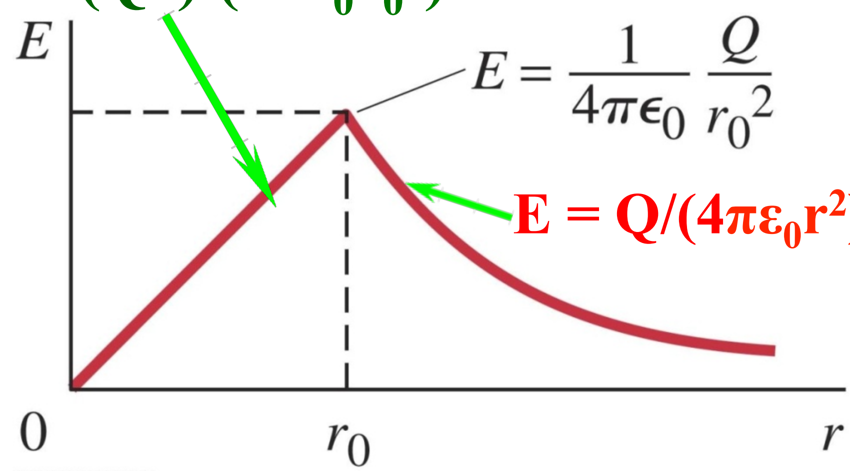


$\mathbf{E = Q/(4\pi\varepsilon_0 r^2)}$

Note!! **E** inside has a ***very different*** **r** dependence than **E** outside!

Example: Nonuniformly Charged Solid Sphere

A solid sphere of radius $\mathbf{r_0}$ contains total charge **Q**. It's volume charge density is nonuniform & given by

$$\rho_E = \alpha r^2$$

where **α** is a constant.

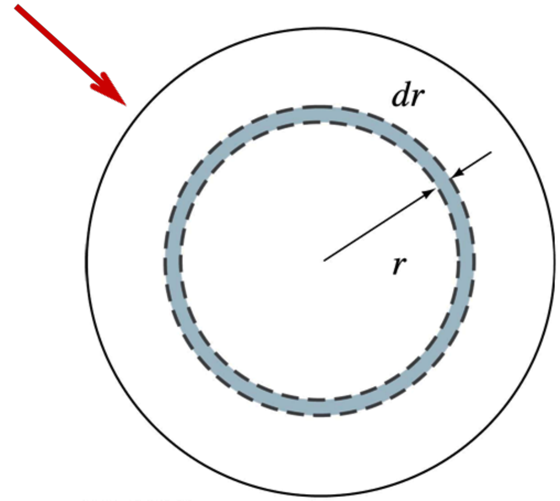
Calculate:

(a) The constant **α** in terms of **Q** & $\mathbf{r_0}$.

(b) The electric field as a function of **r** ***outside*** the sphere.

(c) The electric field as a function of **r** ***inside*** the sphere.

By ***symmetry***, clearly a ***SPHERICAL*** **Gaussian surface** is needed!!



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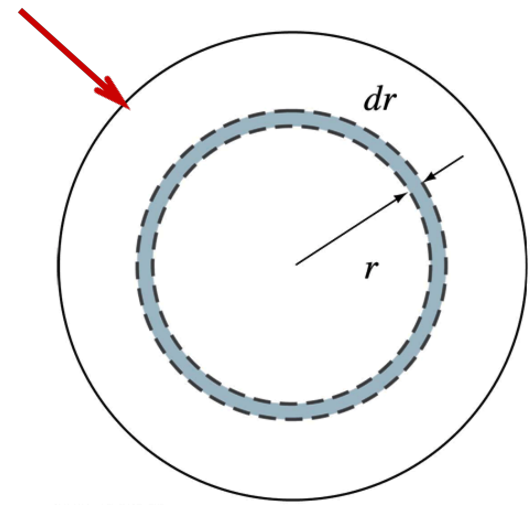


Results

$\alpha = (5Q)/(4\pi_0 r_0^5)$

Outside ($r > r_0$):

$E = Q/(4\pi\varepsilon_0 r^2)$

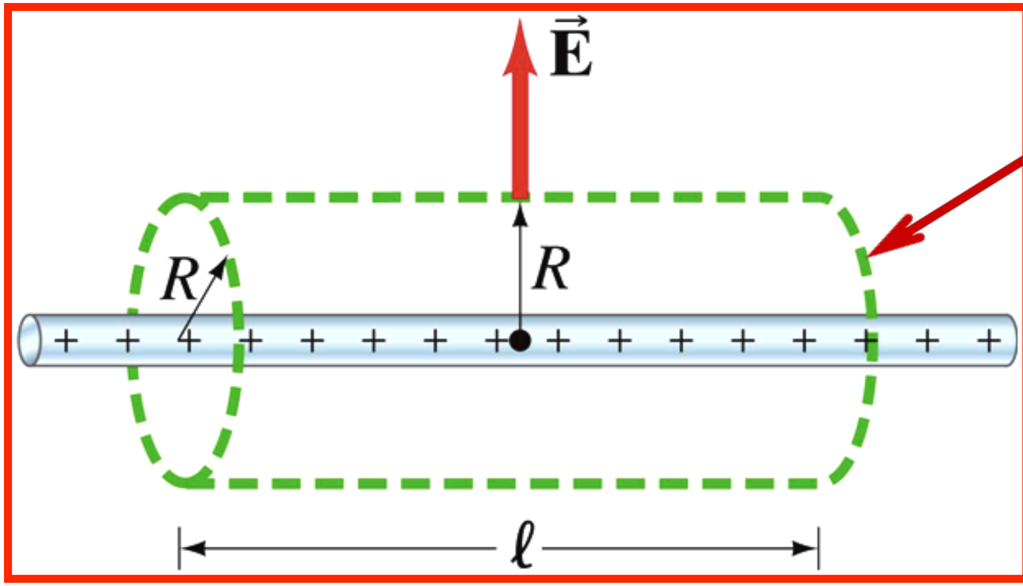
Inside ($r < r_0$):

$E = (Qr^3)/(4\pi\varepsilon_0 r_0^5)$

Example

Long Uniform Line of Charge

- A very long straight (effectively, ℓ ? ?) wire of radius **R** has a uniform positive charge per unit length, **λ**. Calculate the electric field at points near (& outside) the wire, far from the ends.



By ***symmetry***, clearly a ***Cylindrical*** **Gaussian Surface** is needed!!

Note!! **E** for the wire has a ***very different*** **R** dependence than **E** for the sphere!

Results: $E = \lambda/(2\pi\varepsilon_0 R)$

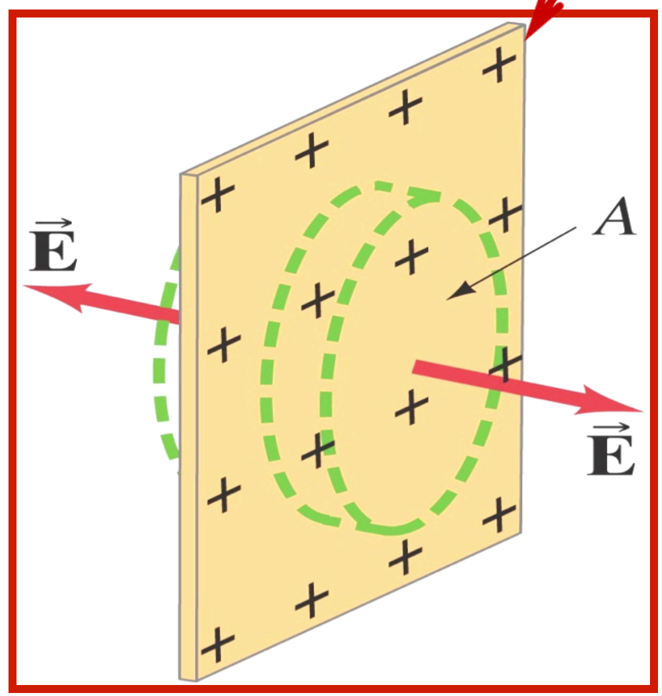
$= 2k\lambda/R$

Example

Infinite Plane of Charge

Charge is distributed uniformly, with a surface charge density **σ** [= charge per unit area = (**dQ/dA**)] over a very large but very thin non-conducting flat plane surface. Calculate the electric field at points near the plane.

A *Cylindrical* **Gaussian surface** was chosen, but here, the shape of the Gaussian surface doesn't matter!! ***The result is independent of that choice!!!***



Example

Infinite Plane of Charge

Charge is distributed uniformly, with a surface charge density **σ** [= charge per unit area = (**dQ/dA**)] over a very large but very thin non-conducting flat plane surface. Calculate the electric field at points near the plane.

Results: $E = \sigma / (2\varepsilon_0)$

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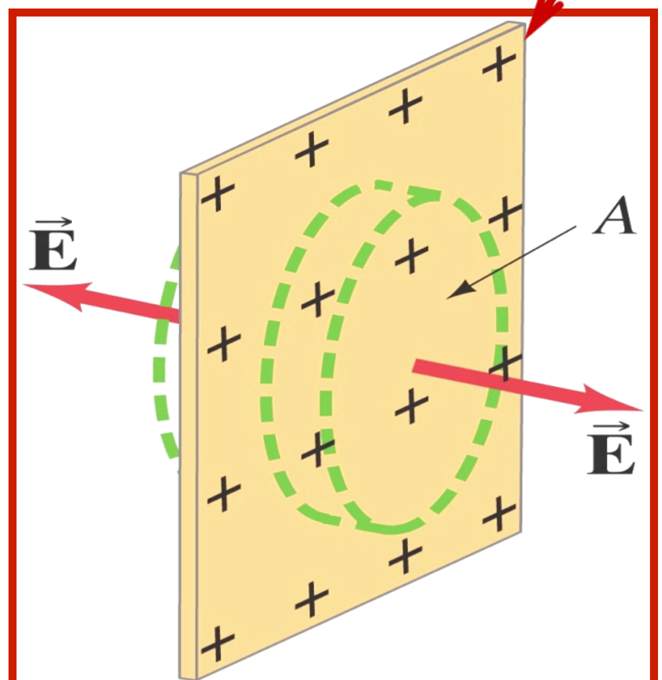


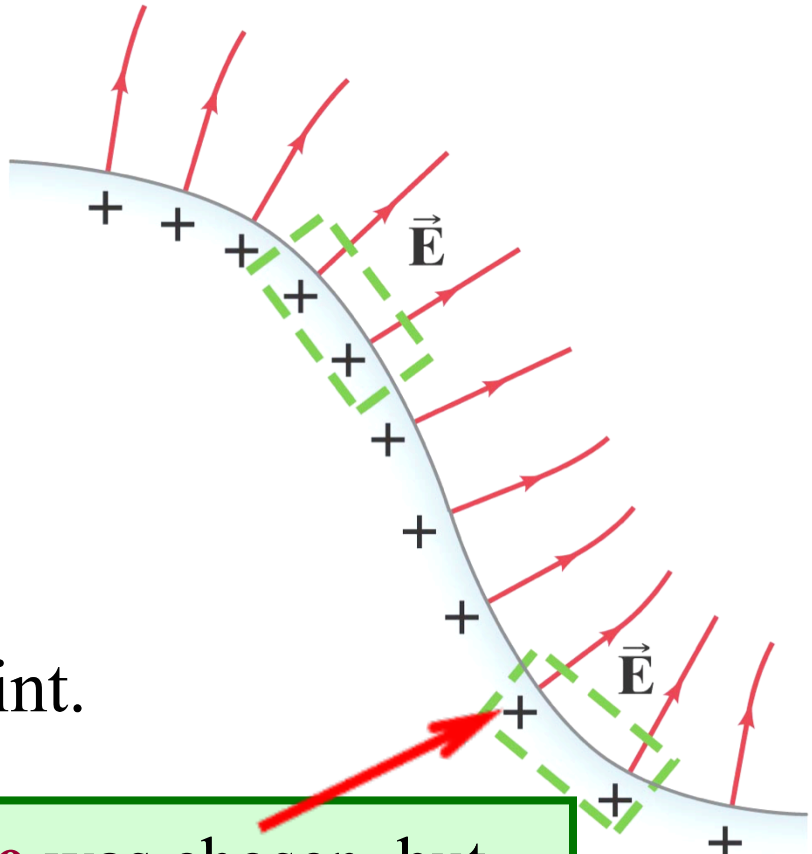
Example

Electric Field Near any Conducting Surface

Show that the electric field just outside the surface of any good conductor of arbitrary shape is given by

$$\mathbf{E} = \sigma/\varepsilon_0$$

where σ is the surface charge density on the surface at that point.

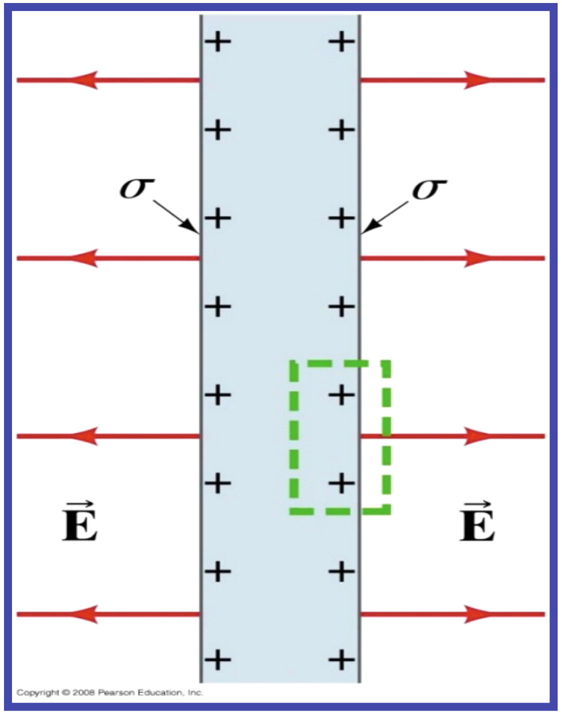


A ***Cylindrical*** **Gaussian Surface** was chosen, but here, the shape of the Gaussian surface doesn't matter!! ***The result is independent of that choice!!!***

- The difference between the electric field ***outside*** a **conducting plane** of charge & ***outside*** a **nonconducting plane** of charge can be thought of in 2 ways:

1. The **E field** ***inside*** the **conductor** is zero, so the flux is all through one end of the Gaussian cylinder.

2. The **nonconducting** plane has a total surface charge density **σ**, but the **conducting** plane has a charge density **σ** on each side, effectively giving it twice the charge density.



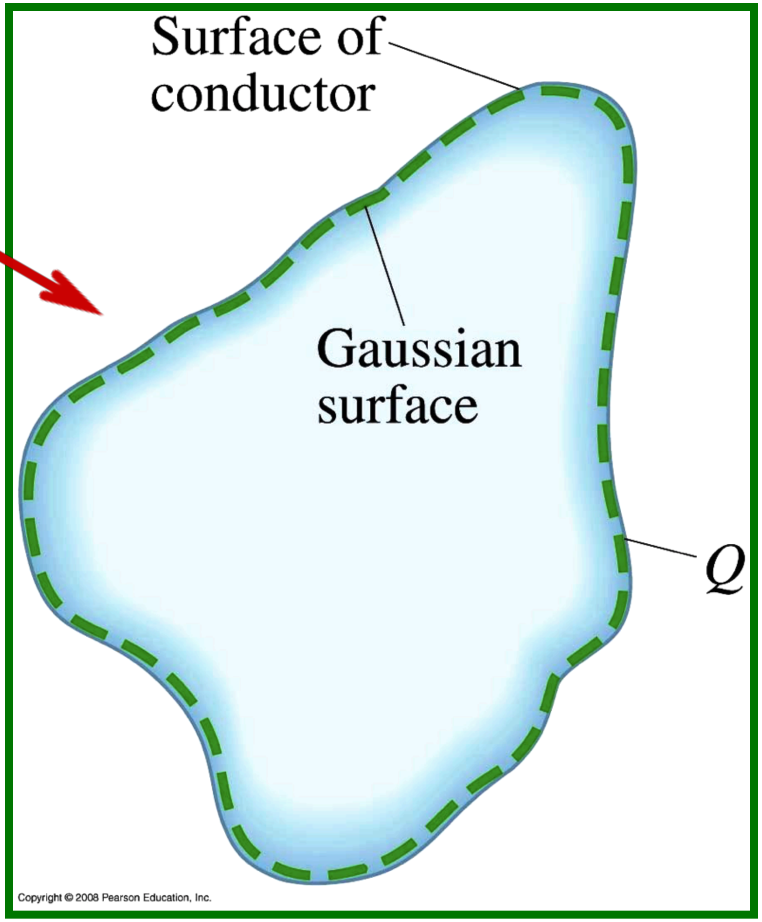
A thin, flat charged conductor with surface charge density **σ** on each surface. For the conductor as a whole, the charge density is

$$\sigma' = 2\sigma$$

Consider a ***static conductor*** of any shape with total charge **Q**.

See figure

Choose a **Gaussian surface** (dashed in figure) just below & infinitesimally close to the conductor surface. We just said that the electric field inside a static conductor must be zero.



By **Gauss's Law**, since the electric field inside the surface is zero, **there must be no charge enclosed. So,**

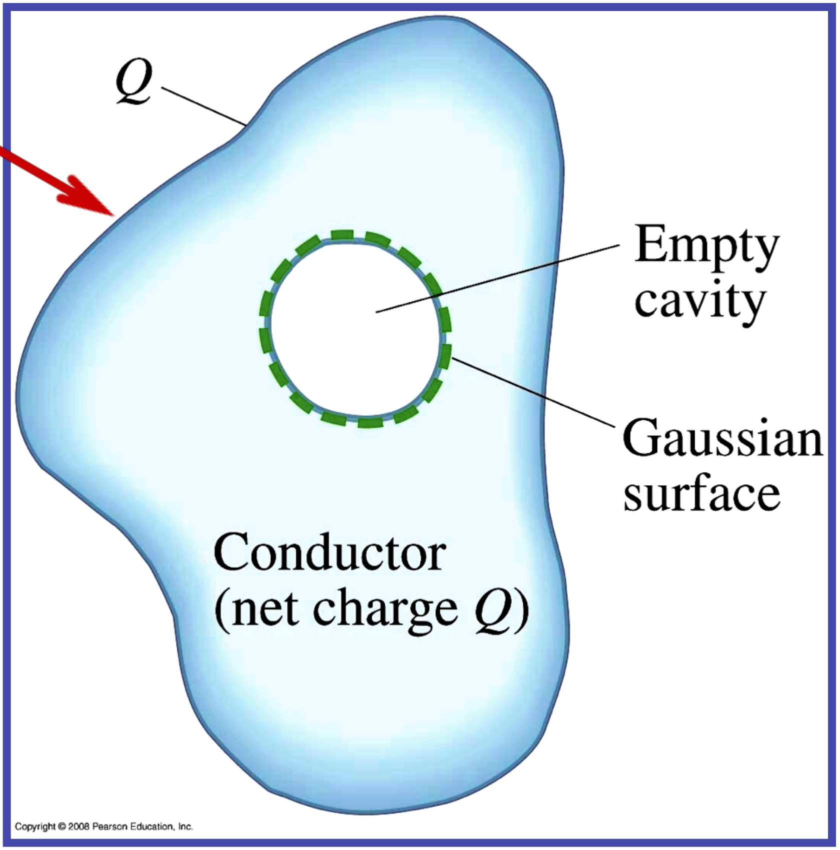
All charge on a static conductor must be on the surface.

Now, consider a ***static conductor*** of any shape with total charge **Q** & an ***empty cavity*** inside

See figure

Choose a Gaussian surface (dashed in the figure) just outside & below, infinitesimally close to the surface of the cavity. Since it is inside the conductor, there can be no electric field there. So, by **Gauss's Law**, there can be no charge there, so there is no charge on the cavity surface &

All charge on a static conductor must be on it's OUTER surface



Outline of Procedure for Solving Gauss' s Law Problems:

1. **Identify the symmetry**, & choose a Gaussian surface that takes advantage of it (with surfaces along surfaces of constant field).
2. **Sketch** the surface.
3. **Use the symmetry** to find the direction of **E**.
4. **Evaluate** the flux by integrating.
5. **Calculate** the enclosed charge.
6. **Solve** for the field.

Summary of the Chapter

- **Electric Flux:** $\Phi_E = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}.$
- **Gauss's Law:** $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{encl}}}{\epsilon_0},$
- **Gauss's Law:** A method to calculate the electric field. It is most useful in situations with a high degree of symmetry.

Gauss's Law: ***Applies in all situations***

- So, it is more general than **Coulomb's Law**. As we'll see, it also applies when charges are moving & the electric field isn't constant, but depends on the time. As we'll see

It is one of the basic equations of Electromagnetism.

It is one of the **4** ***Maxwell's Equations of Electromagnetism***.

Recall the Theme of the Course!