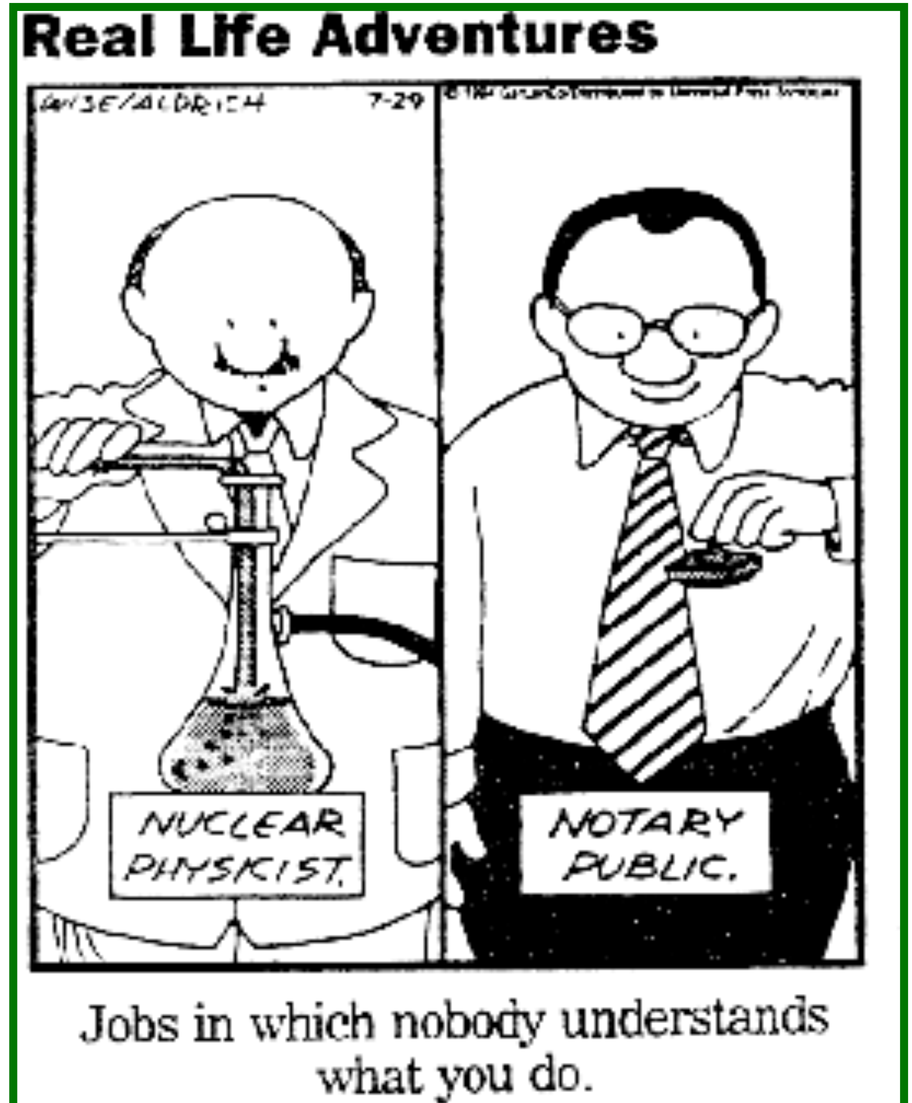
Chapter 25: E. Potential

Electric Field Determined from V



- If the **Electric Field** **E** is known, the **Potential** **V** can be obtained by integrating. Inverting this process, if the **Potential** **V** is known, the **Field** **E** can be obtained by differentiating:

$$E_\ell = -\frac{dV}{d\ell}.$$

This is a ***vector differential equation***.

In Cartesian component form it is:

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad E_z = -\frac{\partial V}{\partial z}.$$

Electrostatic Potential Energy:
The Electron Volt

- The **Electric Potential Energy U** of a charge **q** in an **Electric Potential** **V** is **U = qV**.
- To find the **Electric Potential Energy U** of 2 charges Q_1 & Q_2, imagine bringing each in from infinitely far away.
- The first one takes no work, since there is no external electric field. To bring in the 2nd one, work must be done, since it is moving in the **Electric Field** of the first one; this means that the **Electric Potential Energy** **U** of the pair is:

$$U = Q_2 V = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r_{12}}.$$

- Often, especially for very small individual particles like the electron, it is convenient to use units other than **Joules** to measure electrical energies.
  The ***Electron Volt*** is an often useful unit for this:
- **1 *Electron Volt* (eV)** is ***defined*** as the energy gained by an electron moving through a potential difference of 1 Volt:

$$\mathbf{1\ eV = 1.6 \times 10^{-19}\ J.}$$

.

Example: “Disassembling” a H atom

- Calculate the work needed to “disassemble” a hydrogen atom. Assume that the proton & electron are initially separated by a distance equal to the “average” radius of the hydrogen atom in its ground state. This distance is

$$\mathbf{0.529 \times 10^{-10}\ m}$$

- This distance is called

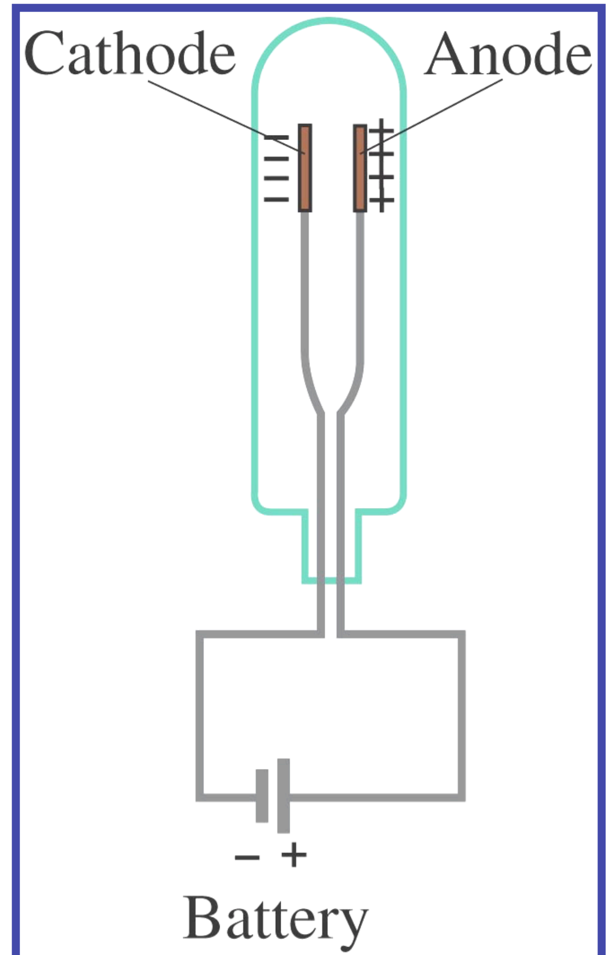
“The Bohr Radius”.

- Assume also that the proton & the electron end up an infinite distance apart from each other.

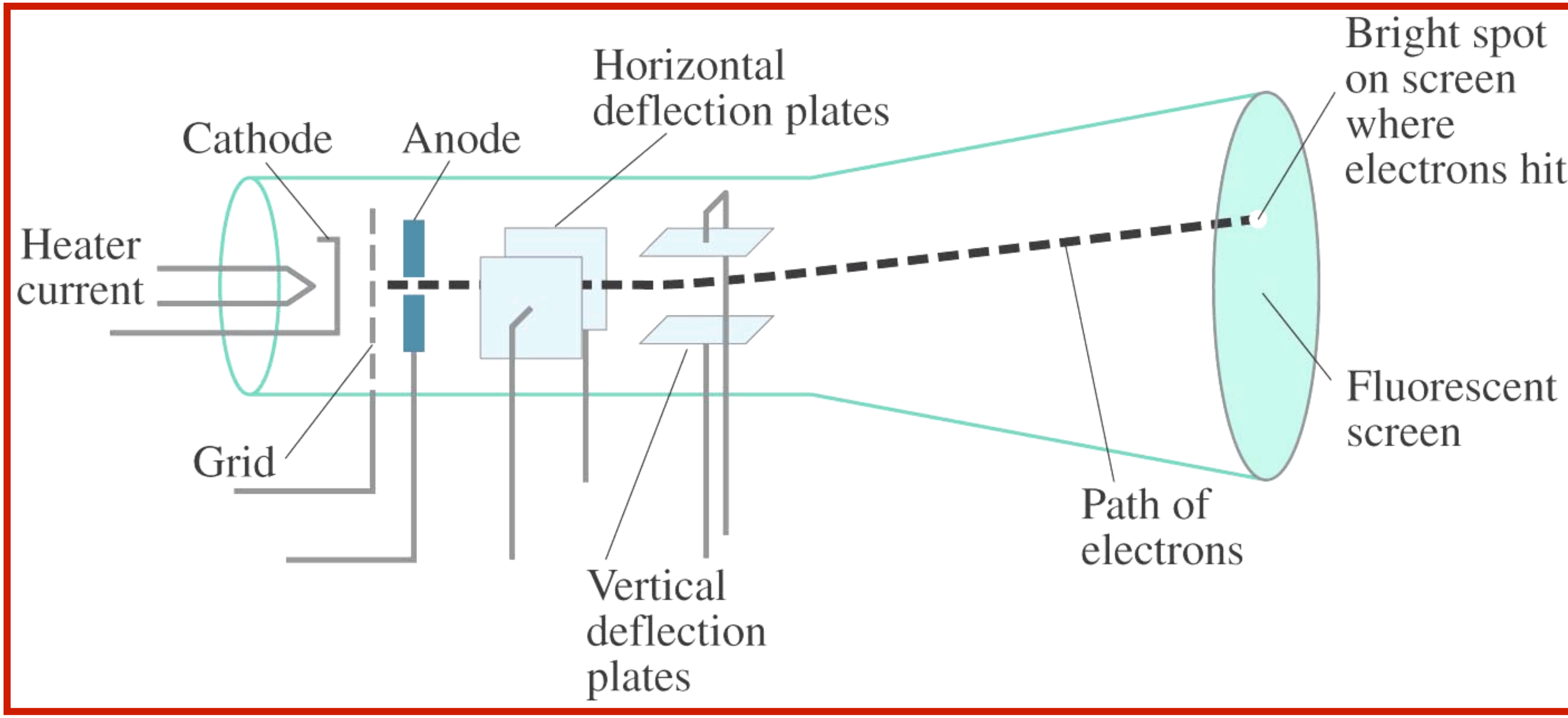
Some Applications

Cathode Ray Tube: TV & Computer Monitors, Oscilloscope

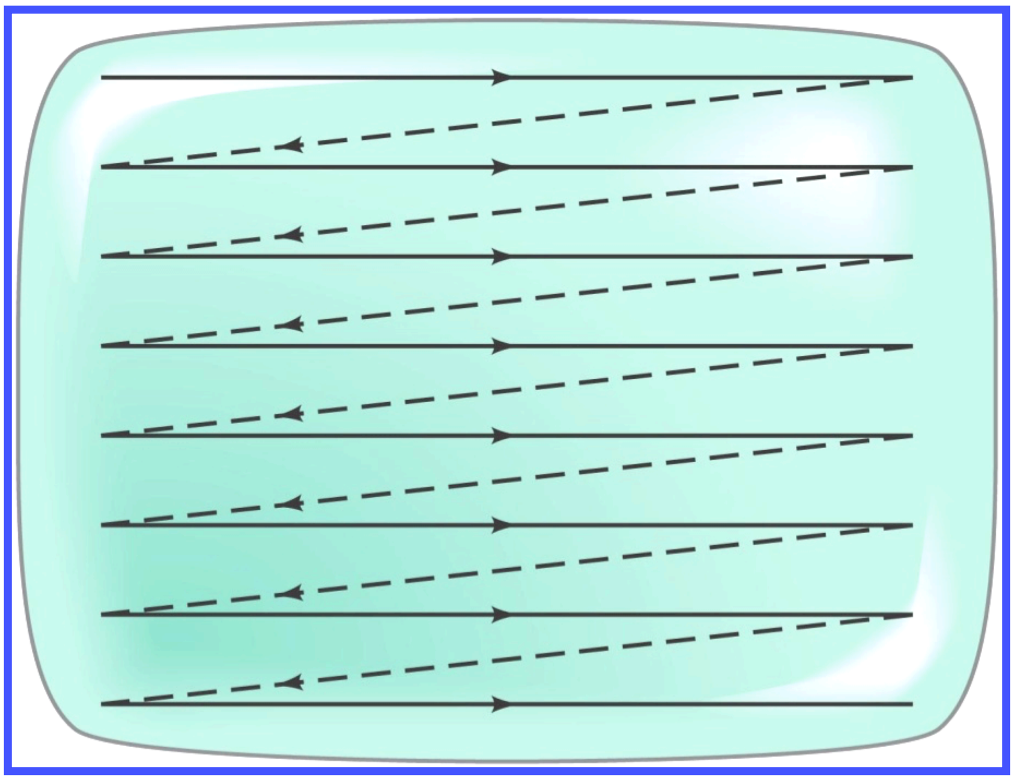
A cathode ray tube contains a wire cathode that, when heated, emits electrons. A voltage source causes the electrons to travel to the anode.



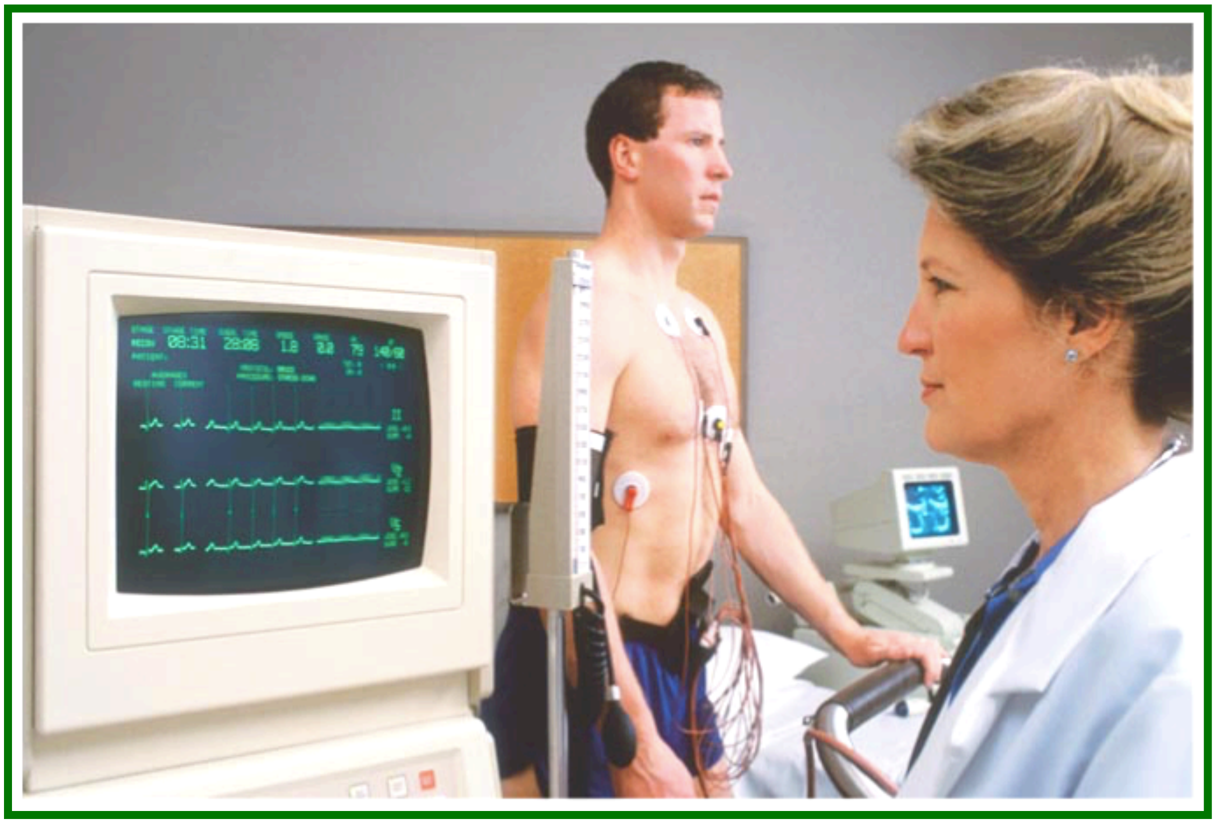
- The electrons can be steered using electric or magnetic fields.



Old fashioned televisions and computer monitors (not LCD or plasma models) have a large cathode ray tube as their display. Variations in the field steer the electrons on their way to the screen.



An oscilloscope displays an electrical signal on a screen, using it to deflect the beam vertically while it sweeps horizontally.



Chapter Summary

- ***Electric potential is potential energy per unit charge:***

$$V_{ba} = \Delta V = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{W_{ba}}{q}.$$

- ***Potential difference between two points is:***

$$V_{ba} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{\ell}}.$$

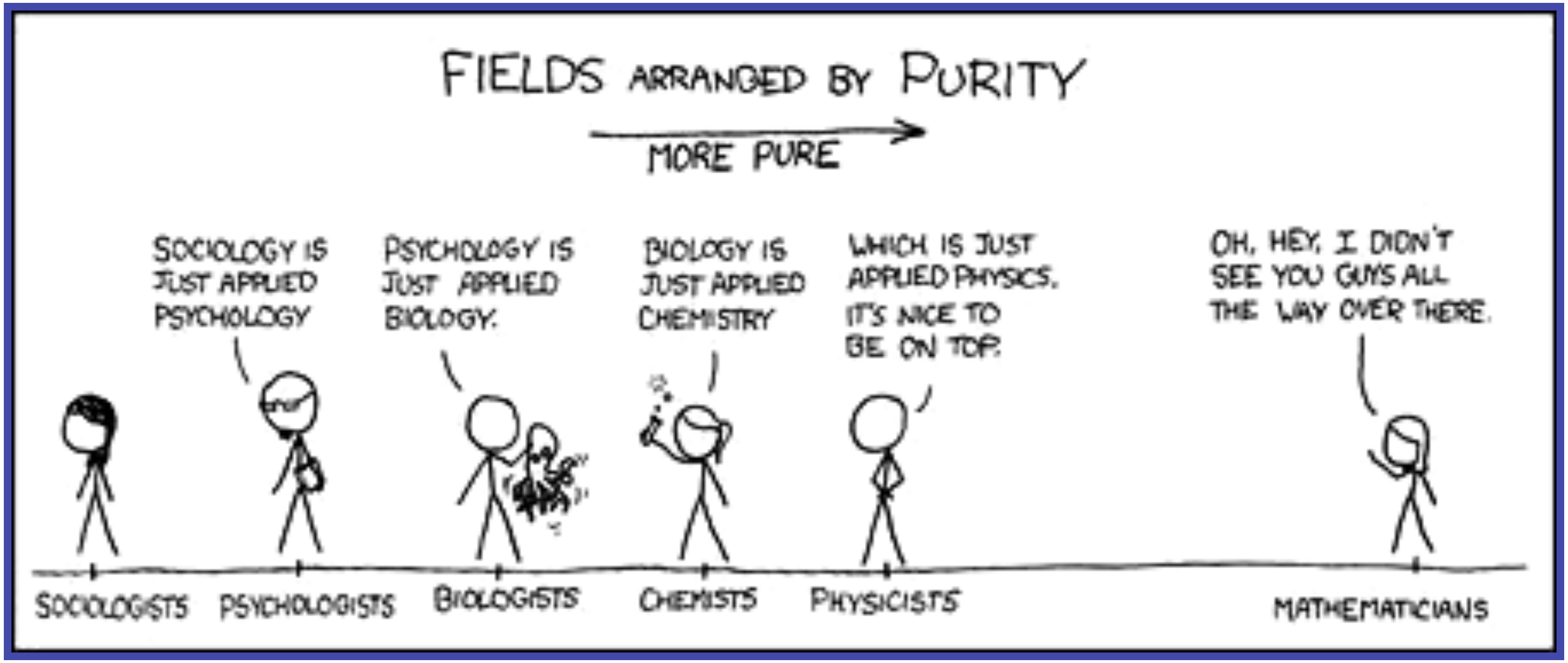
- ***Potential due to a point charge is:***

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}. \qquad \left[\begin{array}{c}\text{single point charge;} \\ V = 0 \text{ at } r = \infty\end{array}\right]$$

- ***An Equipotential*** is a line or surface along which potential is constant.
- An **Electric Dipole potential** is proportional to $\mathbf{1/r^2}$.
- To find the **E** field from the potential **V**, use:

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad E_z = -\frac{\partial V}{\partial z}.$$

Potential Due to An Arbitrary Charge Distribution



Why does *Mathematicians* on the far away edge??

Potential Due to An Arbitrary Charge Distribution

The potential due to an arbitrary charge distribution can be expressed as a sum or integral (if the distribution is continuous):

$$V_a = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{Q_i}{r_{ia}}.$$

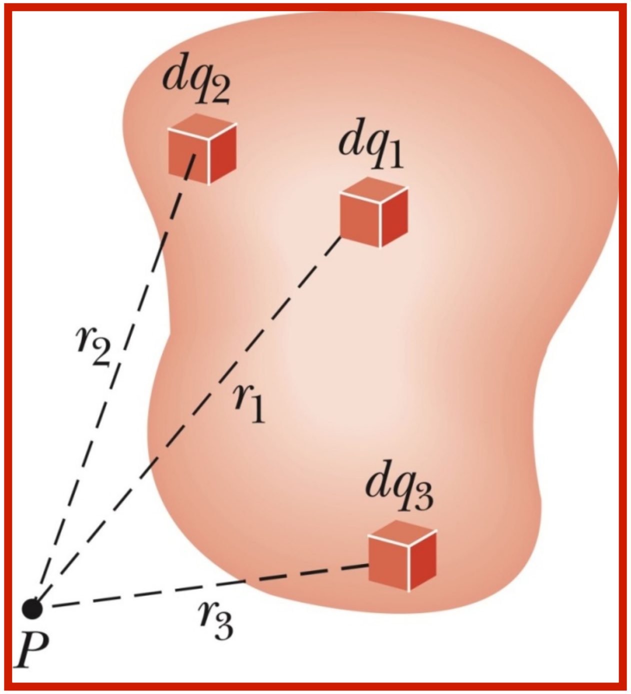
or

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

More Details: Electric Potential for a Continuous Charge Distribution

Method 1

- The charge distribution is known.
- Consider a small charge element **dq.**
  Treat it as a point charge.
- The potential at some point due to this charge element is then:



$$dV = k_e \frac{dq}{r}$$

- To find the total potential, this must be integrated to include the contributions from\ all of the charge elements. This value for **V** uses the reference of **V = 0** when **P** is infinitely far away from the charge distribution.

$$V = k_e \int \frac{dq}{r}$$

V for a Continuous Charge Distribution

Method 2

- If the electric field **E** is already known from other considerations, the potential **V** can be calculated using the original definition:

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- If the charge distribution has sufficient symmetry, first find the field **E** from **Gauss' Law** & then find
the potential difference **ΔV** between any 2 points using the above relation.

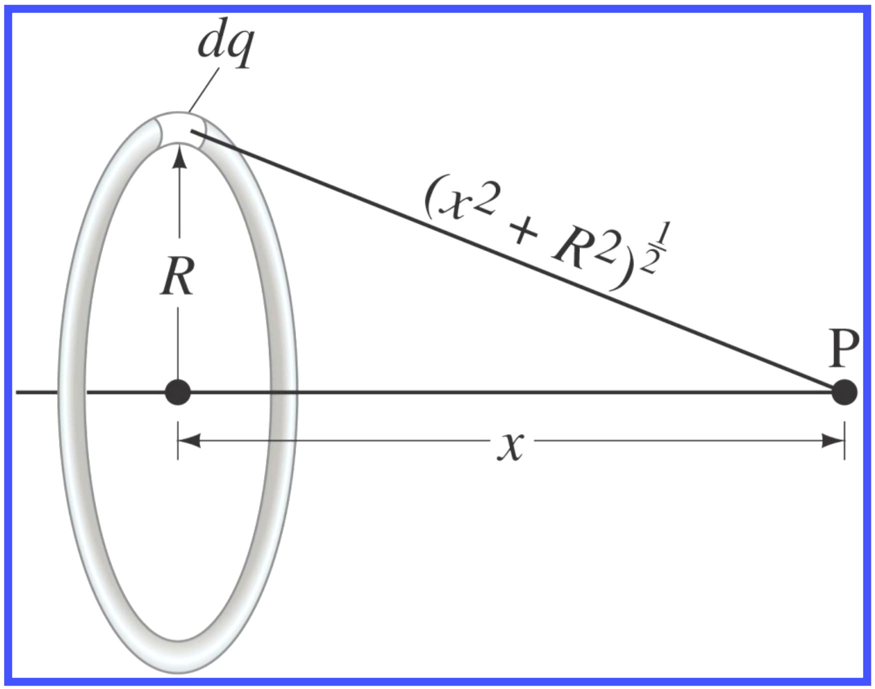
(Choose **V = 0** at some convenient point)

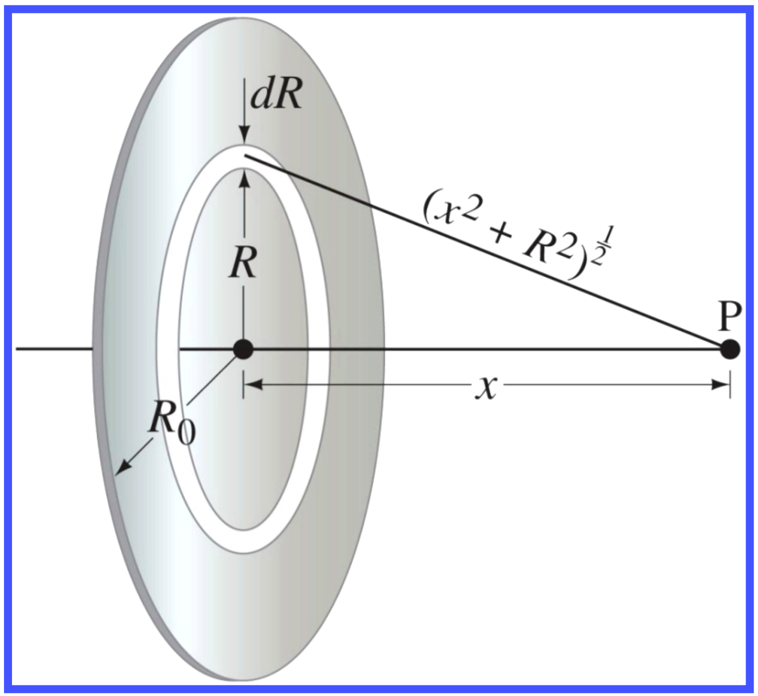
Examples: E for a Ring & for a Disk

Use the known **Electric Potential V** to calculate the **Electric Field E** at point **P** on the axis of

(a) A circular ring of charge.

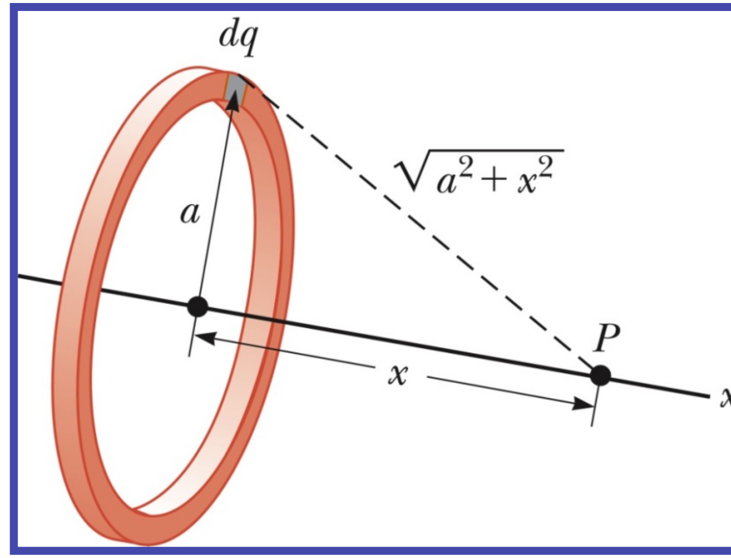
(b) A uniformly charged disk.





V for a Uniformly Charged Ring

- **P** is on the perpendicular central axis of the uniformly charged ring .
- Symmetry means that all charges on the ring are the same distance from Point **P**.
- The ring has a radius **a** and total charge **Q**.
- The potential & field are:

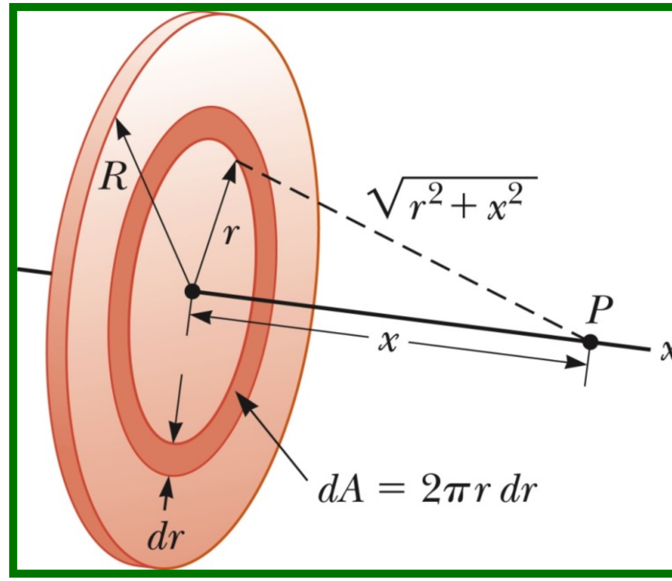


$$V = k_e \int \frac{dq}{r} = \frac{k_e Q}{\sqrt{a^2 + x^2}}$$

$$E_x = \frac{k_e x}{\left(a^2 + x^2\right)^{3/2}} Q$$

V for a Uniformly Charged Disk

- The ring radius is **R** & surface charge density ***σ***. **P** is on the central axis of the disk.
- By symmetry, all points in a given ring are the same distance from **P**. Potential & field are:



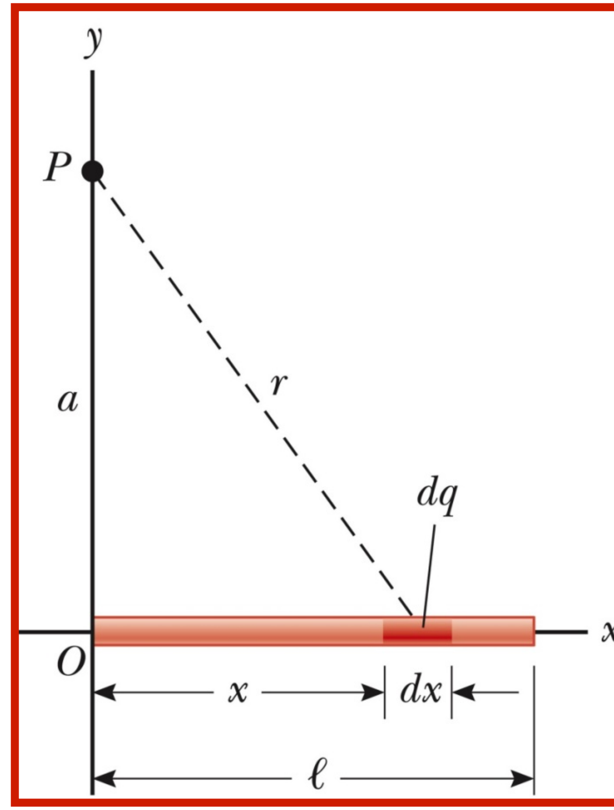
$$V = 2\partial\, k_e \acute{o} \left[\left(R^2 + x^2\right)^{1/2} - x \right]$$

$$E_x = 2\partial\, k_e \acute{o} \left[1 - \frac{x}{\left(R^2 + x^2\right)^{1/2}} \right]$$

V for a Finite Line of Charge

- A rod, length **ℓ** has total charge **Q** & linear charge density **λ**.
- No symmetry to use, but the geometry is simple.

$$V = \frac{k_e Q}{l} \ln\left(\frac{l + \sqrt{a^2 + l^2}}{a}\right)$$



Electric Dipole Potential

The potential due to an **electric dipole** is the sum of the potentials due to each charge, & can be calculated exactly. For distances large compared to the charge separation:

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q\ell\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$$