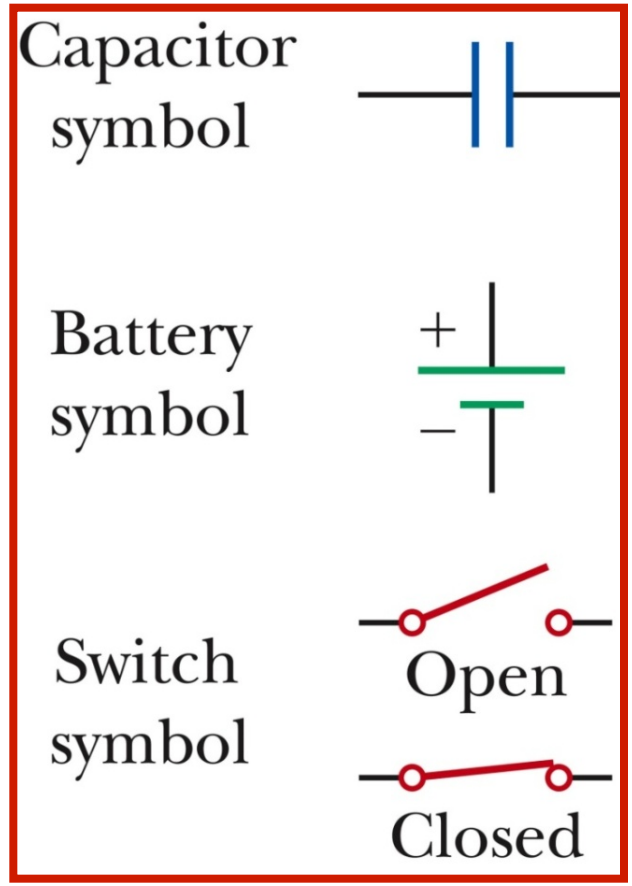
Chapter 26b:

Capacitors in Series & Parallel

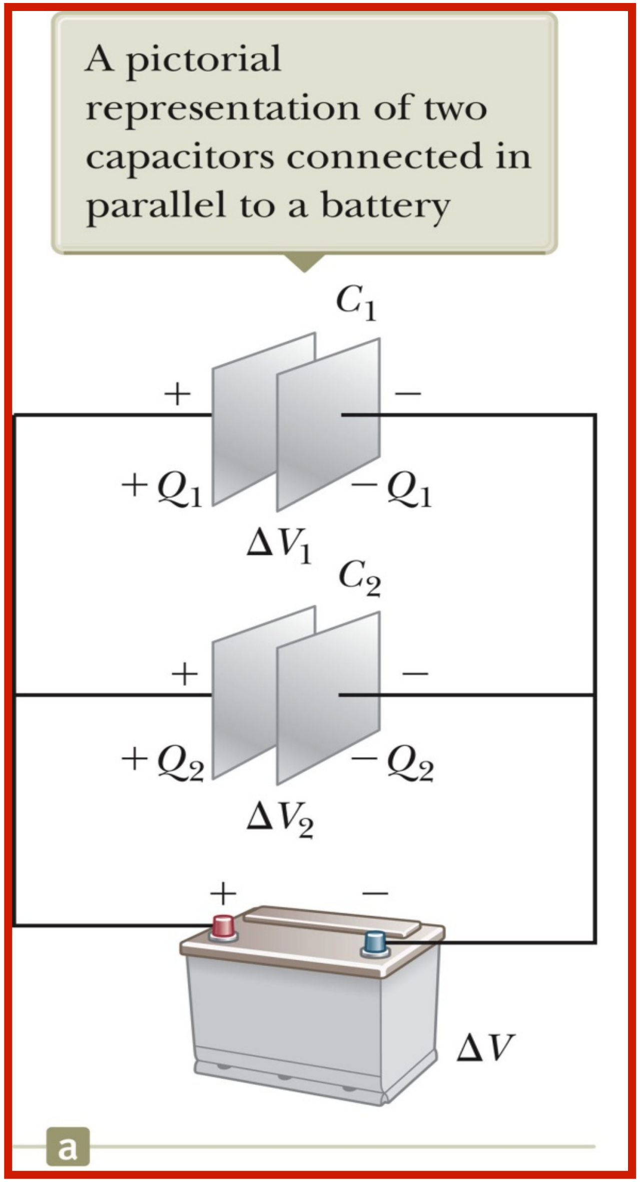
Circuit Symbols

- A circuit diagram is a simplified representation of an actual circuit.
- Circuit symbols are used to represent the various elements. Lines are used to represent wires.
- The battery's positive terminal is indicated by the longer line.



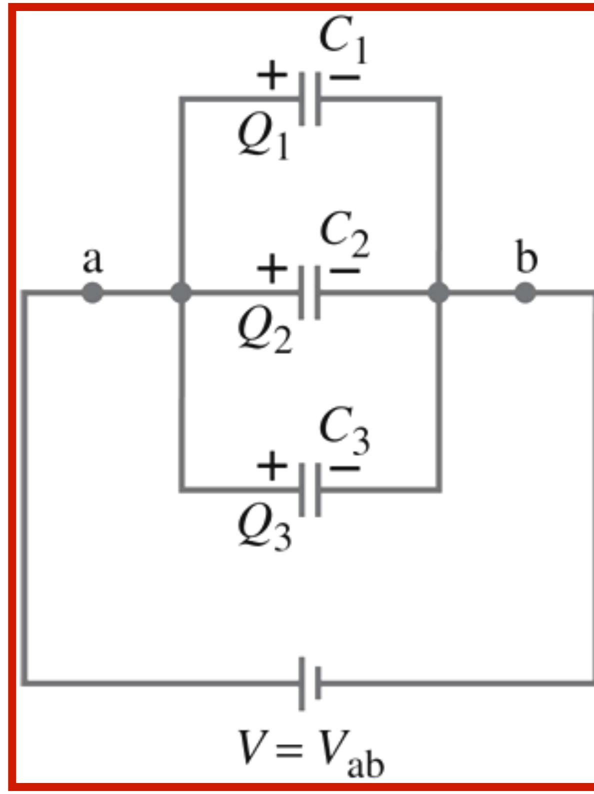
Capacitors in Parallel

- When capacitors are first connected in a circuit, electrons are transferred from the left plate through the battery to the right plate, leaving the left plate positively charged & the right plate negatively charged.



Capacitors in Parallel

- ***Capacitors in Parallel*** have the ***same voltage*** V_{ab} ***across each one***. The equivalent capacitor C_{eq} is one that stores **the same total charge Q** when connected to the same battery.
- That is, in the figure,



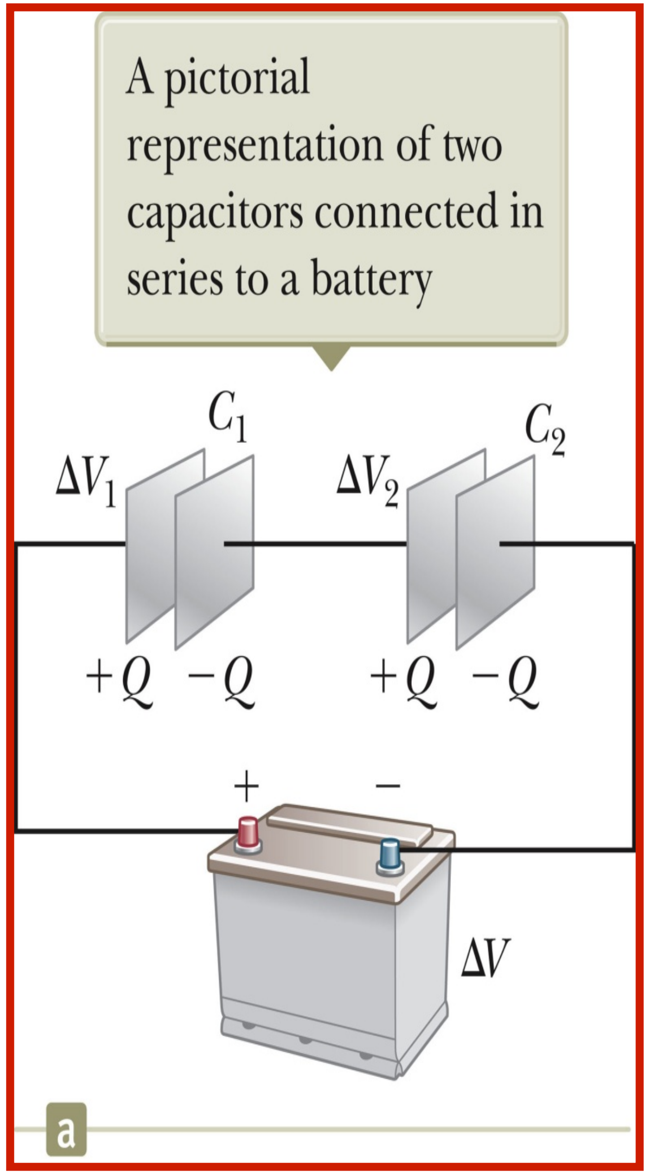
$$Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V$$

Also $Q = C_{eq}V$ so the equivalent capacitance is:

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

Capacitors in Series

- When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery.
- As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge.
- All of the right plates gain charges of $-Q$ & all the left plates have charges of $+Q$.



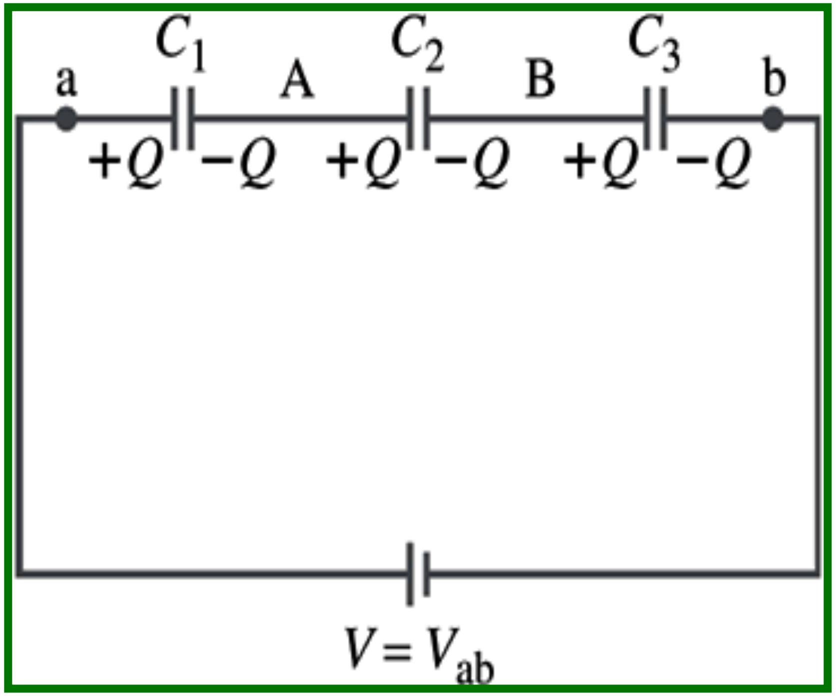
Capacitors in Series

- ***Capacitors in Series*** each have the same charge **Q** their plates. That is in the figure, the equivalent capacitor has the same charge across the total voltage drop: So,

$$\mathbf{V = V_1 + V_2 + V_3 \ \ \& \ \ Q = C_1V_1 = C_2V_2 = C_3V_3 = C_{eq}V}$$

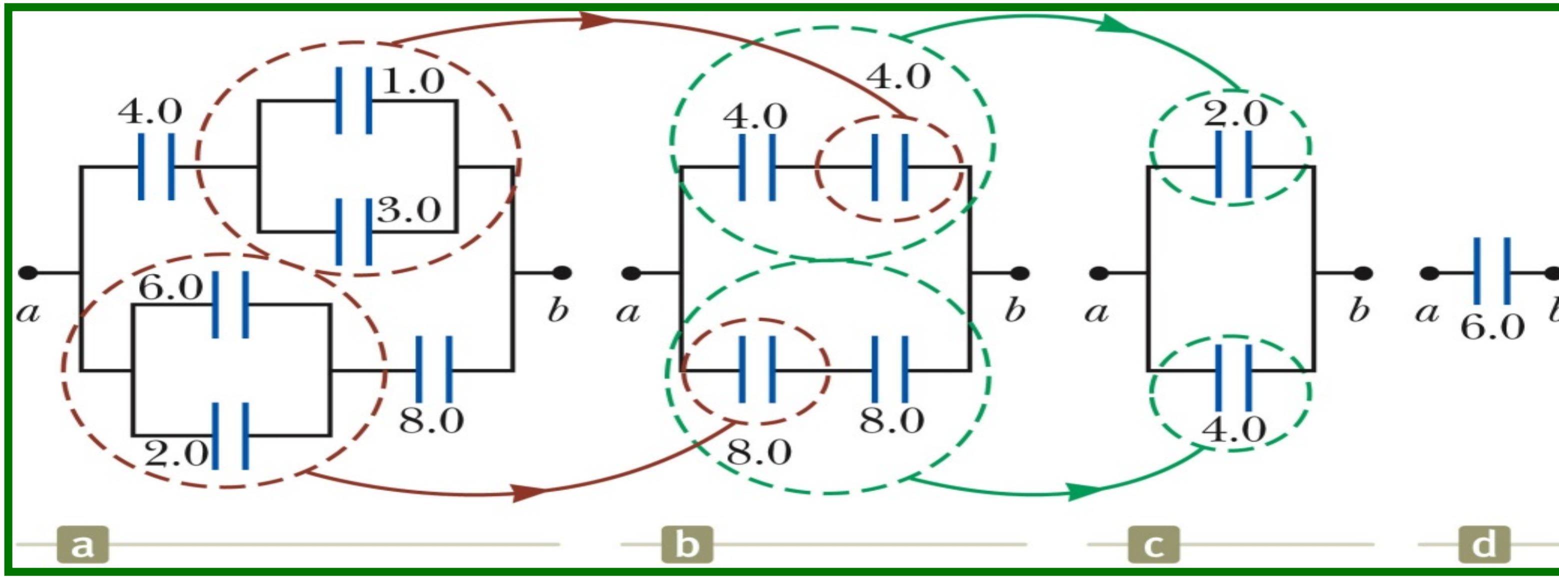
This results in an **equivalent capacitance:**

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.}$$



Note that ***The formula is for the inverse of*** $\mathbf{C_{eq}}$ ***& not for*** $\mathbf{C_{eq}}$ ***itself!***

Equivalent Capacitance, Example 26.3



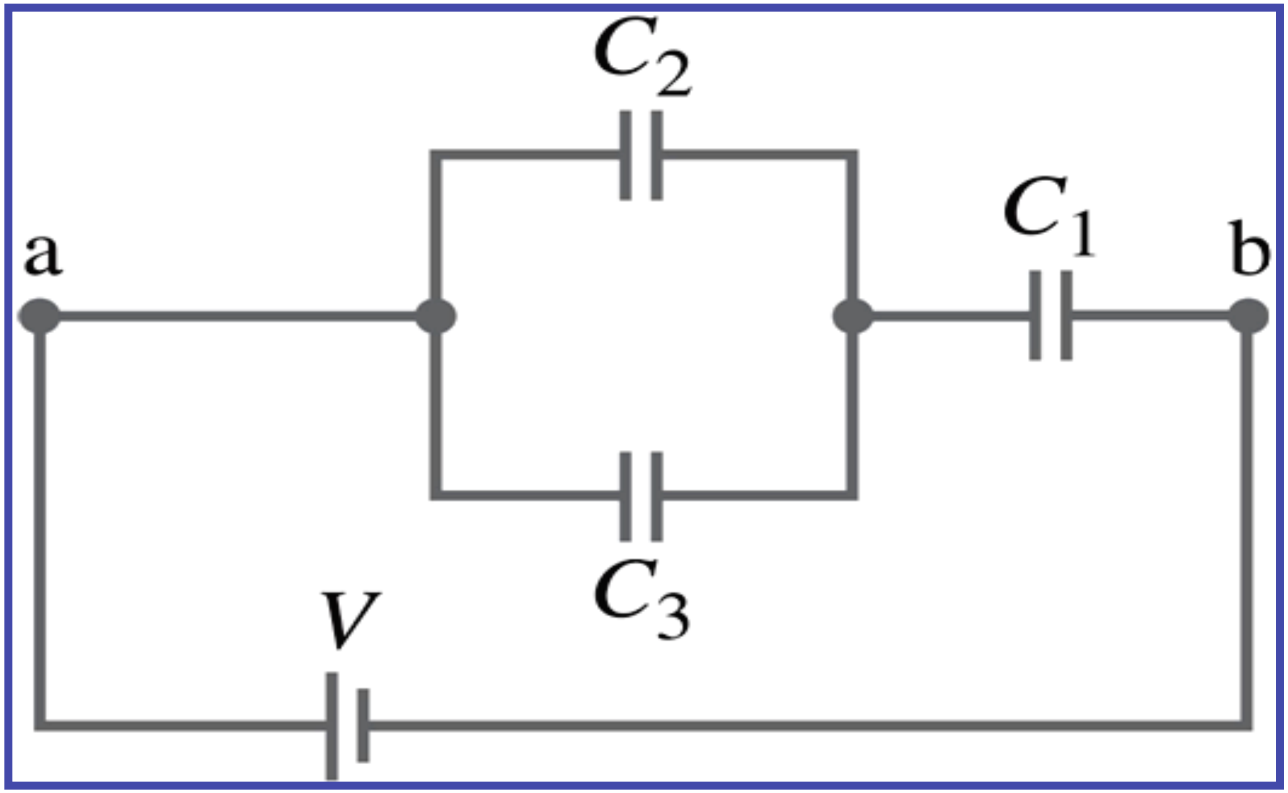
See Figure!

- The **1.0-μF** and **3.0- μF** capacitors are in parallel as are the **6.0- μF** and **2.0-μF** capacitors.
- These parallel combinations are in series with the capacitors next to them.
- The series combinations are in parallel and the final equivalent capacitance can be found.

Example: Equivalent Capacitance

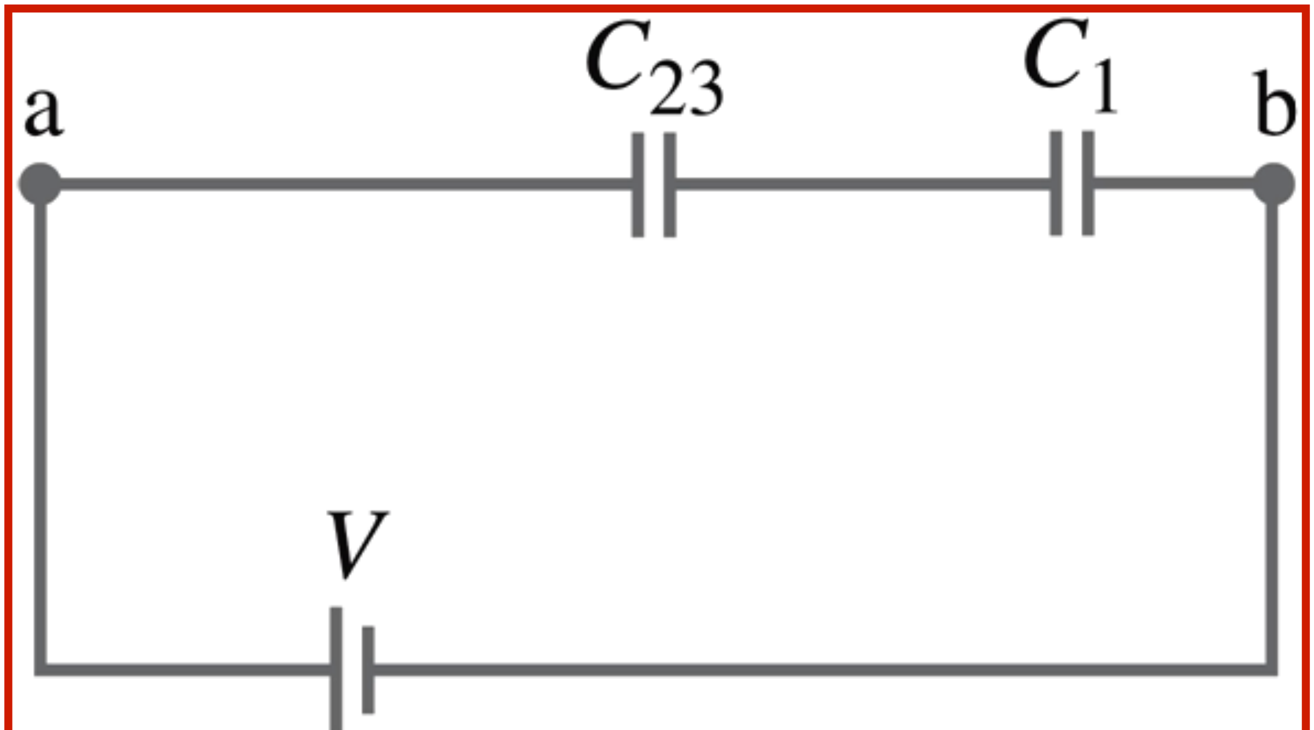
Calculate the capacitance of a single capacitor that will have the same effect as the combination shown.

Let $C_1 = C_2 = C_3 = C$



Example: Charge & Voltage on Capacitors

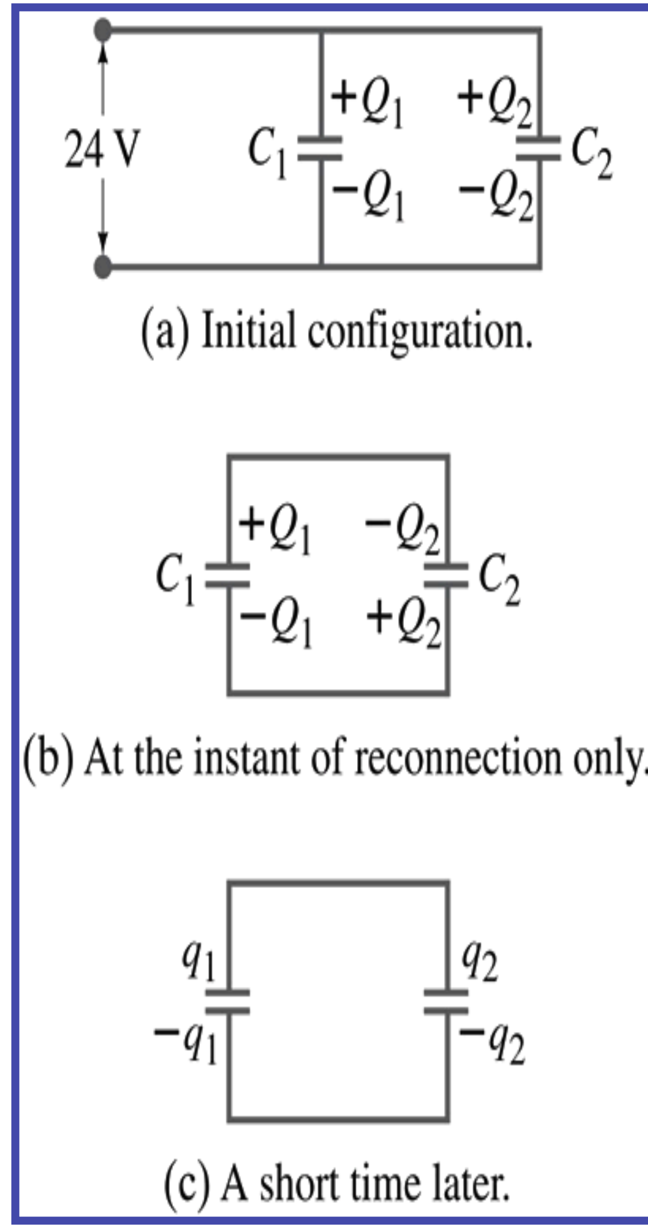
See figure. Calculate the charge on each capacitor & the voltage across each. Let $C_{23} = 3.0\ \mu F$ & the battery voltage $V = 4.0\ V$. Note that the capacitance C_{23} is the capacitance obtained from combining C_2 & C_3.



Example

<u>Capacitors Reconnected</u>

See figure. Two capacitors, **$C_1 = 2.2\ \mu F$ & $C_2 = 1.2\ \mu F$,** are connected in parallel to a **24-V** as in **Fig. a.** After they are charged, they are disconnected from the source & from each other. Shortly afterward, they are reconnected directly to each othe with plates of opposite sign connected together. Calculate the charge on each capacitor & the potential across each after equilibrium is established.



(a) Initial configuration.

(b) At the instant of reconnection only.

(c) A short time later.

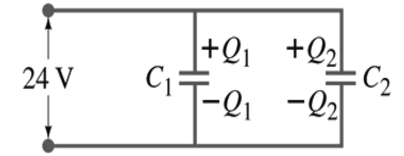
Example: Capacitors Reconnected

Solution: $C_1 = 2.2\ \mu F$ & $C_2 = 1.2\ \mu F$.

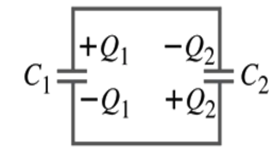
APPROACH We find the charge $Q = CV$ on each capacitor initially. Charge is conserved, although rearranged after the switch. The two new voltages will have to be equal.

SOLUTION First we calculate how much charge has been placed on each capacitor after the power source has charged them fully, using Eq. 24–1:

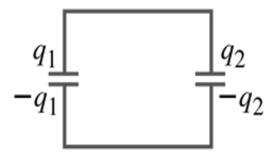
$$Q_1 = C_1 V = (2.2\,\mu\text{F})(24\,\text{V}) = 52.8\,\mu\text{C},$$
$$Q_2 = C_2 V = (1.2\,\mu\text{F})(24\,\text{V}) = 28.8\,\mu\text{C}.$$



(a) Initial configuration.



(b) At the instant of reconnection only.



(c) A short time later.

Example: Capacitors Reconnected

Solution: $C_1 = 2.2\ \mu F$ & $C_2 = 1.2\ \mu F$.

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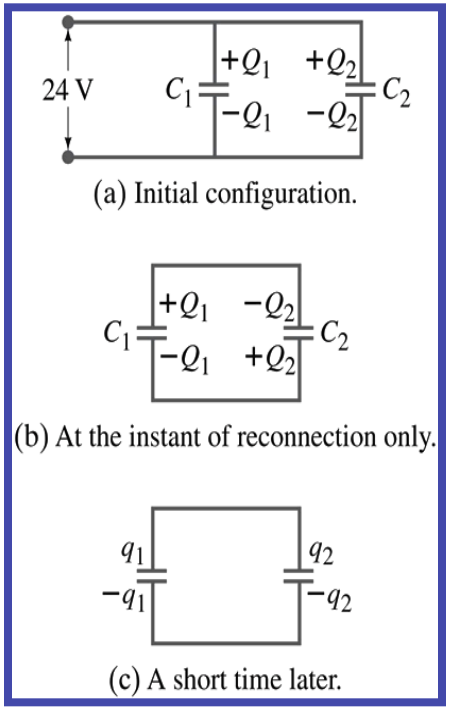
$$Q_2 = C_2 V = (1.2\ \mu\text{F})(24\ \text{V}) = 28.8\ \mu\text{C}.$$

Next the capacitors are connected in parallel, Fig. 24–12b, and the potential difference across each must quickly equalize. Thus, the charge cannot remain as shown in Fig. 24–12b, but the charge must rearrange itself so that the upper plates at least have the same sign of charge, with the lower plates having the opposite charge as shown in Fig. 24–12c. Equation 24–1 applies for each:

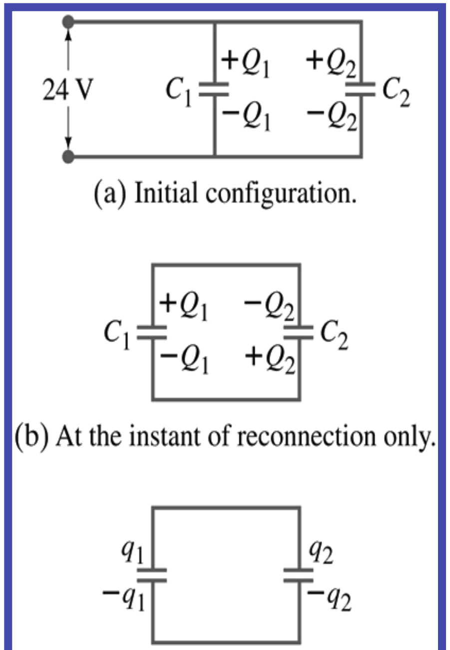
$$q_1 = C_1 V' \qquad \text{and} \qquad q_2 = C_2 V',$$

where V' is the voltage across each capacitor after the charges have rearranged themselves. We don't know q_1, q_2, or V', so we need a third equation. This is



Example: Capacitors Reconnected

Solution: $C_1 = 2.2\ \mu F$ & $C_2 = 1.2\ \mu F$.



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where V' is the voltage across each capacitor after the charges have rearranged themselves. We don't know q_1, q_2, or V', so we need a third equation. This is provided by charge conservation. The charges have rearranged themselves between Figs. 24–12b and c. The total charge on the upper plates in those two Figures must be the same, so we have

$$q_1 + q_2 = Q_1 - Q_2 = 24.0\ \mu C.$$

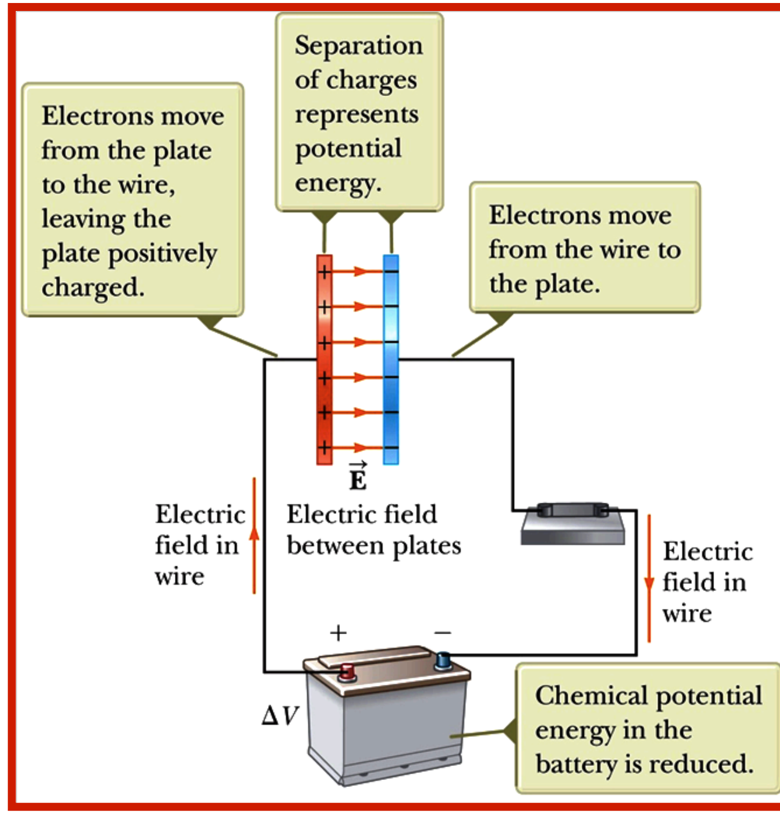
Combining the last three equations we find:

$$V' = (q_1 + q_2)/(C_1 + C_2) = 24.0\ \mu C/3.4\ \mu F = 7.06\ V \approx 7.1\ V$$
$$q_1 = C_1 V' = (2.2\ \mu F)(7.06\ V) = 15.5\ \mu C \approx 16\ \mu C$$
$$q_2 = C_2 V' = (1.2\ \mu F)(7.06\ V) = 8.5\ \mu C$$

where we have kept only two significant figures in our final answers.

Energy in a Capacitor – Overview

- Consider the circuit as a system.
- Before the switch is closed, the energy is stored as chemical energy in the battery.
- When the switch is closed, the energy is transformed from chemical potential energy to electric potential energy.



- The electric potential energy is related to the separation of the positive & negative charges on the plates.
- So, a capacitor can be described as a device that stores energy as well as charge.

Electric Energy Storage

- A useful property of a capacitor is that, if it is charged, it can ***store electric energy***. The **energy stored by a charged capacitor** is equal to ***the work done to charge it.***
- From the potential difference discussion, the work to add an infinitesimal charge **dq** to a capacitor which is at voltage **V** is:

$$\mathbf{dW = Vdq}$$

Electric Energy Storage

$$dW = V dq$$

- So, if a capacitor **C** at voltage **V** is initially uncharged, the work needed to bring charge **Q** to the plates is

$$W = \int V dq$$. (Limits are $q = 0$ & $q = Q$).

- Note that, with charge **q** on the plates, $V = (q/C)$. So

$$W = (1/C) \int q dq = (½)(Q^2/C)$$

- So, the energy stored in capacitor **C** with charge **Q** at voltage **V** is:

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV.}$$

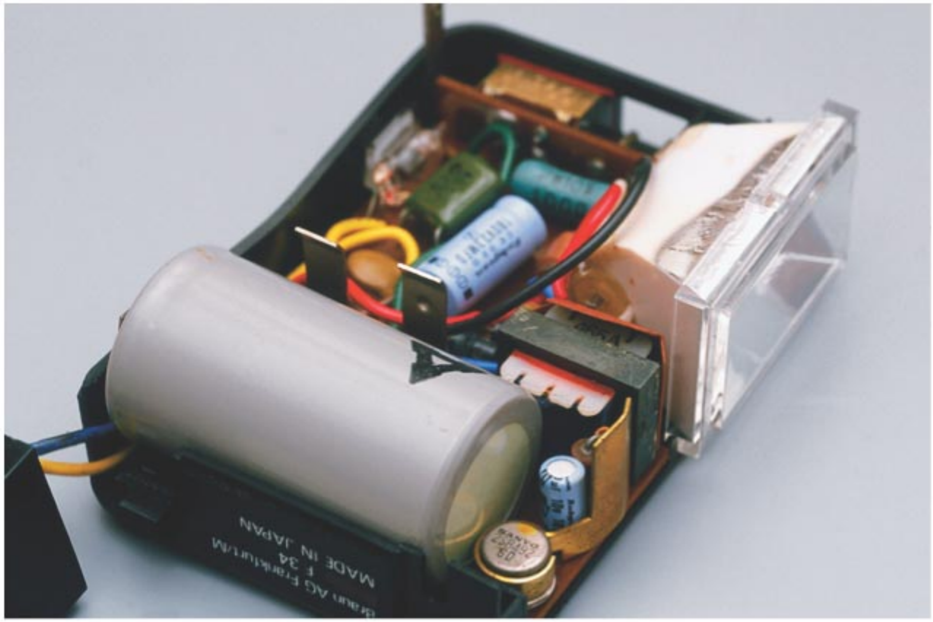
Example: Energy stored in a capacitor

•A camera flash unit stores energy in a **150-μF** capacitor at **200 V**.

Calculate

(a) The energy stored in the capacitor.

(b) The power output if nearly all this energy is released in **1.0 ms**.



Conceptual Example: Capacitor Plate Separation Increased

- A parallel-plate capacitor is given a charge **Q** on its plates & is then disconnected from a battery.
- The 2 plates are initially separated by distance **d**. Suppose the plates are pulled apart until the separation is **2d**. How has the energy stored in this capacitor changed?

Example: Moving Parallel Capacitor Plates

- The plates of a parallel-plate capacitor have area **A**, separation ***x***, & are connected to a battery at voltage **V**. While connected to the battery, the plates are pulled apart until they are separated by **3*x***.

Calculate

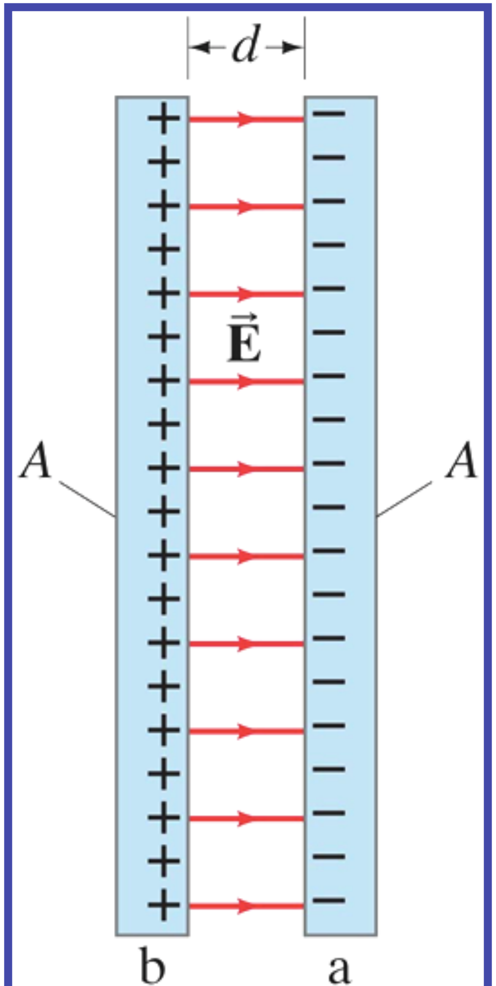
(a) The initial & final energies stored in the capacitor.

(b) The work needed to pull the plates apart (constant speed).

(c) The energy exchanged with the battery.

Energy Density

Parallel Plate Capacitor

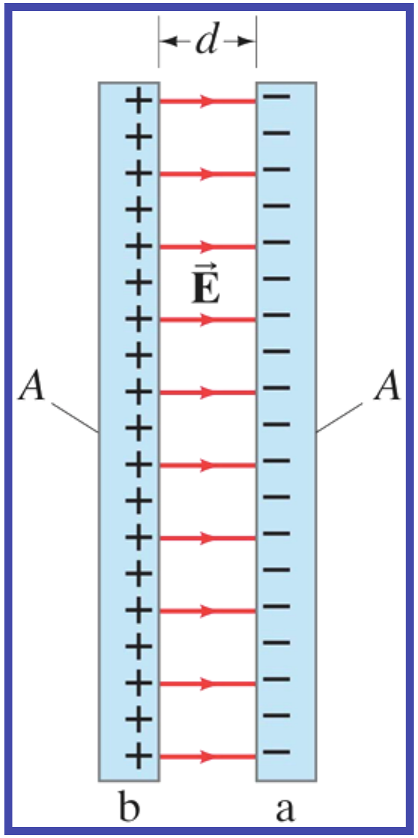


- Its sometimes useful to think of **the energy stored in a capacitor** as being ***stored in the electric field between the plates***.
- Consider a parallel plate capacitor with plate separation **d**. **E =** Electric field between the plates. We' ve seen that the capacitance is

$$\boxed{C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}.}$$

- We' ve also seen that the energy stored in a capacitor **C** with charge **Q** at voltage **V** is:

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV.}$$



Energy Stored in a Capacitor

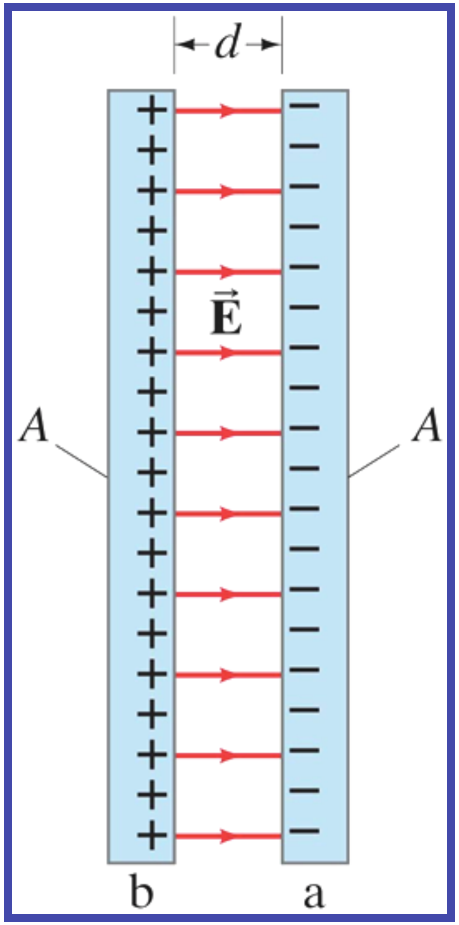
$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV.}$$

- For a parallel plate capacitor, the voltage **V** between the plates is given in terms of electric field **E** as:

$$\mathbf{V = Ed}$$

- Putting this into the expression for **U** gives $\mathbf{U = (½)\varepsilon_0 E^2 Ad}$
- So, the energy density (energy per unit volume) is

$\mathbf{u = [U/(Ad)]}$ or $\mathbf{u = (½)\varepsilon_0 E^2}$



Energy Stored in a Capacitor

$$\boxed{U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV.}$$

- The energy density (energy per unit volume) stored in a parallel plate capacitor is

$$u = (½)\varepsilon_0 E^2$$

- ***NOTE!!*** The sudden discharge of electric energy can be

Harmful or even Fatal!!

So, please be careful!!

- Capacitors can retain charge indefinitely even when not connected to a voltage source!

So, ***PLEASE BE CAREFUL!!!!!!!!***

- This result for the energy density was obtained for a parallel plate capacitor. However, ***the result is actually much more general than that!*** It can be shown that the result

$$\boxed{u = \text{energy density} = \tfrac{1}{2}\epsilon_0 E^2.}$$

is valid anywhere in space where there is an electric field.

- So, in general,

The electric energy stored per unit volume in any region of space is proportional to the square of the electric field in that region.

Heart defibrillators use electric discharge to "jump-start" the heart, and can save lives.