

# Physics 101: Mechanics

## Lecture 9

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# Work-Energy Theorem

- When work is done by a net force on an object and the only change in the object is its speed, the work done is equal to the change in the object's kinetic energy

$$W_{net} = K_f - K_i = \Delta K$$

- Speed will increase if work is positive
- Speed will decrease if work is negative

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

# Work with Varying Forces

- On a graph of force as a function of position, the total work done by the force is represented by the area under the curve between the initial and the final position
- Straight-line motion

$$W = F_{ax}\Delta x_a + F_{bx}\Delta x_b + \dots$$

$$W = \int_{x_1}^{x_2} F_x dx$$

- Motion along a curve

$$W = \int_{P_1}^{P_2} F \cos \phi dl = \int_{P_1}^{P_2} F_{\parallel} dl = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l}$$

Figure 7.7  
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# Work-Energy with Varying Forces

- Work-energy theorem  $W_{tot} = \Delta K$  holds for varying forces as well as for constant ones

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx} \frac{dx}{dt} = v_x \frac{dv_x}{dx}$$

$$W_{tot} = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} ma_x dx = \int_{x_1}^{x_2} mv_x \frac{dv_x}{dx} dx$$

$$W_{tot} = \int_{v_1}^{v_2} mv_x dv_x$$

$$W_{tot} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \Delta K$$

# Spring Force: a Varying Force

- ❖ Involves the *spring constant*,  $k$
- ❖ Hooke's Law gives the force

$$\vec{F} = -k\vec{x}$$

- ❖  $F$  is in the opposite direction of  $x$ , always back towards the equilibrium point.
- ❖  $k$  depends on how the spring was formed, the material it is made from, thickness of the wire, etc

Figure 7.10  
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# Measuring Spring Constant

- ❖ spring at its natural equilibrium length.
- ❖ Hang a mass on spring and let it hang to distance  $d$  (stationary)
- ❖ From  $F_x = kx - mg = 0$ 
$$k = \frac{mg}{d}$$
- ❖ get spring constant.

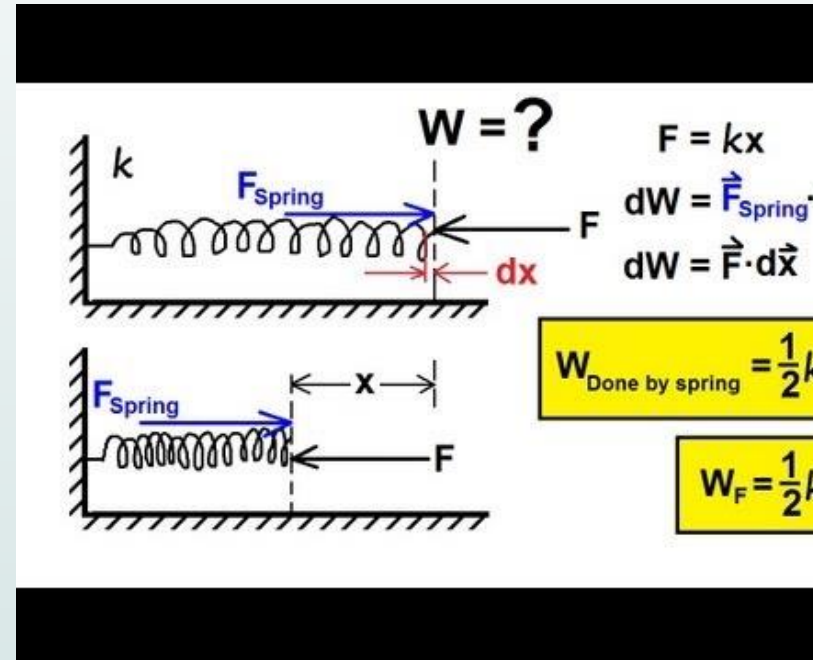
Figure 7.12  
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# Work done on a Spring

- ❖ We have to do work to extend the spring
- ❖ We apply equal and opposite forces to the ends of spring and gradually increase the forces
- ❖ The work we must do to stretch the spring from  $x_1$  to  $x_2$

$$W = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} kx dx = \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2$$

- ❖ Work done on a spring is not equal to work done by a spring



# Power

- ❖ Work is not tied to work time interval
- ❖ The transfer rate of energy is important in practical device design and use.
- ❖ The time ratio of energy transfer is called power
- ❖ The average power is given by

$$\bar{P} = \frac{W}{\Delta t}$$



# Instantaneous Power

❖ Power is the time rate of energy transfer. Power applies to any energy transfer vehicle

❖ Other expression

$$\bar{P} = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = F\bar{v}$$

❖ A more general definition of instantaneous power

$$P = \lim_{\Delta t \rightarrow 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$P = \vec{F} \cdot \vec{v} = Fv\cos\theta$$

# Units of Power

- The SI unit of power is called the watt
  - 1 watt = 1 joule / second =  $1 \text{ kg} \cdot \text{m}^2 / \text{s}^3$
- Units of power can also be used to express units of work or energy
  - 1 kWh =  $(1000 \text{ W})(3600 \text{ s}) = 3.6 \times 10^6 \text{ J}$

# Power Delivered by an Elevator Motor

- ❖ 1000 kg lift, carries a maximum load of 800 kg. The continuous friction force of 4000 N retards the upward movement. What minimum power should the engine use to lift the fully loaded elevator at a constant speed of 6 m / s?

$$F_{net,y} = ma_y$$

$$T - f - Mg = 0$$

$$T = f + Mg = 2.16 \times 10^4 \text{ N}$$

$$P = Fv = (2.16 \times 10^4 \text{ N})(3 \text{ m / s})$$

$$= 13 \times 10^4 \text{ W}$$

Figure 7.19  
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