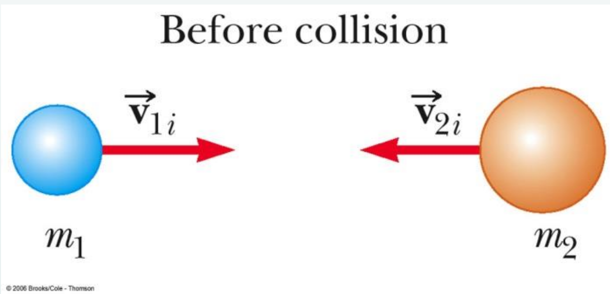
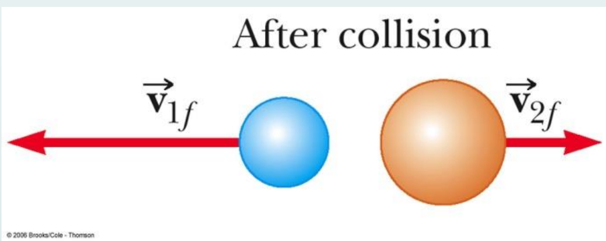
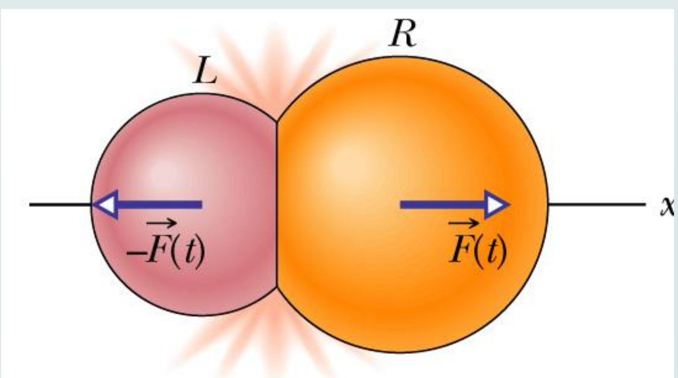
Physics 101: Mechanics
Lecture 13

Baris EMRE

Conservation of Momentum







- Start from impulse-momentum theorem

$$\vec{F}_{21}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$$

$$\vec{F}_{12}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$$

- Since $\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$

- Then $m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$

- So $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Conservation of Momentum

- The total momentum of the system remains constant over time if there is no external force effect on a system consisting of two bodies colliding with each other

$$\vec{F}_{net}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

- When $\vec{F}_{net} = 0$ then $\Delta\vec{p} = 0$
- For an isolated system

$$\vec{p}_f = \vec{p}_i$$

- Specifically, the total momentum before the collision will equal the total momentum after the collision

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

The Archer

- An archer on a frictionless ice and scans an arrow of 0.5 kg horizontally at 50.0 m / s. Archer and bow combined weight is 60.0 kg. At what speed does the archer go over the ice after you shoot the archer?

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 = 60.0kg, m_2 = 0.5kg, v_{1i} = v_{2i} = 0, v_{2f} = 50m/s, v_{1f} = ?$$

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1f} = -\frac{m_2}{m_1} v_{2f} = -\frac{0.5kg}{60.0kg}(50.0m/s) = -0.417\, m/s$$

Types of Collisions

- Momentum is conserved in any collision
- **Inelastic collisions:** *rubber ball and hard ball*
  - Kinetic energy is not conserved
  - **Perfectly inelastic** collisions occur when the objects stick together
- **Elastic collisions:** *billiard ball*
  - both momentum and kinetic energy are conserved
- **Actual collisions**
  - Most collisions fall between elastic and perfectly inelastic collisions

Collisions Summary

- In an elastic collision, both momentum and kinetic energy are conserved
- In a non-elastic collision, momentum is preserved, but kinetic energy is absent. Moreover, objects do not stick together
- In a perfect nonelastic collision, momentum is preserved, kinetic energy is absent, and two objects collide after colliding, so the final speeds are the same
- Elastic and excellent inelastic collisions are limiting behaviors, and most actual collisions are between these two types
- The momentum is preserved in all collisions

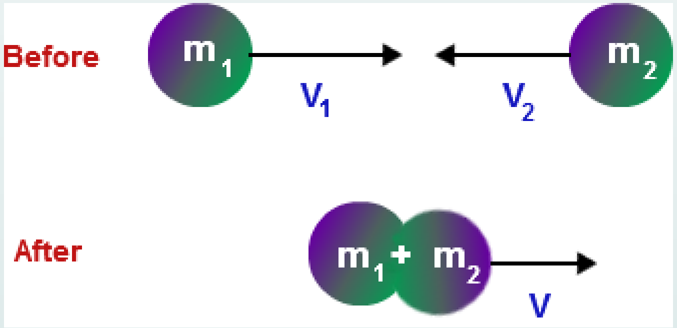
Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision
- Conservation of momentum

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

- Kinetic energy is NOT conserved

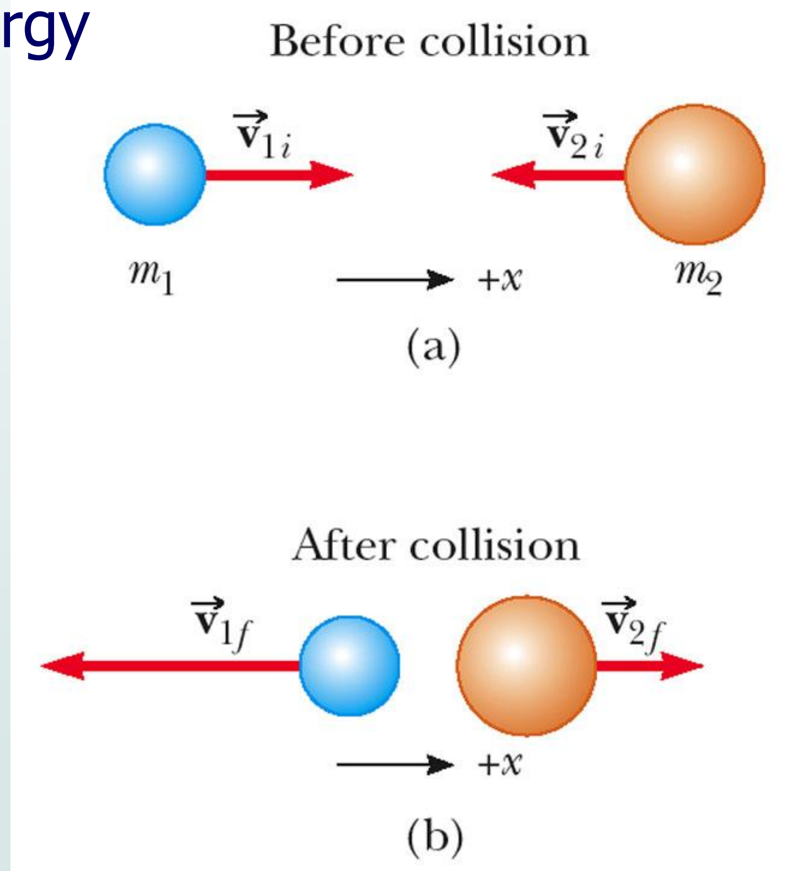


More About Elastic Collisions

- Both momentum and kinetic energy are conserved

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$



- Typically have two unknowns
- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs

Elastic Collisions

- A simpler equation can be used in place of the KE equation

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i})$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

Summary of Types of Collisions

- In an elastic collision, both momentum and kinetic energy are conserved

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- In an inelastic collision, momentum is conserved but kinetic energy is not

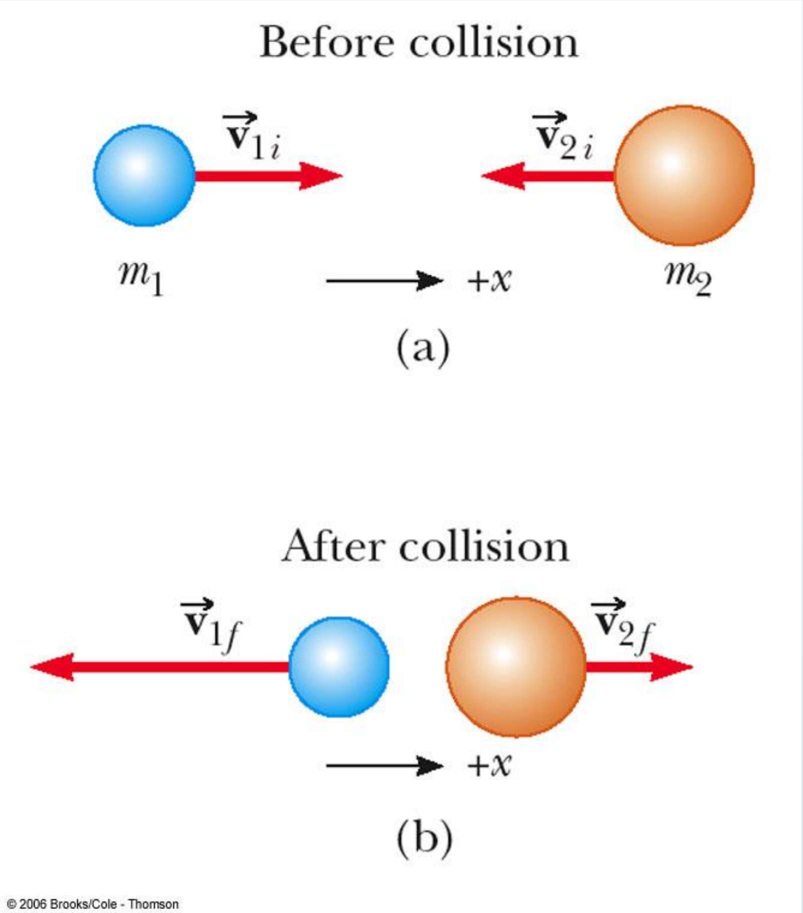
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Problem Solving for 1D Collisions, 1

- **Coordinates:** Set up a coordinate axis and define the velocities with respect to this axis
  - It is convenient to make your axis coincide with one of the initial velocities
- **Diagram:** In your sketch, draw all the velocity vectors and label the velocities and the masses



Two-Dimensional Collisions

- For a general collision of two objects in two-dimensional space, the conservation of momentum principle implies that the *total momentum of the system in each direction is conserved*

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$