

In-Class Assignment

Consider the system $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 2r(t)$ where initial conditions are $y(0)=1$, $\frac{dy(0)}{dt} = 0$ and $r(t)=1$ $t>0$. a) Find $Y(s)$, b) Time response c) steady state response from final value theorem and d) steady state response from time response

$$\frac{d^n f(t)}{dt^n} \xleftrightarrow{\mathcal{L}} s^n F(s) - s^{(n-1)} f(0^-) - s^{(n-2)} \dot{f}(0^-) - \dots - s f^{(n-2)}(0^-) - s^{(n-2)} f^{(n-1)}(0^-)$$

$$a) [s^2 Y(s) - s] + 4[sY(s) - 1] + 3Y(s) = 2R(s)$$

$$Y(s)[s^2 + 4s + 3] = \frac{2}{s} + 4 + s$$

$$Y(s) = \frac{s^2 + 4s + 2}{s(s^2 + 4s + 3)}$$

$$b) Y(s) = \frac{k_1}{s} + \frac{k_2}{s+3} + \frac{k_3}{s+1}$$

$$k_1 = \frac{2}{3}$$

$$k_2 = -1/6$$

$$k_3 = \frac{1}{2}$$

$$Y(s) = \frac{2/3}{s} - \frac{1/6}{s+3} + \frac{1/2}{s+1}$$

$$y(t) = \left(\frac{2}{3} - \frac{1}{6} e^{-3t} + \frac{1}{2} e^{-t} \right)$$

$$c) \lim_{t \rightarrow \infty} y(t) = \frac{2}{3}$$

$$d) \lim_{s \rightarrow 0} sF(s) = \frac{2}{3}$$