Lecture 7 :

Continuous charge distributions and retarded potentials : In nonstatic case it is not the present status of the source which matters, but its old position at some earlier time is important. Retarded time is defined by :

$$t_r = t - \frac{|\boldsymbol{r} - \boldsymbol{r}'|}{c}$$

Therefore the retarded potentials are given by :

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}', t_r)}{\mathscr{r}}\, d\tau', \quad \mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t_r)}{\mathscr{r}}\, d\tau'.$$

However the same argument does not work for the fields.

So one should work them out starting from their definitions in terms of the potentials.

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

Substituting the retarded expressions for the potentials above and taking the differentials properly (because both the retarded time t_r and $\mathscr{r} = |\boldsymbol{r} - \boldsymbol{r}'|$ depend on position also) one arrives the retarded Jefimenko equations for the electric and magnetic fields

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int \left[\frac{\rho(\mathbf{r}', t_r)}{\mathscr{r}^2} \hat{\boldsymbol{\mathscr{r}}} + \frac{\dot{\rho}(\mathbf{r}', t_r)}{c\mathscr{r}} \hat{\boldsymbol{\mathscr{r}}} - \frac{\dot{\mathbf{J}}(\mathbf{r}', t_r)}{c^2 \mathscr{r}} \right] d\tau'.$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \left[\frac{\mathbf{J}(\mathbf{r}', t_r)}{\mathscr{r}^2} + \frac{\dot{\mathbf{J}}(\mathbf{r}', t_r)}{c\mathscr{r}} \right] \times \hat{\boldsymbol{\mathscr{r}}}\, d\tau'.$$

Homeworks :

Study and solve the following exercises and problems from the textbook by D.Griffiths's "Introduction to Electrodynamics"

Study Exercises 10.1 and 10.2

Solve Problem 10.9

Solve Problem10.10

Solve Problem10.11