Lecture 9 :

Aharonov-Bohm effect and other topological issues :

As we have seen before via the gauge transformations one can alter the potentials without changing the electric and magnetic fields which are the only measured physical quantities in the lab. So in a way the potentials play an auxiliary role in classical electrodaynamics.

Aharonov-Bohm effect (1959) :

Logically it seems that this is the general picture valid in whole of the physics.

However in quantum mechanics we encounter several topological consideretions which force us to change our previous conclusions.

Suppose a particle is moving through a region where **B** field is zero but there exists a nonzero vector potential **A**. The time dependent Schröndinger equation is

$$\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A}\right)^2 + V\right]\Psi = i\hbar\Psi$$

To recover the invariance under gauge transformations the phase of the wave function must have a geometric phase factor

$$\Psi = e^{ig}\,\Psi'$$

$$g(\boldsymbol{r}) = \frac{q}{\hbar}\int_0^r \boldsymbol{A}(r')\cdot\boldsymbol{dr}'$$

Now consider a long solenoid and electron beam which split in two such that two beams pass from the opppsite sides of the solenoid. In the region they pass there is no magnetic field and classically no magnetic deflection. However if the total phase factor is evaluated over a closed contour around the long solenoid (which gives the interference term between the two electron beams arriving the screen) one can obtain using the Stokes' theorem

$$g = \frac{q}{\hbar}\oint \boldsymbol{A}(r')\cdot\boldsymbol{dr}' = \frac{q\Phi}{\hbar}$$

This phase shift leads to a measurable interference on the screen which was first observed by Chambers (1960).

Conclusion : Altough there is no interaction between magnetic field and charged particle the vector potential has certainly a surprising measurable physical consequence.

The above AB geometric set-up migth be varied to investigate other types of such topological effects.

An analogue of the AB geometry leads us to the Aharonov- Casher effect. In this new scenario the magnetic solenoid is replaced by a line electric charge and the electron is replaced obviously by the neutron having a nonzero magnetic moment.

Reference : D.Griffiths, Introduction to Quantum Mechanics, 2016, Cambridge University Press.