Lecture 11 :

Radiation from arbitrary sources : Charges at rest and steady currents do not radiate. But accelerating charges radiate. On the other hand a vibrating spherical shell charge distribution does not radiate either; due to the Gauss law, if during the vibration the charge distribution keeps its spherical symmetry.

AS we have seen before the retarded scalar potential has the following form :

$$V(\boldsymbol{r},t)=\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\boldsymbol{r}',t_r)}{|\boldsymbol{r}-\boldsymbol{r}'|}d\tau'$$

We simply replace these in the retarded scalar potential expression and need some suitable approximations in order to carry out the calculations :

a) $r' \ll r$

b) $r' \ll \frac{c}{|\ddot{\rho}/\dot{\rho}|}$; $\frac{c}{|\dddot{\rho}/\dot{\rho}|^{1/2}}$; $\frac{c}{|\dddot{\rho}/\dot{\rho}|^{1/3}}$ namely $r' \ll \lambda$ for an oscillating system

c) keep only those terms that have 1/r dependence

The retarded scalar potential for an arbitrary charge distribution becomes :

$$V(\mathbf{r},t)\cong\frac{1}{4\pi\epsilon_0}\left[\frac{Q}{r}+\frac{\hat{\mathbf{r}}\cdot\mathbf{p}(t_0)}{r^2}+\frac{\hat{\mathbf{r}}\cdot\dot{\mathbf{p}}(t_0)}{rc}\right]$$

A similar analysis can be done for the retarded vector potential **A** (**r**,t) :

$$\mathbf{A}(\mathbf{r},t)\cong\frac{\mu_0}{4\pi}\frac{\dot{\mathbf{p}}(t_0)}{r}.$$

The fields are derived from the potentials as in the usual case :

$$\boldsymbol{B}=\nabla\times\boldsymbol{A}$$

$$\boldsymbol{E}=-\nabla V-\frac{\partial\boldsymbol{A}}{\partial t}$$

but now keeping only terms with 1/r behaviour :

$$\mathbf{E}(r,\theta,t)\cong\frac{\mu_0\ddot{p}(t_0)}{4\pi}\left(\frac{\sin\theta}{r}\right)\hat{\boldsymbol{\theta}},$$

$$\mathbf{B}(r,\theta,t)\cong\frac{\mu_0\ddot{p}(t_0)}{4\pi c}\left(\frac{\sin\theta}{r}\right)\hat{\boldsymbol{\phi}}.$$

The Poynting vector $\boldsymbol{S}$ and total power radiated are :

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B} = \left(\frac{\mu_0\left[\ddot{p}(t_0)\right]^2}{16\pi^2 c}\right)\frac{\sin^2\theta}{r^2}\hat{r}$$

The total power radiated can be found by integrating it over a sphere of radius :

$$P = \int \boldsymbol{S}\cdot d\boldsymbol{a} = \frac{\mu_0\,\ddot{p}^2\,\omega^4}{6\pi c}$$

Homeworks :

Study and solve the following exercises and problems from the textbook by D.Griffiths's "Introduction to Electrodynamics"

Study Exercises 11.2 (Larmor formula for a single charge)

Solve Problem 11.8

Solve Problem11.9

Solve Problem 11.10

Solve Problem 11.11

The same problem might be reexamined by starting directly from the formulas (obtained in the previous chapter) for the **E** and **B** fields of a charge q in arbitrary motion. For velocities comparable to that of the light the total power radiated turns out to be :

$$P = \frac{\mu_0 q^2\gamma^6}{6\pi c}\left(a^2 - \left|\frac{\mathbf{v}\times\mathbf{a}}{c}\right|^2\right)$$

This result is an extension of the Larmor theory and is called **Liénard generalization**.

Synchrotron radiation : We have seen the radiation due to a relativistic charged particle in an arbitrary motion. Now one can obtain the radiation emitted by a particle in a circular motion, in this case the radiation is called Synchrotron radiation.

Study Example 11.3

Solve Problem 11.16

in Griffiths' Textbook "Introduction to Electrodynamics"