Lecture 14 :

Magnetic monopoles : Presently there is no experimental evidence for the presence of the of the magnetic charges (monopoles). Why is it so ? On the contrary we know perfectly that there exist electric charges (monopoles) in nature. So why the symmetry between electric and magnetic sources of electromagnetism is so badly broken ? P.A.M. Dirac once said "One would be surprised if Nature had made no use of it". The brilliant theoretical argument due to Dirac gives a surprising answer and in addition it also explains why the electric charge is quantized. Today the search for magnetic monopoles is an active and interesting research area in elementary particle physics; as the colliders' energy reach increases every decade the search goes on. Even the rock minerals brought from the Moon or found in the meteorites have been investigated and the research is enlarged to cover the cosmic scale, the whole universe.

Duality transformations : Let us suppose that magnetic charges exist. So one can write the Maxwell's equations including electric and magnetic charge densities and as well as electric and magnetic current densities as follows :

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_e}{\varepsilon_0} \qquad \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}_e + \mu_0\, \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = \mu_0\, \rho_m \qquad \nabla \times \boldsymbol{E} = -\,\mu_0 \boldsymbol{J}_m - \frac{\partial \boldsymbol{B}}{\partial t}$$

One can show that the above generalized Maxwll's equations are invariant under the following duality transformations :

$$\begin{aligned}
\mathbf{E}' &= \mathbf{E}\cos\alpha + c\mathbf{B}\sin\alpha, \\
c\mathbf{B}' &= c\mathbf{B}\cos\alpha - \mathbf{E}\sin\alpha, \\
cq_e' &= cq_e\cos\alpha + q_m\sin\alpha, \\
q_m' &= q_m\cos\alpha - cq_e\sin\alpha,
\end{aligned}$$

Electric and magnetic current densities transform in the same way as the charges. If all the particles have the same magnetic to electric charge ratios then one can choose the parameter α such that all the particles have only electric charge but no magnetic charge.

Dirac quantization of electric charge (P.A.M.Dirac, See also A.S.Goldhaber , Phys.Rev. vol.140, 1965 and D.Jackson' "Classical Elctrodynamics" Section 6.11.and 6.12)

Considering the the quantum mechanics of a an electron in the vicinity of a magnetic monopole Dirac obtained the quantization condition for the electric charges :

$$\frac{e\,g}{\hbar} = 2\pi n \qquad (n = 0,\ \pm 1, \pm 2, \pm 3, ...)$$

here g is the magnetic charge of the magnetic monopole.

For further discussion :

1) John Preskil, “Magnetic Monolopes”, Ann. Rev. Nucl. Part. Sci. 1984. 34:461-530
2) Prof. Steven Errede Lecture Notes, UIUC Physics 435 EM Fields & Sources I
   Chapter 18 “Magnetic Monoples”
3) Doris Teplitz, “Electromagnetism : Paths to Research” Chapter 8 (1982) Springer Pub.