FDE 205 FLUID MECHANICS

2.6 Overall Mass Balance and Continuity Equation

Simple Mass Balances:

- Fluid dynamics   fluid in motion
- They are moved from place to place
  - by means of mechanical devices (pumps),
  - by gravity head,
  - by pressure
- The first step in the solution of flow problems is generally to apply the principles of conservation of mass to the whole system or to any part of the system

$$\text{INPUT} - \text{OUTPUT} + \text{GENERATION} = \text{ACCUMULATION}$$

$$\begin{array}{c} \text{Rate of} \\ \text{mass} \\ \text{input} \end{array} - \begin{array}{c} \text{Rate of} \\ \text{mass} \\ \text{output} \end{array} + \begin{array}{c} \text{Rate of} \\ \text{mass} \\ \text{generated} \end{array} = \begin{array}{c} \text{Rate of mass} \\ \text{accumulation} \end{array}$$

For systems in steady state and without reaction:

$$\begin{array}{c} \text{Rate of} \\ \text{mass} \\ \text{input} \end{array} = \begin{array}{c} \text{Rate of} \\ \text{mass} \\ \text{output} \end{array}$$

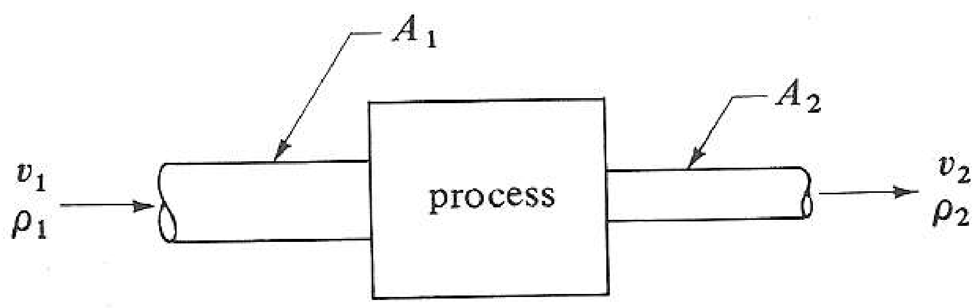


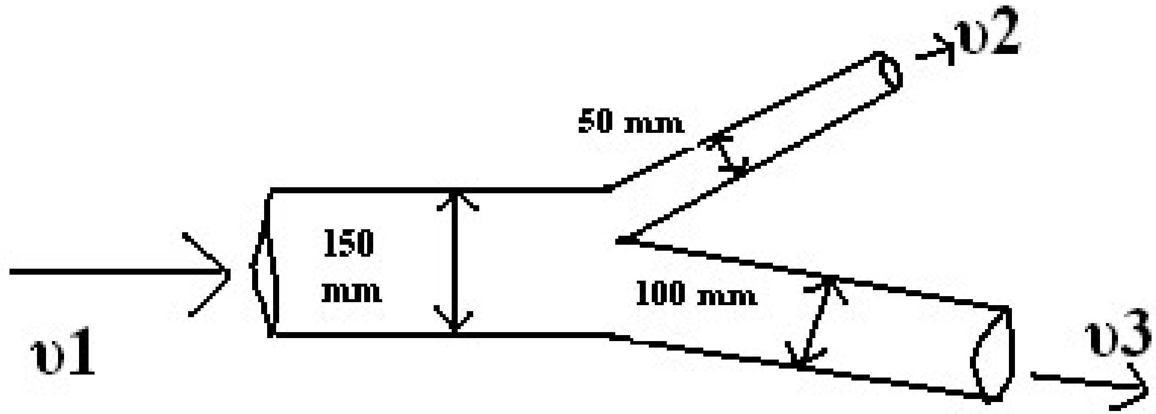
FIGURE 2.6-1. *Mass balance on flow system.*

- In the figure above, the fluid enters section 1 with an average velocity υ_1(m/s) and density ρ_1(kg/m^3). The fluid leaves section 2 with an average velocity υ_2(m/s) and density ρ_2(kg/m^3). Corresponding sectional areas are A_1 and A_2. Apply the mass balance and find a relation between these parameters.

$$\dot{m} = \rho_1 A_1 \upsilon_1 = \rho_2 A_2 \upsilon_2$$

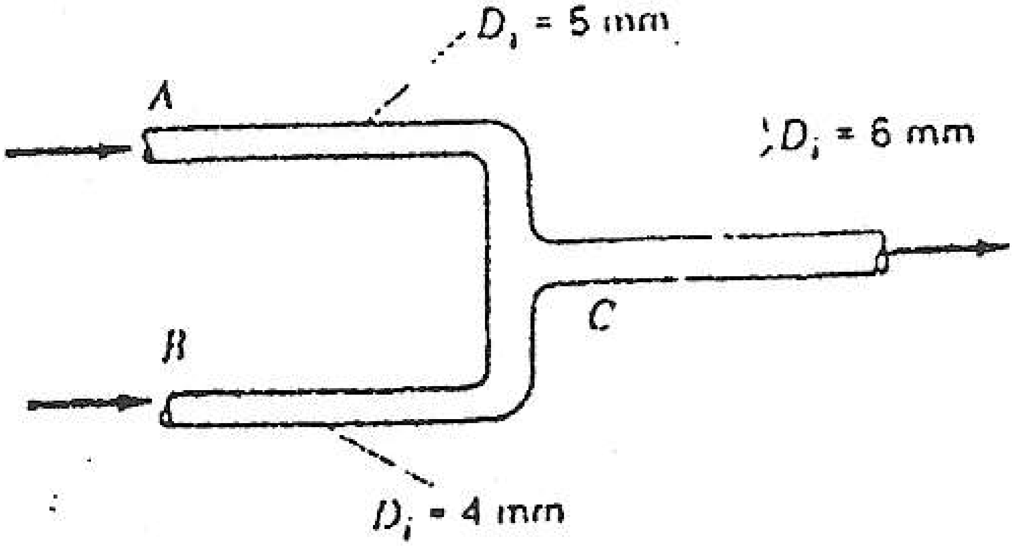
Example

Water is flowing trough the piping arrangement shown in figure below at a total rate of 0.072 m^3/s entering pipe 1(d=150 mm). The flow is divided into two branches as in figure. If the velocity of the water flowing through pipe with a 50 mm diameter is 12 m/s, what should be the velocity of the other branch?



Example

Water is flowing trough the piping arrangement shown in figure below. The flow rate of the water in pipe A is 2X10-3 L/s. If we want to achieve a laminar flow in pipe C, what would be the maximum fluid velocity (m/s) in pipe B? What are the fluid flow types in pipe A and B? (Assume water is at 20C)



For systems at unsteady state:

- There should be an accumulation term in the general equation.

$$\dot{m}_{in} - \dot{m}_{out} = \underbrace{\frac{dM}{dt}}_{\text{The change of total mass in the system with time}}$$

Example

- A tank with a diameter of 0.914 m and a height of 1.219 m is full of water initially. When the drainage whole (d=1.27 cm) at the bottom of the tank is opened, the water discharged at an average velocity of $\sqrt{2gh}$.
- h is height of the water (from the center of the whole),
- g is the gravitational acceleration.
- Determine the time passed for the water level to drop to 0.609 m height in the tank.

Overall Energy Balance

- The second property to be considered in the overall balances on a control volume is energy.
- We will apply the principle of the conservation of energy to a control volume fixed in space (similar to conservation of mass)
- This will then be combined with the first law of thermodynamics to obtain the final overall energy-balance equation.

- First Law of Thermodynamics:

Bir sistemin iç enerjisindeki artış, sisteme verilen ısı ile, sistemin çevresine uyguladığı iş arasındaki farktır.

$$\Delta E = Q - W$$

- ΔE: is the total energy change in the system per unit mass of fluid
- Q: is the heat absorbed per unit mass of fluid
- W: is the work of all kinds done per unit mass of fluid upon the surroundings.
- The unit of each term is (J/kg).

$$\Delta E = Q - W$$

$$\left\{ \boxed{\begin{array}{l}\text{The total energy of fluid}\\ \text{leaving the system}\end{array}} - \boxed{\begin{array}{l}\text{The total energy of fluid}\\ \text{entering the system}\end{array}} \right\}$$

$$\Delta E = \underbrace{Q}_{} - W$$

is the heat absorbed per
unit mass of fluid

$$\Delta E = Q - \underbrace{W}$$

is the work of all kinds done per unit mass of fluid upon the surroundings

- In fluid mechanics we deal with shaft work (Ws).
- There are machines like pumps, turbines, fans and compressors in many fluid systems. The work done by these machines upon the fluid is called shaft work.

- There are 4 types of energy (E) a fluid carries within :
  1) Potential energy (PE)
  2) Kinetic energy (KE)
  3) Internal energy (U)
  4) Pressure energy (PV)

1) Potential Energy (P.E.): is the energy present because of the position of the mass in a gravitational field. It's unit is J/kg.

PE= (z.g)

(potential energy per unit mass of fluid)

z: relative height (m) from a reference plane

2) Kinetic Energy (K.E.): is the energy present because of translational or rotational motion of the mass. It's unit is J/kg.

$KE = (v^2/2\alpha)$

(kinetic energy per unit mass of fluid)

v : velocity (m/s) of the fluid.

α : Kinetic energy velocity correction factor

It is equal to ½ for laminar flow and equal to 1.0 for turbulent flow.

3) Internal Energy (U): is all of the other energy present, such as rotational and vibrational energy in chemical bonds. It’s unit is J/kg.
4) Pressure Energy (PV): Every fluid has a specific volume. PV can be interpreted as the energy you need to "create room" for the system. (Akışkan bir basınç uygulayarak borunun içine itilir)

   Pressure X Volume

- Hence the total energy carried with a unit mass of fluid is:

$$E = z.g + v^2/2\alpha + \underbrace{U + PV}_{H \text{ : enthalpy}}$$

Overall Energy Balance for Steady-State System

$$\Delta E = Q - W$$

E= z.g+ $v^2/2\alpha$+ U+ PV

$$\Delta H + \Delta\left(\frac{\upsilon^2}{2\alpha}\right) + g\Delta z = Q - W_s$$

α : Kinetic energy velocity correction factor

It is equal to ½ for laminar flow and equal to 1.0 for turbulent flow.

Overall Energy Balance for Steady-State System

$$\Delta H + \Delta\left(\frac{\upsilon^2}{2\alpha}\right) + g\Delta z = Q - W_s$$

$$H_2 - H_1 + \frac{1}{2\alpha}\left(\upsilon_2^2 - \upsilon_1^2\right) + g\left(z_2 - z_1\right) = Q - W_s$$

α : Kinetic energy velocity correction factor

It is equal to ½ for laminar flow and equal to 1.0 for turbulent flow.

Example 2.7.1

- Water enters a boiler at 18.33°C and 137.9 kPa through a pipe at an average velocity of 1.52 m/s . Exit superheated steam at a height of 15.2m above the liquid inlet leaves at 137.9 kPa, 148.9°C, and 9.14 m/s in the outlet line. At steady state, how much heat must be added per kg mass of steam(J/kg)? The flow in the two pipes is turbulent.

Saturated vs superheated steam

- In a boiler, energy from the fuel is transfered to liquid water in order to create steam. At first, cold water gets warmer and receives energy in the form of “sensible heat”, right until the boiling point.
- Once the boiling point is reached, the water’s temperature ceases to rise and stays the same until all the water is vaporized. The water goes from a liquid state to a vapour state and receives energy in the form of “latent heat of vaporization”. As long as there’s some liquid water left, the steam’s temperature is the same as the liquid water’s. Steam is then called saturated steam.
- When all the water is vaporized, any subsequent addition of heat raises the steam’s temperature. Steam heated beyond the saturated steam level is called superheated steam.

Saturated vs superheated steam

- Industries normally use saturated steam for heating, cooking, drying or other procedures.
- Superheated steam is used almost exclusively for turbines. The various types of steam have different energy exchange capacities and this justifies their different uses.
- Appendix A.2.9 and A2.10 can be used for the properties of saturated and superheated steam.
- How can you understand if a steam is saturated or superheated?

Example 2.7-2

- Water at 85°C is being stored in a large, insulated tank at atmospheric pressure. It is being pumped at steady state from this tank at point 1 by a pump at the rate of 0.567 m^3/min. The motor driving the pump supplies energy at the rate of 7.45 kW. The water passes through a heat exchanger, where it gives up 1408 kW of heat. The cooled water is then delivered to a second large open tank at point 2, which is 20 m above the first tank. Calculate the final temperature of the water delivered to the second tank. Neglect any kinetic energy changes since the initial and final velocities in the tank are essentially zero.

Example 2.7-2