FDE 205 FLUID MECHANICS

Shell Momentum Balance and Velocity Profile in Laminar Flow

- So far we analyzed momentum balances using an overall, macroscopic control volume.
- From this we obtained the total or overall changes in momentum crossing the control surface.
- This overall momentum balance did not tell us the details of what happens inside the control volume (inside the pipe).
- In this lecture we will analyze a small control volume and then shrink this control volume to differential size.
- Shell(kabuk): a shell is a differential element of flow. (akışın çok küçük boyutta bir kesiti)

- The most important reason for establishing a shell momentum balance (kabuk momentum denkliği) is to determine the velocity profile (hız profili) of the fluid inside the pipe.
- Velocity profile (hız profili): is the equation which helps us to determine the local velocity of the fluid inside the system.

  (sistemin içerisindeki bir noktada akışkanın hızını hesaplamamıza yarayan bir denklem)

Shell Momentum Balance Inside a Pipe

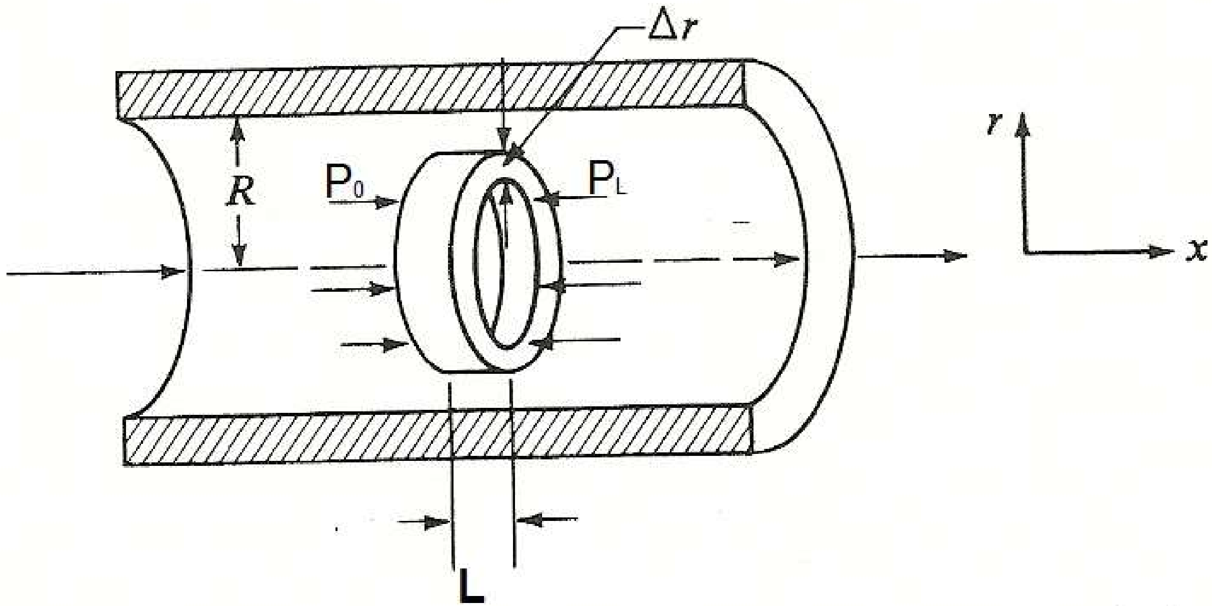


FIGURE 2.9-1. *Control volume for shell momentum balance on a fluid flowing in a circular tube.*

Shell Momentum Balance Inside a Pipe

- Assumption:

1) Incompressible, newtonian fluid
2) One directional flow
3) Steady State
4) Laminar flow
5) Fully developed flow : tam gelişmiş akış (akış borunun giriş kısmında değildir. Girişten kaynaklanan etkiler yok) ($V_x \neq f(x)$)
6) No slip boundary condition : sınır koşullarında kayma yok (akışkanın boruyla temas ettiği noktalarda hız=0)

Shell Momentum Balance Inside a Pipe

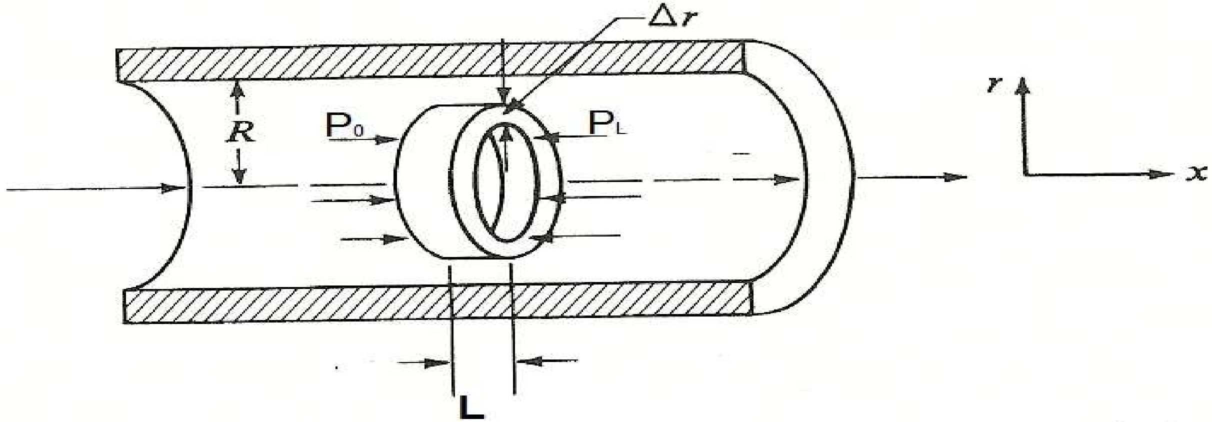


FIGURE 2.9-1. *Control volume for shell momentum balance on a fluid flowing in a circular tube.*

- Let's assume the cylindrical control volume is a shell with an inside radius r, thickness Δr and lenght L.

  (Boru içerisindeki sıvıdan iç çapı r, kalınlığı Δr ve uzunluğu L olan çok küçük bir kesit aldığımızı düşünelim .)

- Momentum Balance at steady state:

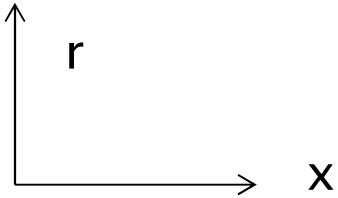
Forces acting on system=
Rate of momentum out−Rate of momentum in

- Forces acting on the fluid:

1) Pressure Forces(P.A)
2) Gravitational Forces(M.g)

- Momentum Transfer:

1. Convective momentum transfer (Akışkanın hızından kaynaklanan) ($\dot{m}\upsilon$)
2. Molecular momentum transfer (Moleküler aktarımla momentum transferi) ($\vec{\tau}_{rx}A$)



- Molecular momentum transfer:
  - If there is a fluid flowing in x direction, there will be a molecular momentum transport between the layers of the fluid in r direction.
  - This molecular momentum transport is caused by the shear stress

τ_{rx} –> x yönünde uygulanan kuvvet sebebiyle oluşan r yönündeki kayma gerilimi

τ_{rx} –> momentum akısı olarak da bilinir
(Momentum akısı: birim zamanda birim alandan akan momentum)

Forces acting on control volume =

(rate of momentum out−rate of momentum in)

$$\text{Pressure Forces} + \text{Gravitational Force} = \left[\begin{array}{c}\text{Rate of Mom. out by convective transport}\end{array} - \begin{array}{c}\text{Rate of Mom. in by convective transport}\end{array}\right] + \left[\begin{array}{c}\text{Rate of Mom. out by molecular transport}\end{array} - \begin{array}{c}\text{Rate of Mom. in by molecular transport}\end{array}\right]$$

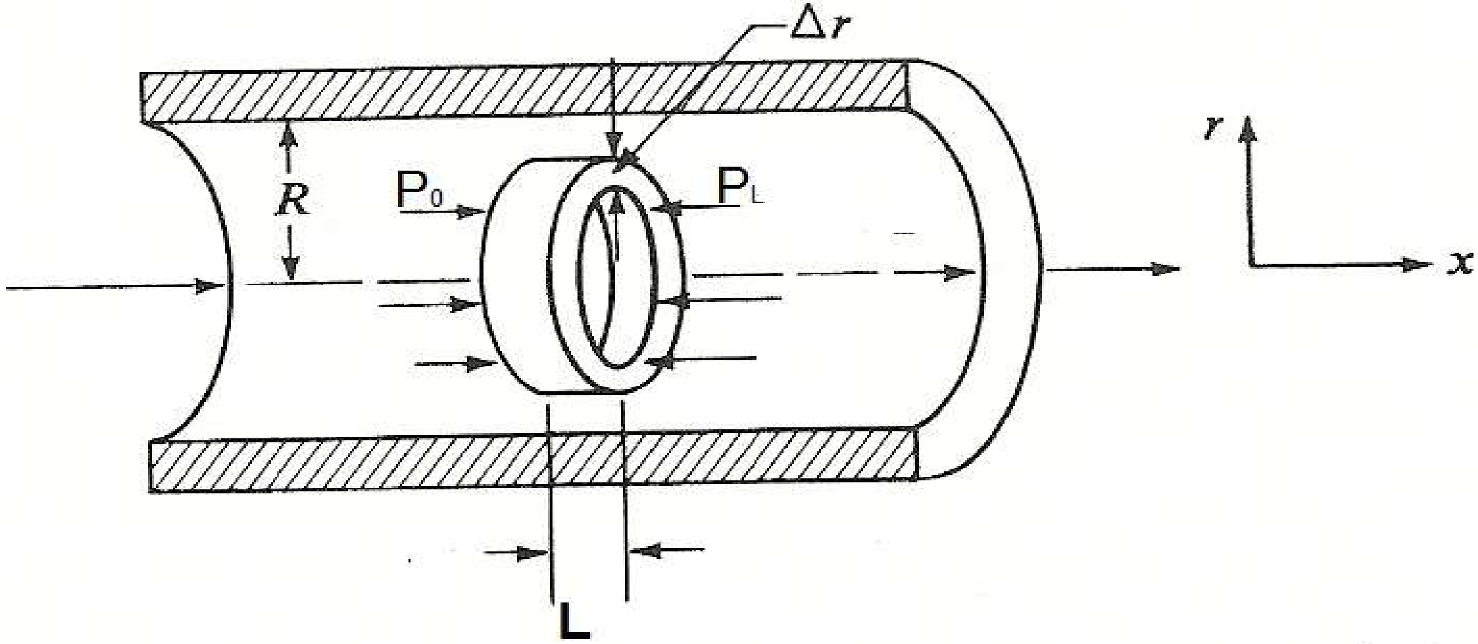


FIGURE 2.9-1. *Control volume for shell momentum balance on a fluid flowing in a circular tube.*

Velocity Profile

- For laminar flow at steady state, the velociy profile inside a pipe is a parabolic curve:

$$V_x = \left(\frac{P_0 - P_L}{4\mu L} \right) R^2 \left(1 - \frac{r^2}{R^2} \right)$$

Maximum velocity

- r=0 V=Vmax

$$V_x = \left(\frac{P_0 - P_L}{4\mu L} \right) R^2$$

Average velocity

- Average velocity for a cross section is found by summing up all the velocities over the cross section and dividing by the cross-sectional area

$$V_{x,average} = \left(\frac{P_0 - P_L}{8\mu L} \right) R^2 = \left(\frac{P_0 - P_L}{32\mu L} \right) D^2$$

- Hagen-Poiseuille equations, relates the pressure drop and average velocity for laminar flow in horizontal pipe.

Pressure Drop and Friction Loss in Pipes

- The pressure drop in Laminar flow in a pipe can be calculated by Hagen-Poiseuille equation.

$$\Delta P = \frac{8\mu L Q}{\pi R^4}$$

$$\Delta P = \frac{32\mu L < v >}{D^2}$$