Numerical Differentiation and Integration

Lecture 8

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In the previous lecture, we have learned how to calculate the derivative using Taylor Series Expansion.

In this lecture, we will learn using Finite-Divided Difference Formulas to calculate the derivative of a function numerically.

We will see three types of Finite-Divided Difference (FDD) formulas:

- Forward FDD
- Backward FDD
- Centered FDD

Forward Finite-Divided Difference Formulas:

First Derivative:

$$f^{(1)} = \frac{f(x_{i+1}) - f(x_i)}{h}, \qquad O(h)$$

$$f^{(1)} = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h}, \qquad O(h^2)$$

Second Derivative:

$$f^{(2)} = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}, \qquad O(h)$$

$$f^{(2)} = \frac{-f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)}{h^2}, \qquad O(h^2)$$

Backward Finite-Divided Difference Formulas:

First Derivative:

$$f^{(1)} = \frac{f(x_i) - f(x_{i-1})}{h}, \qquad O(h)$$

$$f^{(1)} = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h}, \qquad O(h^2)$$

Second Derivative:

$$f^{(2)} = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^2}, \qquad O(h)$$

$$f^{(2)} = \frac{2f(x_i) - 5f(x_{i-1}) + 4f(x_{i-2}) - f(x_{i-3})}{h^2}, \qquad O(h^2)$$

Centered Finite-Divided Difference Formulas:

First Derivative:

$$f^{(1)} = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}, \qquad O(h^2)$$

$$f^{(1)} = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}, \qquad O(h^4)$$

Second Derivative:

$$f^{(2)} = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1})}{h^2}, \qquad O(h^2)$$

$$f^{(2)} = \frac{f(x_{i+2}) + 16f(x_{i+1}) - 30f(x_i) + 16f(x_{i-1}) - f(x_{i-2})}{h^2}, \quad O(h^4)$$

Numerical Integration:

Trapezoidal Rule:

To calculate a definite integral numerically, one of the methods to be used is Trapezoidal Rule, which uses linear approximations of a function with equal intervals:

$$\int_a^b f(x)dx \approx \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n))$$

$$\Delta x = \frac{b-a}{n}$$

$$x_i = a + i\Delta x$$

Simpson's Rule:

Another method to calculate a finite integral numerically is Simpson's Rule. Rather than using straight lines like Trapezoidal Rule, Simpson's Rule makes use of quadratic polynomials:

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + ... + 4f(x_{n-1}) + f(x_n))$$

$$\Delta x = \frac{b-a}{n}$$

$$x_i = a + i\Delta x$$