

FEEDBACK CONTROL SYSTEMS

LECTURE NOTES-2/12

Electrical Network Modeling

Electric Circuits

Capacitor

$$v(t) = \frac{1}{C} \int i(t) dt$$

$$i(t) = C \frac{dv(t)}{dt}$$

$$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{Cs}$$

Resistor

$$v(t) = Ri(t)$$

$$i(t) = \frac{1}{R} v(t)$$

$$Z(s) = \frac{V(s)}{I(s)} = R$$

Inductor

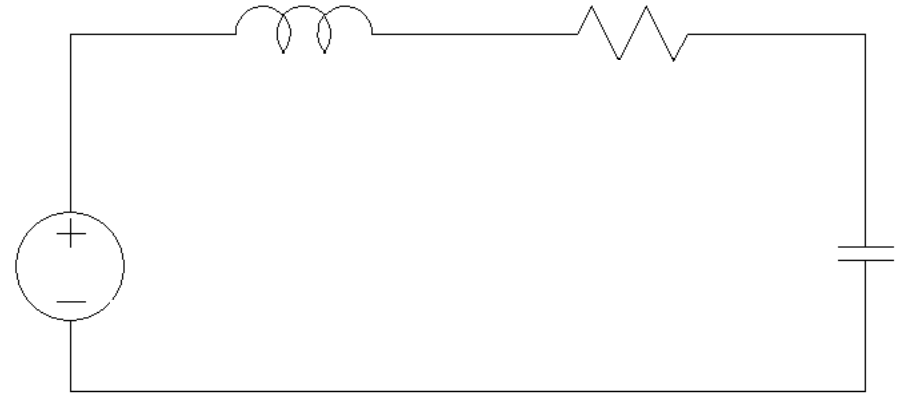
$$v(t) = L \frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L} \int v(t) dt$$

$$Z(s) = \frac{V(s)}{I(s)} = Ls$$

Example:

Obtain the transfer function $V_c(s)/V(s)$



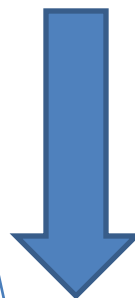
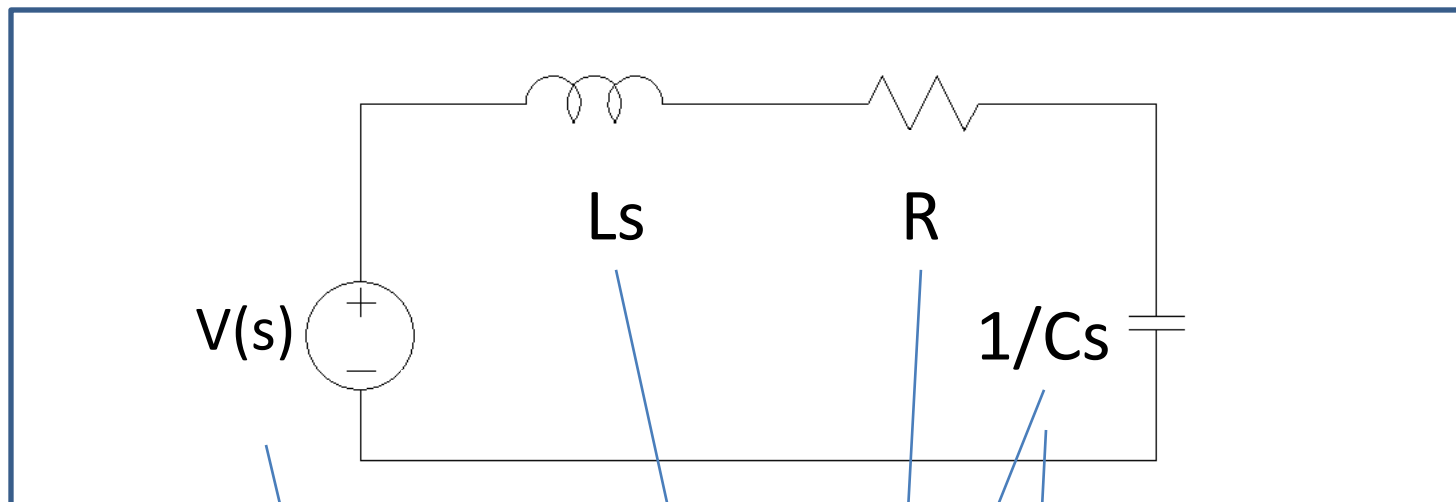
$$L \frac{di(t)}{dt} + Ri(t) + \frac{1}{C} \int_0^t i(\tau) d\tau = v(t) \quad \text{I}$$

$$LsI(s) + RI(s) + \frac{1}{C} \frac{1}{s} I(s) = V(s)$$

$$\frac{1}{C} \int_0^t i(\tau) d\tau = v_c(t) \quad \text{II}$$

$$\frac{1}{C} \frac{1}{s} I(s) = V_c(s)$$

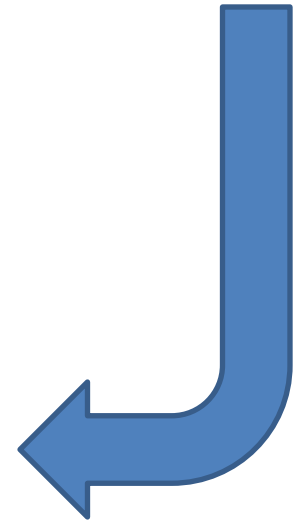
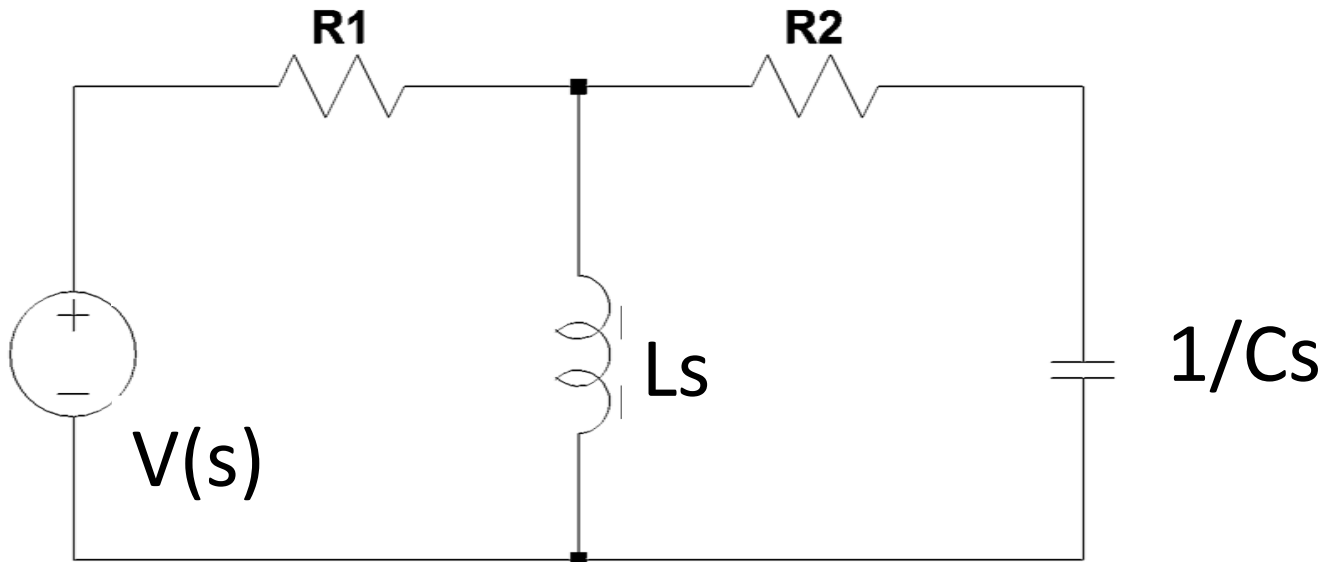
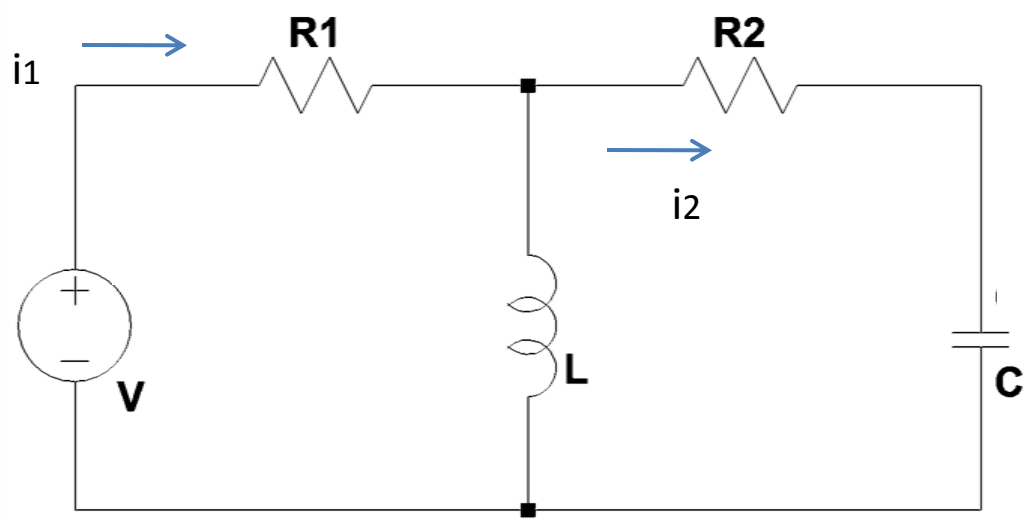
$$\frac{V_c(s)}{V(s)} = \frac{\frac{1}{C} \frac{1}{s} I(s)}{I(s) \left(Ls + R + \frac{1}{C} \frac{1}{s} \right)} \longrightarrow \frac{V_c(s)}{V(s)} = \frac{1}{(CLs^2 + RCs + 1)}$$

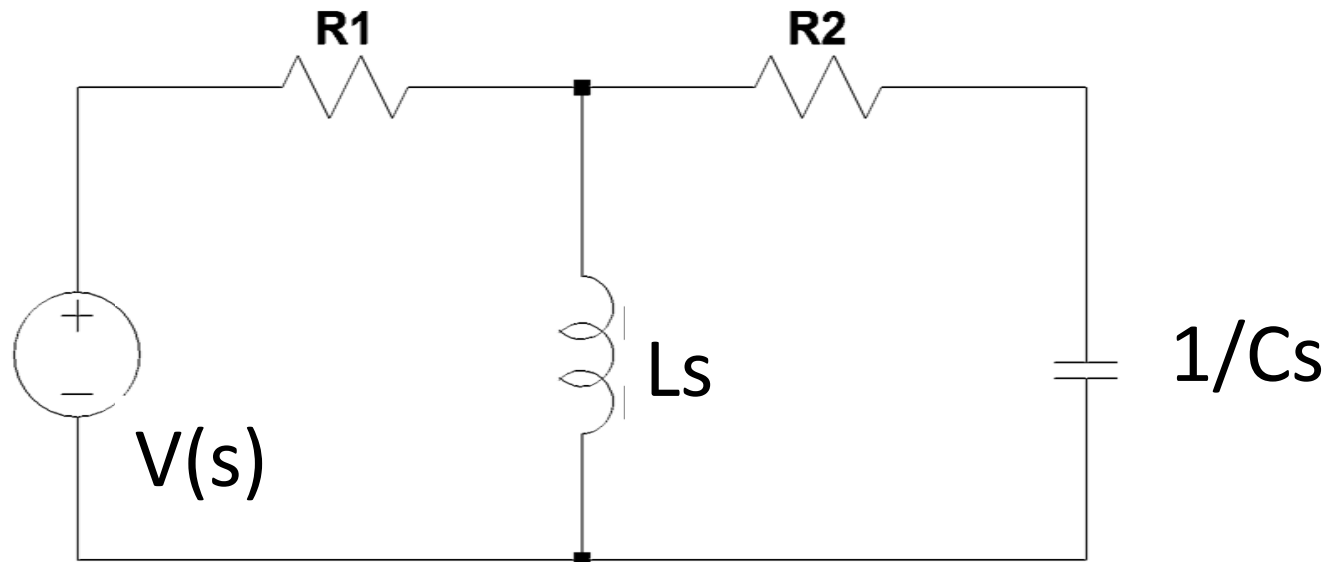


$$\frac{V_C(s)}{V(s)} = \frac{\frac{1}{C} \frac{1}{s} I(s)}{I(s) \left(Ls + R + \frac{1}{C} \frac{1}{s} \right)}$$

Example:

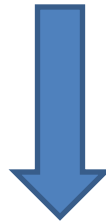
Obtain the transfer function $I_2(s)/V(s)$





$$(R_1 + Ls)I_1(s) - LsI_2(s) = V(s)$$

$$-LsI_1(s) + \left(Ls + R_2 + \frac{1}{Cs} \right) I_2(s) = 0$$



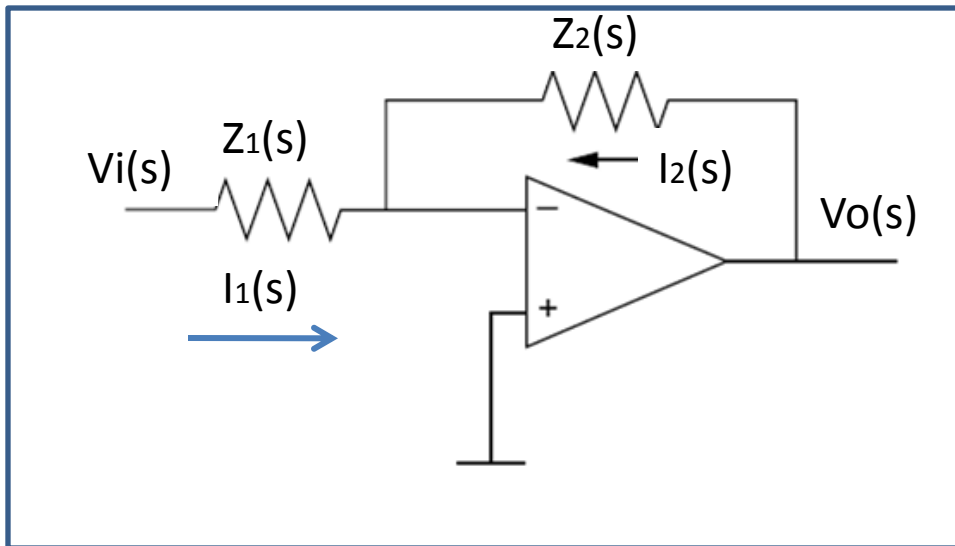
$$G(s) = \frac{I_2(s)}{V(s)} = \frac{LCs^2}{(R_1 + R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

Operational Amplifiers (OpAmp)

1. Differential input, $V_2(t)-V_1(t)$
2. High input impedance, $Z_i=\infty$
3. Low output impedance $Z_o=0$
4. High Constant gain amplification $A=\infty$

$$v_o(t) = A(v_2(t) - v_1(t))$$

Inverting OpAmp



$$I_1(s) = -I_2(s)$$

$$I_1(s) = \frac{V_i(s)}{Z_1(s)}$$

$$-I_2(s) = -\frac{V_o(s)}{Z_2(s)}$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

Mechanical System Modeling

Spring

$$f(t) = K \int v(t) dt$$

$$f(t) = Kx(t)$$

$$Z(s) = \frac{F(s)}{X(s)} = K$$

Viscous Damper

$$f(t) = f_v v(t)$$

$$f(t) = f_v \frac{dx(t)}{dt}$$

$$Z(s) = \frac{F(s)}{X(s)} = f_v s$$

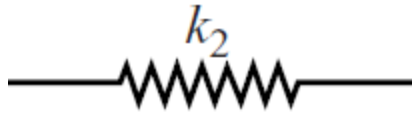
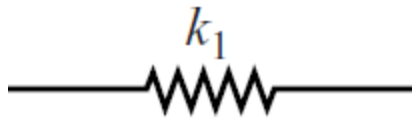
Mass

$$f(t) = M \frac{dv(t)}{dt}$$

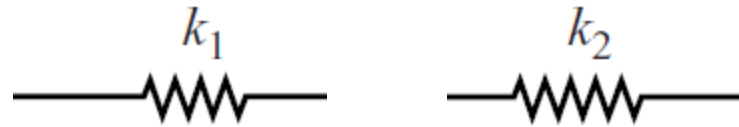
$$f(t) = M \frac{d^2 x(t)}{dt^2}$$

$$Z(s) = \frac{F(s)}{X(s)} = Ms^2$$

Spring

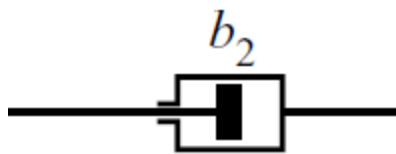
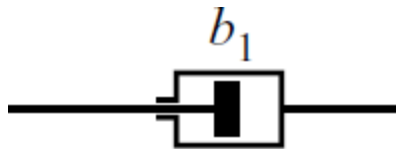


$$k_1 + k_2$$

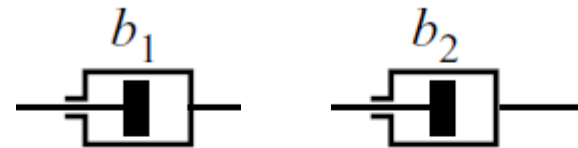


$$\frac{1}{\frac{1}{k_1} + \frac{1}{k_2}}$$

Damper



$$b_1 + b_2$$



$$\frac{1}{\frac{1}{b_1} + \frac{1}{b_2}}$$

Rotational Mechanical System Modeling

Spring

$$T(t) = K \int w(t) dt$$

$$T(t) = K\theta(t)$$

$$Z(s) = \frac{T(s)}{\theta(s)} = K$$

Damper

$$T(t) = Dw(t)$$

$$T(t) = D \frac{d\theta(t)}{dt}$$

$$Z(s) = \frac{T(s)}{\theta(s)} = Ds$$

Inertia

$$T(t) = J \frac{dw(t)}{dt}$$

$$T(t) = J \frac{d^2\theta(t)}{dt^2}$$

$$Z(s) = \frac{T(s)}{\theta(s)} = Js^2$$