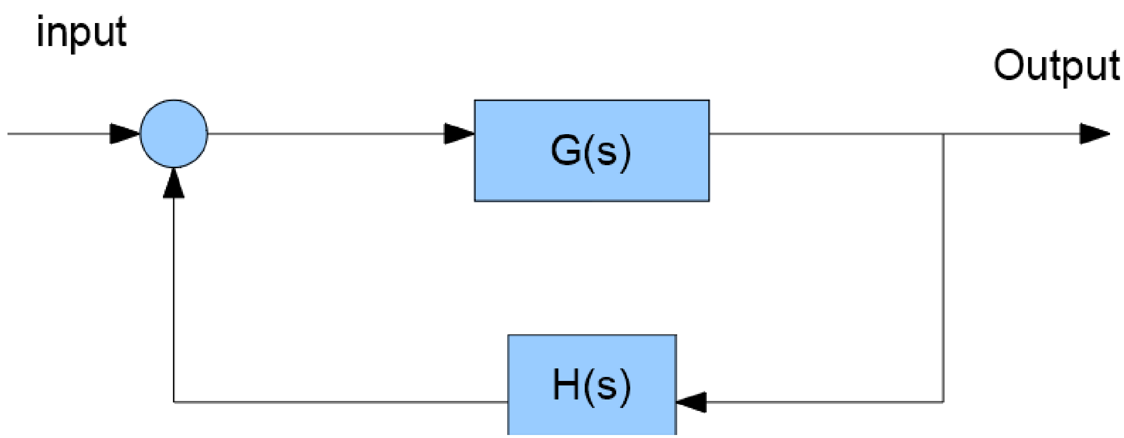
FEEDBACK CONTROL SYSTEMS

LECTURE NOTES-3/12

Feedback Control System Characteristics

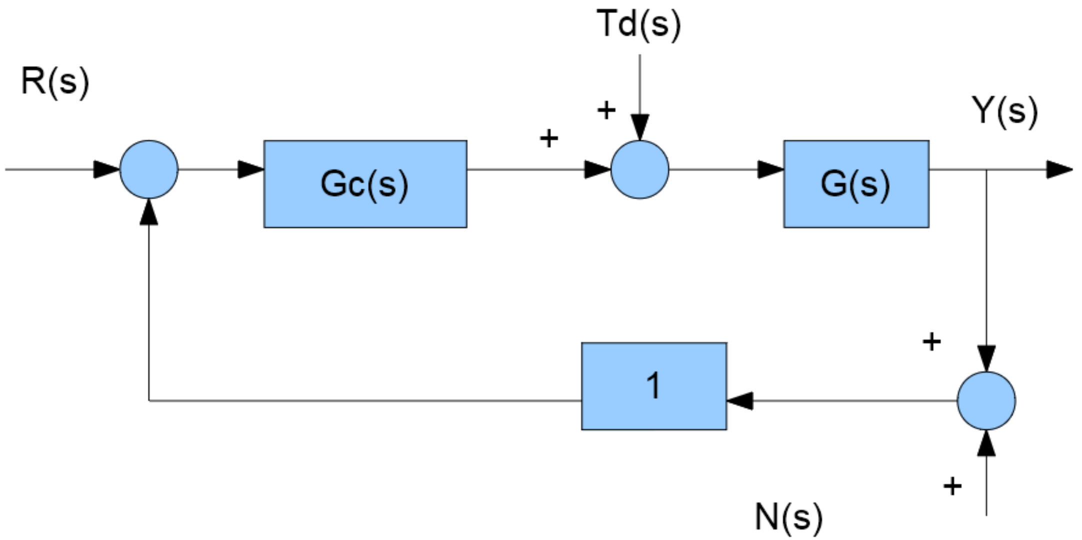
The role of error signal, sensitivity to parameter change explored
Aim is to minimize the error signal



A closed loop system uses a measurement from sensor of the output signal and compared with the desired reference signal to generate error that is used by controller to adjust the plant

Closed-loop feedback control advantages:

Decrease sensitivity
Improved rejection to disturbance
Improved reduction to steady state error
Easy control and adjustment the transient response



Inputs : R(s), Td(s) disturbance and N(s) measurement noise
Output: Y(s)

Tracking Error E(s) $E(s) = R(s) - Y(s)$

$$Y(s) = \frac{Gc(s)G(s)}{1+Gc(s)G(s)}R(s) + \frac{G(s)}{1+Gc(s)G(s)}Td(s) - \frac{Gc(s)G(s)}{1+Gc(s)G(s)}N(s)$$

$$E(s) = \frac{1}{1+Gc(s)G(s)}R(s) - \frac{G(s)}{1+Gc(s)G(s)}Td(s) + \frac{Gc(s)G(s)}{1+Gc(s)G(s)}N(s)$$

$$Y(s) = \frac{Gc(s)G(s)}{1+Gc(s)G(s)}R(s) + \frac{G(s)}{1+Gc(s)G(s)}Td(s) - \frac{Gc(s)G(s)}{1+Gc(s)G(s)}N(s)$$

If Gc(s)G(s) >>1 then Y(s)=R(s) that cause highly oscillatory and unstable system

Effect of Uncertainty:

Assume no disturbance on the system

$$E(s) = \frac{1}{1+Gc(s)G(s)}R(s)$$

$$E(s) + \Delta E(s) = \frac{1}{1+Gc(s)\left(G(s)+\Delta G(s)\right)}R(s)$$

$$\Delta E(s) = \frac{-Gc(s)\Delta G(s)}{\left(1+Gc(s)G(s)+Gc(s)\Delta G(s)\right)\left(1+Gc(s)G(s)\right)}R(s)$$

$$\Delta E(s) \approx -\frac{1}{L(s)}\frac{\Delta G(s)}{G(s)}R(s)$$

-Larger magnitude L(s)
→ Small S(s)
→Small change at E(s)

System Sensitivity

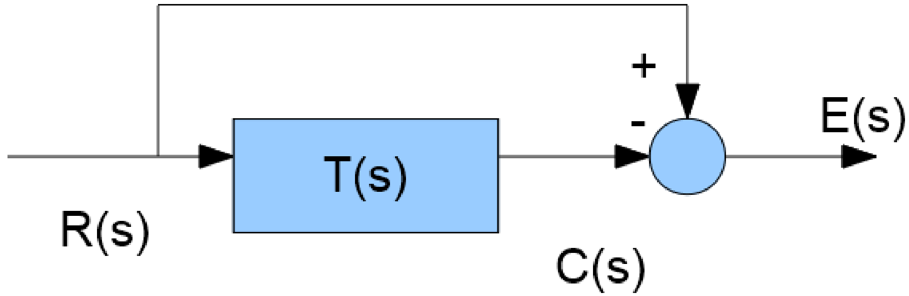
Ratio of percentage change in system transfer function to Percentage change of process transfer function

System sensitivity is the ratio of the change (differentiation) with respect to the change of a process for a small incremental change

$$T(s) = \frac{Y(s)}{R(s)}$$

$$S = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{\partial T}{\partial G}\frac{G}{T}$$

$$S = \frac{\frac{\Delta T(s)}{T(s)}}{\frac{\Delta G(s)}{G(s)}}$$



Apply final value theorem

$$C(s) = R(s)T(s)$$

$$E(s) = R(s) - C(s)$$

$$E(s) = R(s)(1 - T(s))$$

$$e(\infty) = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

$$e(\infty) = \lim_{s \to \infty} sR(s)(1 - T(s))$$

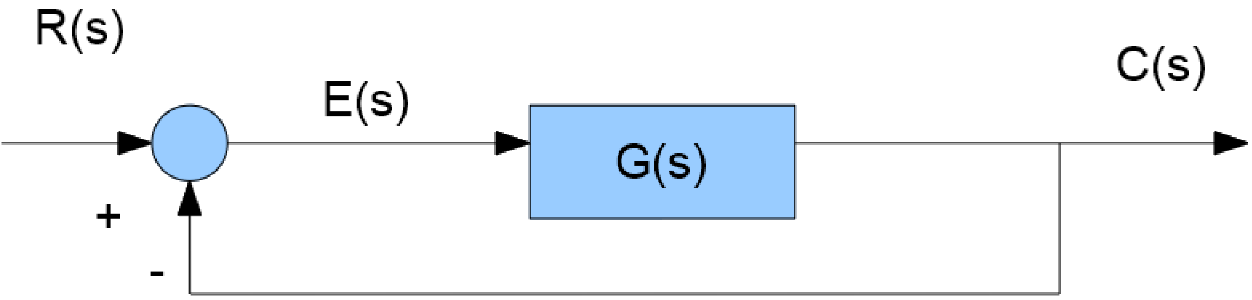
Example: Input is unit step

$$T(s) = \frac{5}{s^2 + 7s + 10}$$

$$R(s) = \frac{1}{s}$$

$$E(s) = \frac{s^2 + 7s + 5}{s(s^2 + 7s + 10)}$$

$$e(\infty) = 0.5$$



$$E(s) = R(s) - C(s)$$

$$C(s) = E(s)G(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)}$$

$$e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}$$

Step Input

$$e(\infty) = \lim_{s \to 0} \frac{s\frac{1}{s}}{1 + G(s)} = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

Ramp Input

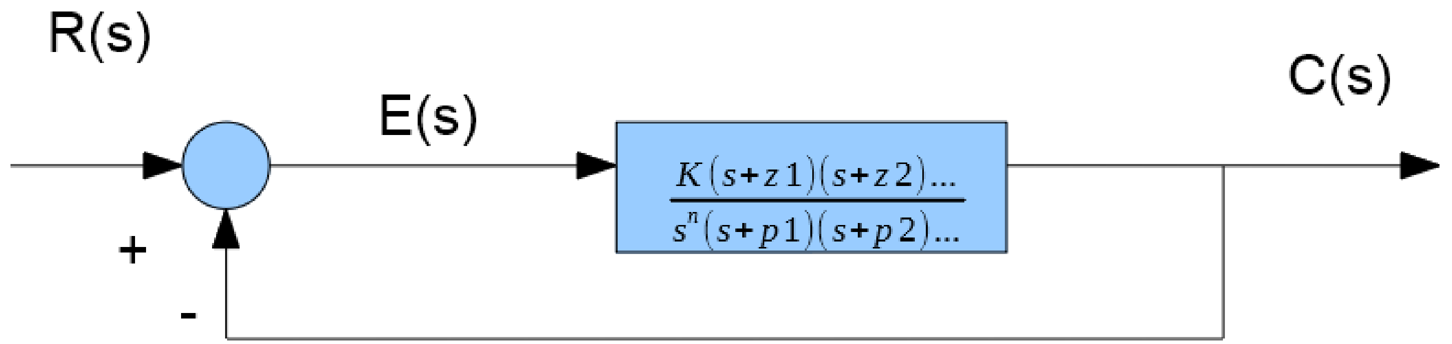
$$e(\infty) = \lim_{s \to 0} \frac{s\frac{1}{s^2}}{1 + G(s)} = \frac{1}{\lim_{s \to 0} sG(s)}$$

Parabolic Input

$$e(\infty) = \lim_{s \to 0} \frac{s\frac{1}{s^3}}{1 + G(s)} = \frac{1}{\lim_{s \to 0} s^2G(s)}$$

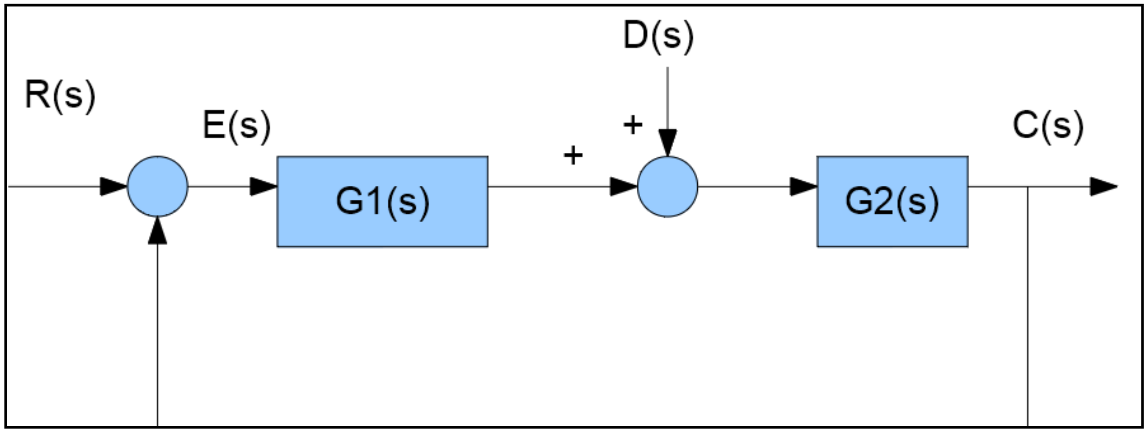
System Type and Static Error Constants

Step Input	$e(\infty) = \dfrac{1}{1 + \lim\limits_{s \to 0} G(s)}$	Position constant	$Kp = \lim\limits_{s \to 0} G(s)$
Ramp Input	$e(\infty) = \dfrac{1}{\lim\limits_{s \to 0} sG(s)}$	Velocity constant	$Kv = \lim\limits_{s \to 0} sG(s)$
Parabolic Input	$e(\infty) = \dfrac{1}{\lim\limits_{s \to 0} s^2 G(s)}$	Acceleration constant	$Ka = \lim\limits_{s \to 0} s^2 G(s)$



	Type 0	Type 1	Type 2
	Static Error Constant	Static Error Constant	Static Error Constant
Step Input	Kp=Constant	Kp=inf	Kp=inf
Ramp Input	Kv=0	Kv=Constant	Kv=inf
Parabola Input	Ka=0	Ka=0	Ka=Constant

Steady State Error for Distrubances



$$C(s) = E(s)G_1(s)G_2(s) + D(s)G_2(s)$$
$$C(s) = R(s) - E(s)$$

$$E(s) = \frac{1}{1+G_1(s)G_2(s)}R(s) - \frac{G_2(s)}{1+G_1(s)G_2(s)}D(s)$$

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{s}{1+G_1(s)G_2(s)}R(s) - \lim_{s\to 0}\frac{sG_2(s)}{1+G_1(s)G_2(s)}D(s)$$

$$e_R(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{s}{1+G_1(s)G_2(s)}R(s)$$

$$e_D(\infty) = \lim_{s\to 0} sE(s) = -\lim_{s\to 0}\frac{sG_2(s)}{1+G_1(s)G_2(s)}D(s)$$