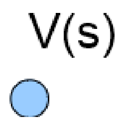
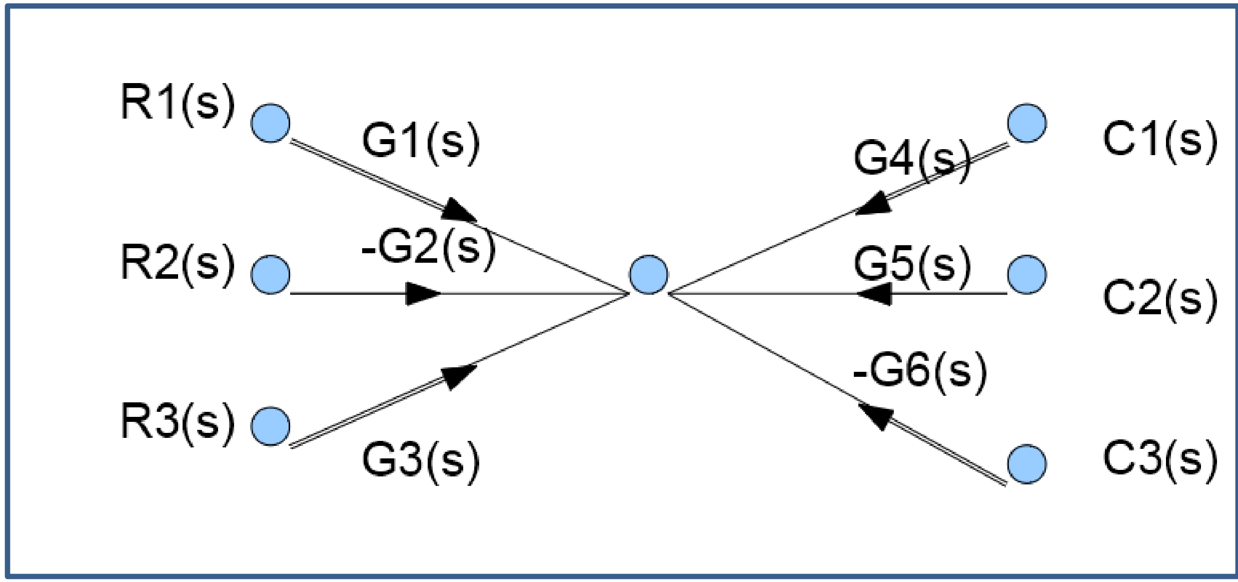
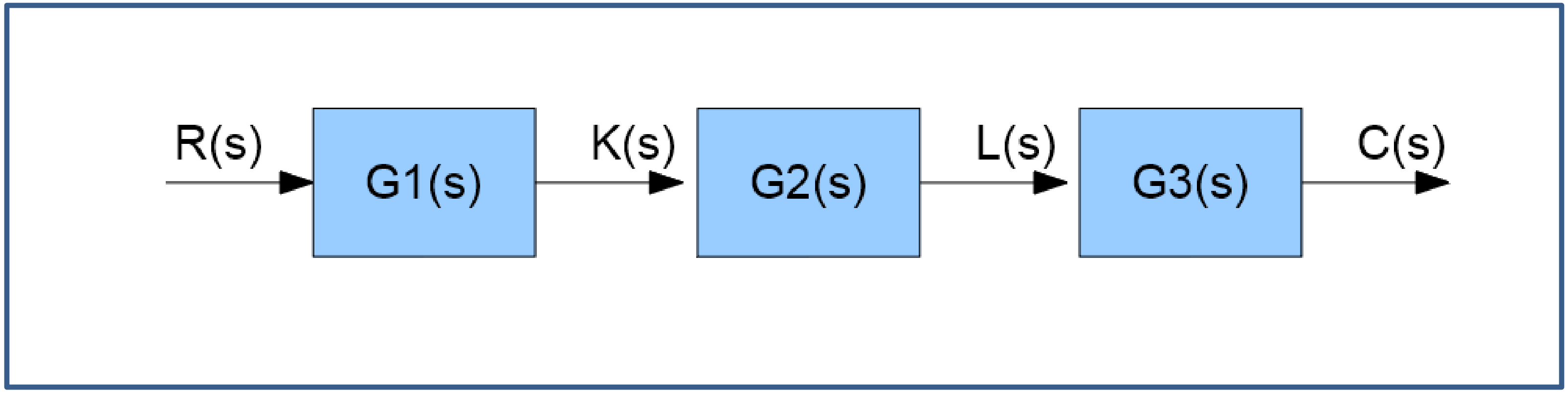
FEEDBACK CONTROL SYSTEMS

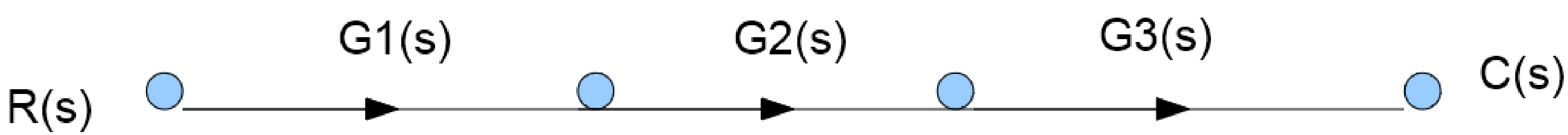
LECTURE NOTES-5/12

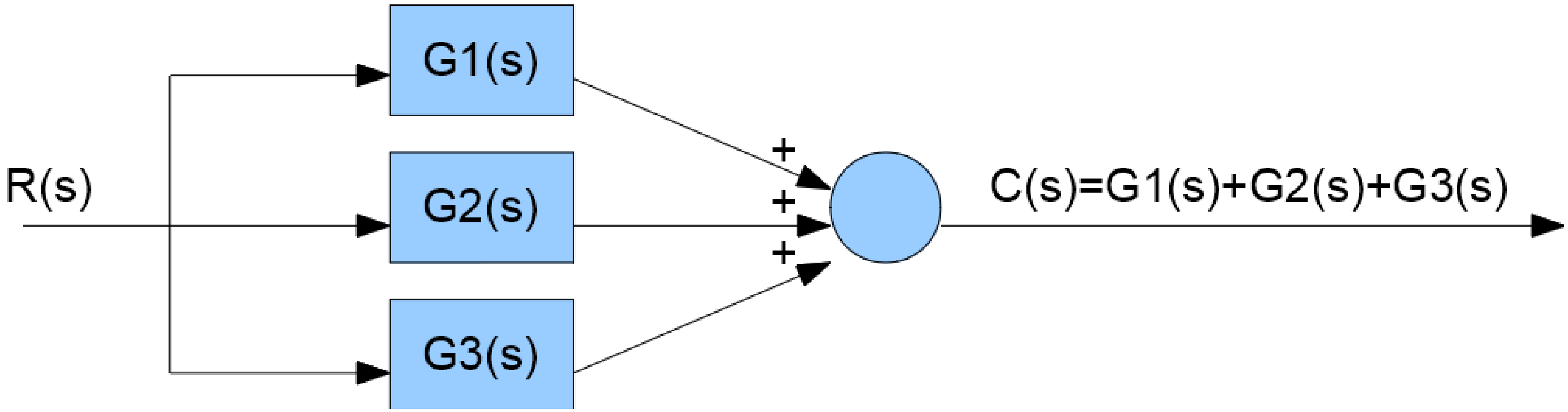
Signal-Flow Diagram

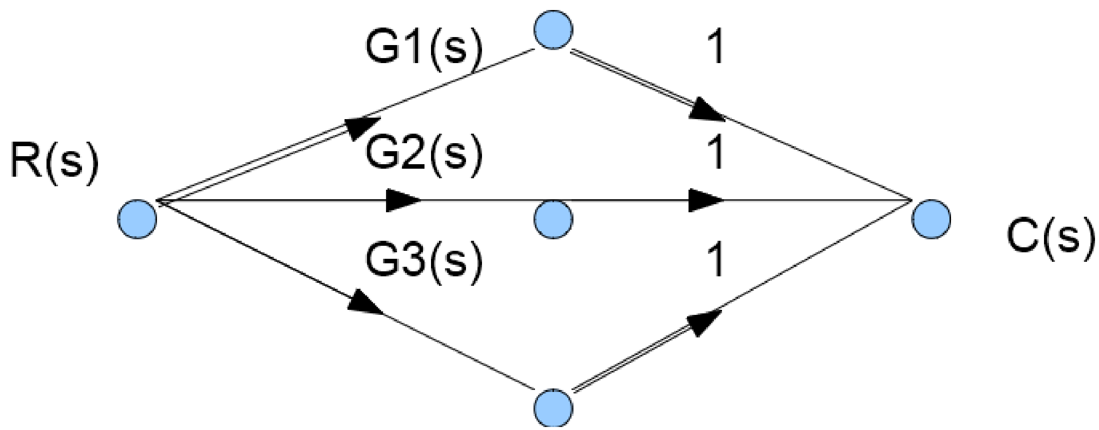


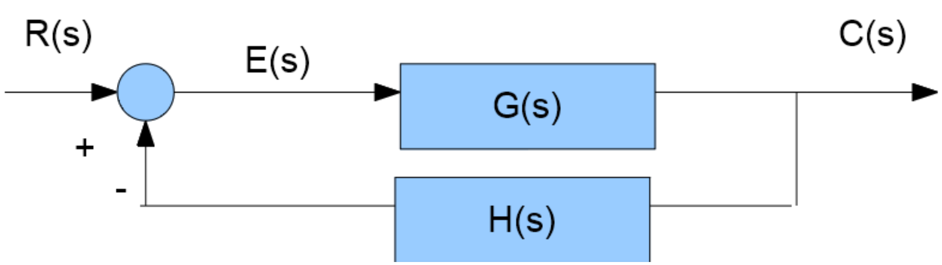


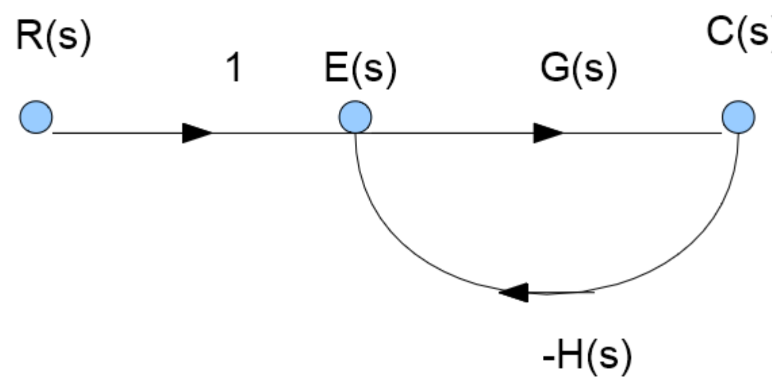




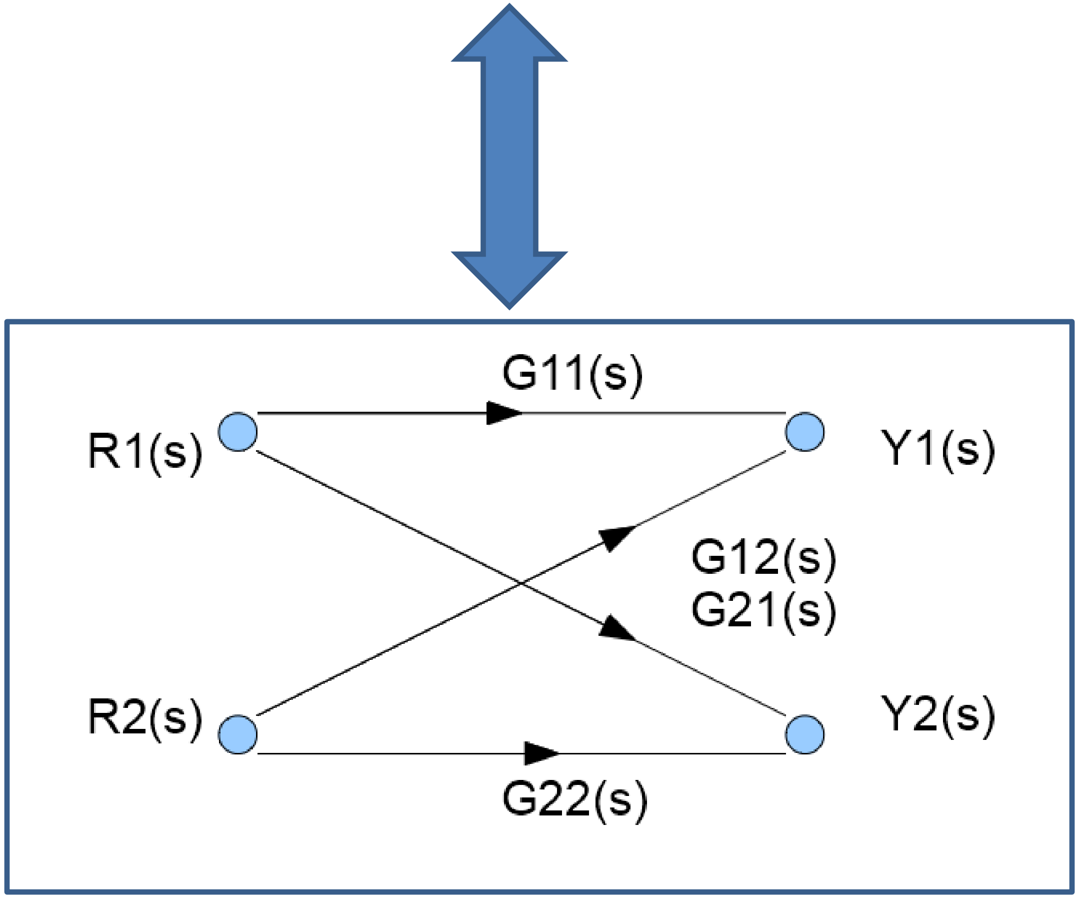

G1(s)
G2(s)
G3(s)
R(s)
+
+
+
C(s)=G1(s)+G2(s)+G3(s)


G1(s)
1
R(s)
G2(s)
1
C(s)
G3(s)
1


R(s)
E(s)
C(s)
G(s)
+
-
H(s)

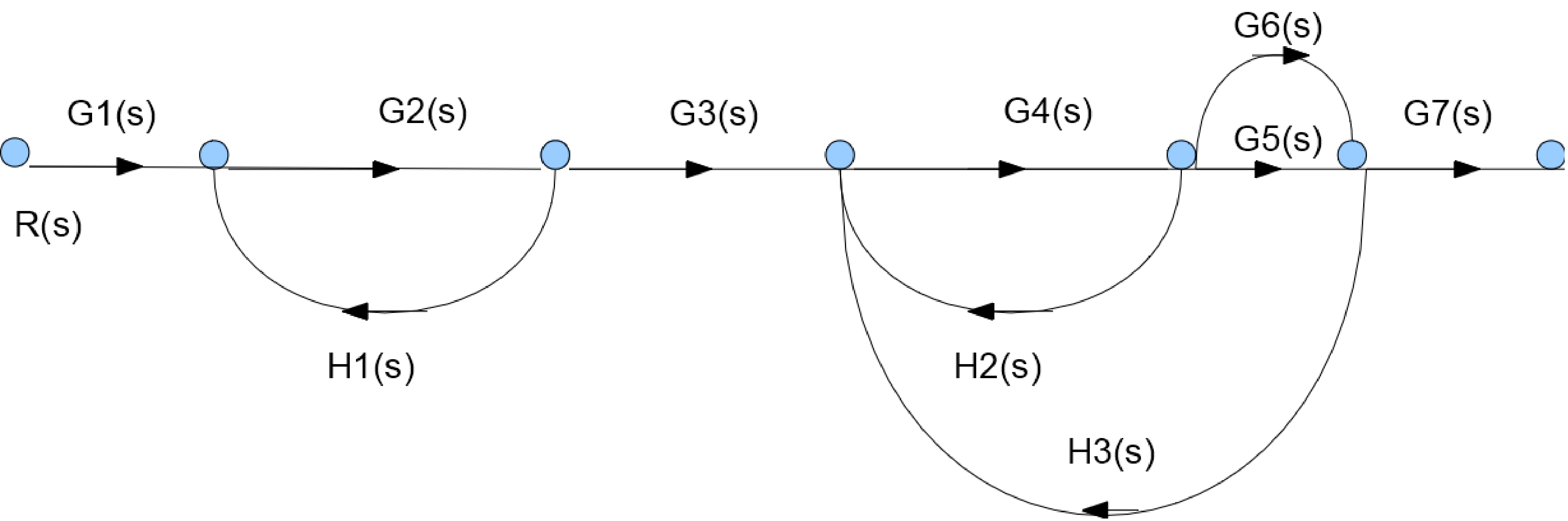

R(s)
1
E(s)
G(s)
C(s)
-H(s)

$$Y_1(s) = G_{11}(s)R_1(s) + G_{12}(s)R_2(s)$$
$$Y_2(s) = G_{21}(s)R_1(s) + G_{22}(s)R_2(s)$$



Mason's Rule – From Signal flow to Transfer Function

Loop Gain: Starts at a node and ends at the same node following the direction of the Signal flow without passing through any other node more than once.



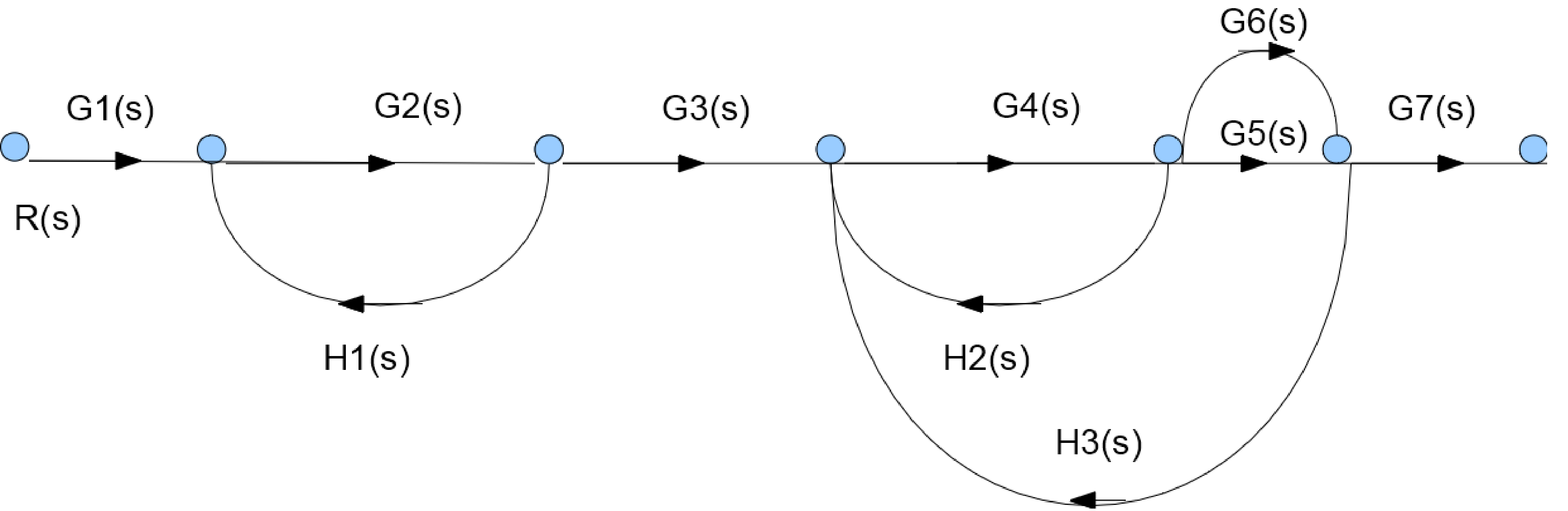
$$1. G_2(s)H_1(s)$$

$$2. G_4(s)H_2(s)$$

$$3. G_4(s)G_5(s)H_3(s)$$

$$4. G_4(s)G_6(s)H_3(s)$$

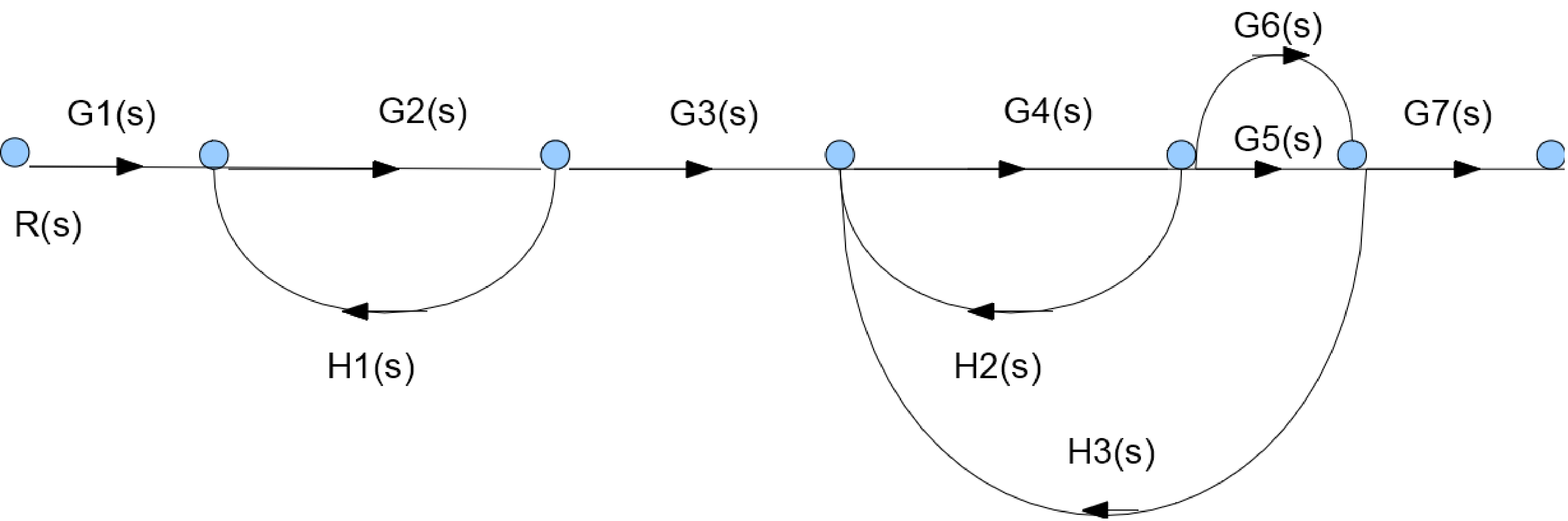
Forward-path Gain: Passing a path from the input to the output node of the signal Flow graph in the direction of signal flow.



$$1. G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$$

$$2. G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$$

Nontouching Loops: Loops that do not have any nodes in common



1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

Mason's Rule:

The transfer function C(s)/R(s) of a system is represented by

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{k} Ti\Delta i}{\Delta}$$

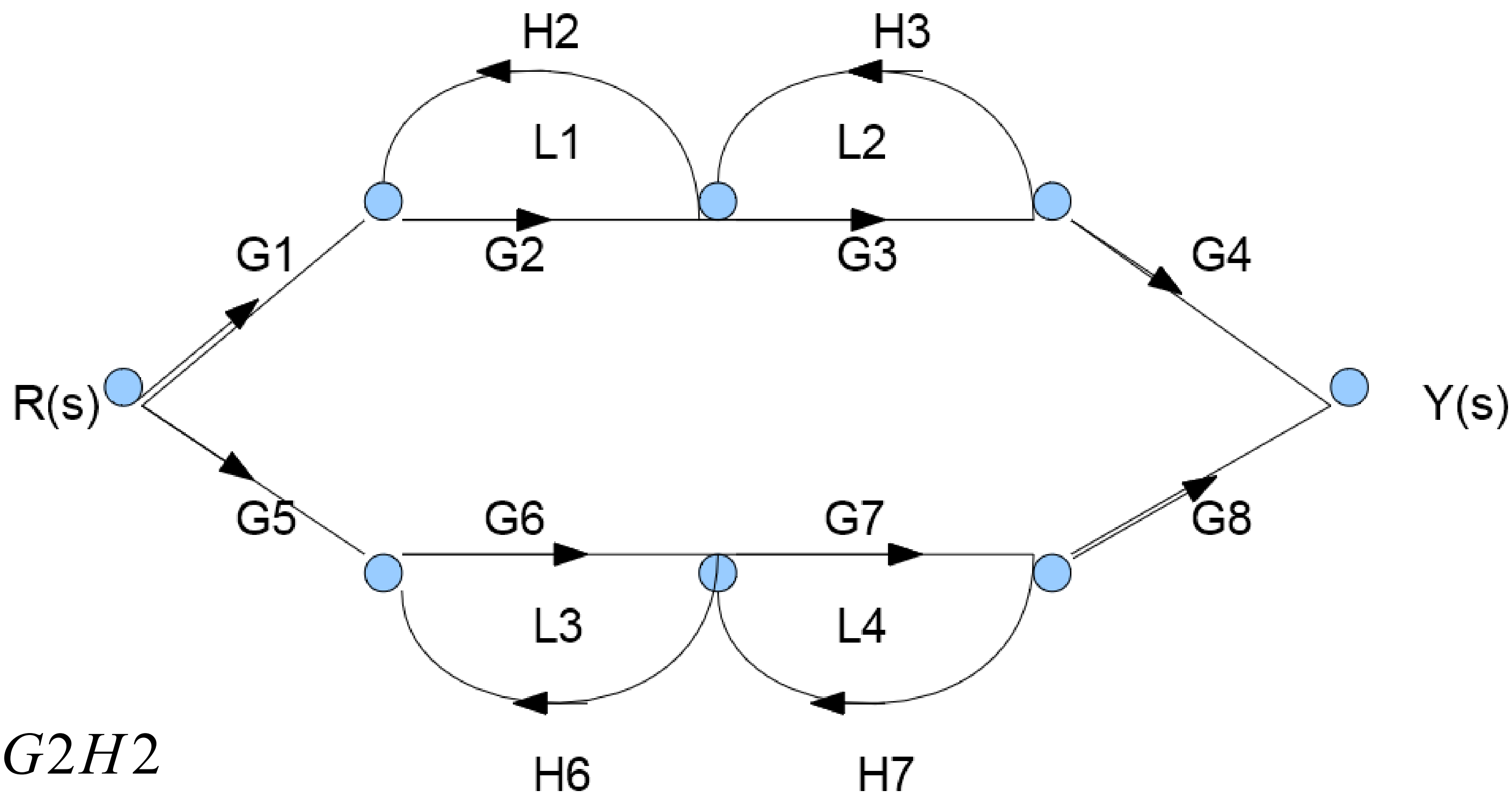
•k=number of forward paths

•Ti=the ith forward path gain

•Δ=1- (Sum)loop gains + (Sum)nontouching-loop gains taken two – (Sum)non-touching-loop gains take three + …

•Δi=Δ-(Sum)loop gain terms in Δ that touch the ith forward path

Example: Find the transfer function Y(s)/R(s) for the signal flow graph



$$L1 = G2H2$$

$$L2 = H3G3$$

$$L3 = G6H6$$

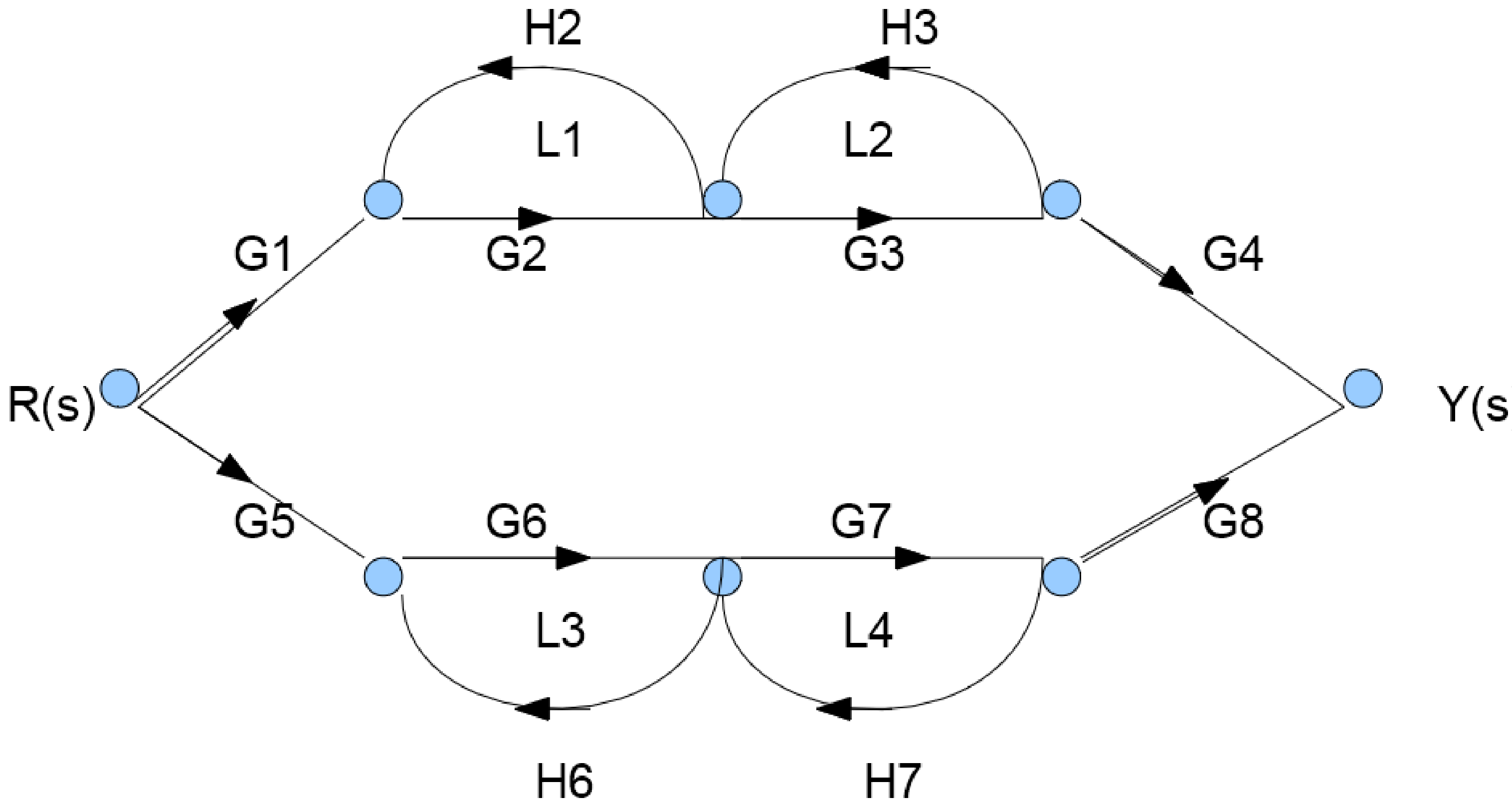
$$L4 = G7H7$$

L1 and L2 do not touch L3 and L4

$$\Delta = 1 - (L1 + L2 + L3 + L4) + (L1L3 + L1L4 + L2L3 + L2L4)$$

$$\Delta 1 = 1 - (L3 - L4)$$

$$\Delta 2 = 1 - (L1 + L2)$$



$$T(s) = \frac{Y(s)}{R(s)} = \frac{P1\Delta 1 + P2\Delta 2}{\Delta}$$

$$T(s) = \frac{G1G2G3G4(1 - L3 - L4) + G5G6G7G8(1 - L1 - L2)}{1 - L1 - L2 - L3 - L4 + L1L3 + L1L4 + L2L3 + L2L4}$$