FEEDBACK CONTROL SYSTEMS

LECTURE NOTES-10/12

Remember:

1) Number of branches = number of poles
2) Symmetrical about the real axis
3) Real-axis segments are to the left of an odd number of finite poles or zeros
4) Root locus begins with poles and ends at zeros
5) Asymptotes interceptions and angles

$$\sigma_a = \frac{\text{sum of finite poles - sum of finite zeros}}{\text{number of finite poles - number of finite zeros}}$$

$$\theta_a = \frac{(2m+1)\pi}{\text{number of finite poles - number of finite zeros}}$$

$$m = 0, \pm 1, \pm 2, \ldots$$

6) Real-axis break-in and breakaway points

$$K(\sigma) = -\frac{1}{G(\sigma)H(\sigma)}$$

$$\frac{dK(\sigma)}{d\sigma} = 0$$

7) Imaginary axis crossings

Remember: **Root Locus**

Example: Sketch the root locus for the system and find the following:

a. The exact point and gain where the locus crosses the jw-axis
b. The breakaway point on the real axis
c. The range of K within which the system is stable

$$G(s) = \frac{K(s^2 - 4s + 20)}{(s+2)(s+4)}$$

$$H(s) = 1$$

$$\sigma_a = \frac{\text{sum of finite poles - sum of finite zeros}}{\text{number of finite poles - number of finite zeros}}$$

$$\theta_a = \frac{(2m+1)\pi}{\text{number of finite poles - number of finite zeros}}$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Use the Routh-Hurwitz criterion such that a row of zeros will yield the gain

$$-\theta_1 + \theta_2 + \theta_3 - \theta_4 - \theta_5 + \theta_6 = (2k+1)180$$

$$KG(s)H(s) = -1$$

Zero=+
Pole=-

$$\frac{dK}{d\sigma} = 0$$

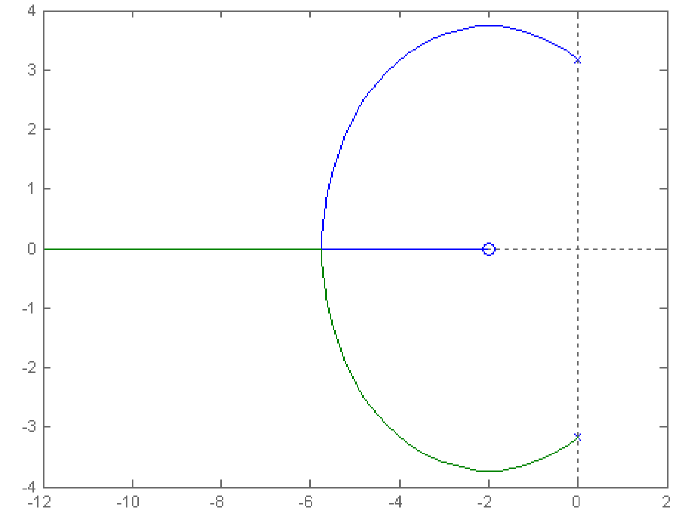
Matlab and Root Locus

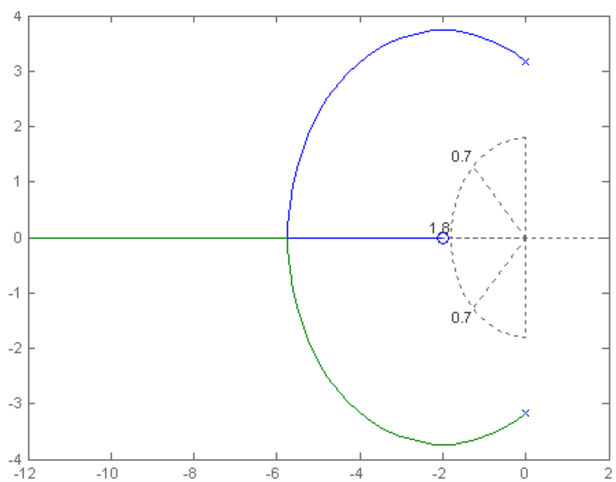
Remember:

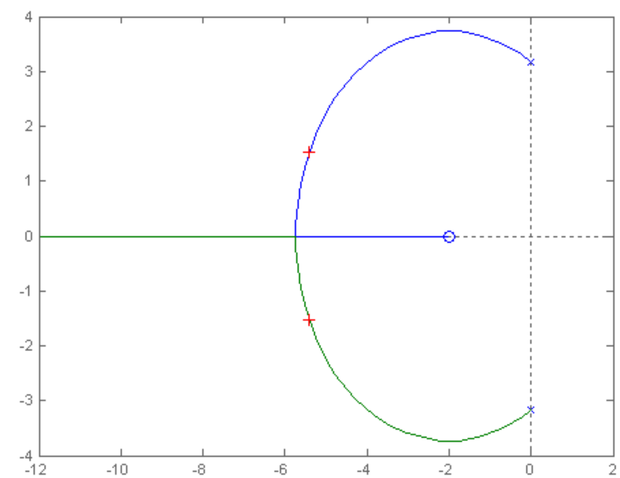
$$\frac{s + 2}{s^2 + 10}$$

```
>> sys = tf([1 2],[1 0 10])
>> rlocus(sys)
>> rlocfind(sys)

>> rlocus(sys)
>> Zeta = 0.7;
>> Wn = 1.8;
>> sgrid(Zeta,Wn)
```







Solution: 1) replace the all system with another one (not possible or expensive)

2) *Compansate* with additional poles and zeros (open-loop)

Disavantage: Increase the system order.

Question: What should be the proper locations of the additional poles and/or zeros to yield the **desired** second order **closed-loop poles**?
(Compansators not only used to improve transient response but also improve steady state error)

Improving Steady-State Error

Since the transient response and the static error constant were related to the gain.

The higher the gain, the smaller the steady-state error, but the larger the percent overshoot.

On the other hand, reducing gain to reduce overshoot increased the steady-state error.

Compansator use pure integration for improving steady-state error
or pure differentiation for improving transient response

} ideal compensators

Ideal
Compensators
Implementation

Active:
Electric networks, active amplifiers
Additional source

Steady state error → zero

Passive:
Resistors and capacitors

Less expensize
Steady state not driven zero

What improvement can we expect in the steady-state error?

$$K_{VN} = K_{VO} \frac{z_c}{p_c} > K_{VO}$$

a) In order to keep the transient response unchanged, compensator pole and zero must be close to each other

b) The ration of zc to pc can be large for improvement in steady state at the same time pole and zero close to each other. For example pc=-0.001 and zc=-0.01

Improving Transient Response via Cascade Compensation

Design a system with a given overshoot and settling time

1) Ideal derivative compensation: pure differentiator is added
Requires active network → proportional-plus-derivative (PD) controller

2)Passive network: adding zero and more distant pole –> lead compensator

Ideal Derivative Compensation (PD)

Begin with choosing an appropriate closed-loop location on s-plane

Root-locus must be re-shaped → poles and zeros are added

Speed-up original system by adding a single zero

$$G_c(s) = s + z_c$$

Improving Steady-State Error and Transient Response

Combining steady-state error and transient response techniques

(1) Improve steady state (2) improve transient response
Disadvantage: improvement transient response cause deterioration is steady-state error

(1) improve transient response (2) Improve steady state

First active PD controller then active PI controller = PID Controller

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_1 s + K_2 + K_3 s^2}{s}$$

$$G_c(s) = \frac{K_3\left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K3}\right)}{s}$$

a) Determine how much improvement need in transient response

b) Design Pıfor steady state error

c) Determine gains

Lag compensator

Velocity constant Kv is $$Kv = \lim_{s \to 0} sG(s)$$

$$e(\infty) = e_{ramp}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)}$$

$$G(s) = \frac{192.1}{s(s+6)(s+10)}$$

The static error constant is inversely proportional to the steady state error which is equal to 3.201

$$G_{LC}(s) = \frac{1977}{s(s+10)(s+29.1)}$$

The static error constant is inversely proportional to steady state error which is 6.794

The additional lead compensator has improved the steady state by 4.713

We can arbitrary choose the lag compensator pole at 0.01 and zero at 0.04713

$$G_{lag}(s) = \frac{s+0.04713}{s+0.01}$$