FEEDBACK CONTROL SYSTEMS

LECTURE NOTES-11/12

Stability, Gain Margin, Phase Margin

The gain margin is found by using the phase plot to find the frequency, where the phase angle is 180.

The magnitude plot to determine the gain margin, GM, which is the gain required to raise the magnitude curve to 0 dB.

The phase margin is found by using the magnitude curve to find the frequency, where the gain is 0 dB.

On the phase curve at that frequency, the phase margin is the difference between the phase value and 180.

negative margins in an open-loop system indicate instability

Design Procedure

1. Set the gain K to the value that satisfies the steady state error specification and plot the Bode magnitude and phase diagrams for this value of gain

2. Find the frequency where the phase margin is 5 and 12 greater that the phase margin that yields the desired transient response. This step compensates for the fact that the phase of the lag compensator may still contribute anywhere from -5 to -12 of phase at phase-margin frequency

3. Select a lag compensator whose magnitude response yields a composite Bode magnitude diagram that goes through 0 dB at the frequency found in step 2 as: Draw the compensator high frequency asymptote to yield 0 dB for the compensated system at the frequency found in step 2. If the gain at the frequency is 20logKpm, than compensator high frequency asymptote will be set at -20logKpm; select the upper break frequency to be 1 decade below the frequency found in step 2 and select the low frequency asymptote to be at 0 dB, connect the compensator high and low frequency asymptotes with a -20dB/decade line to locate the lower break frequency

4. Reset the system gain K to compensate for any attenuation in the lag network in order to keep the static error constant the same as that found in step 1

Consider the system

$$G(s) = \frac{1}{s(s+1)(0.5s+1)}$$

It is desired to compansate the system so that the static velocity error constant Kv is 5

Phase margin is at least 40 and the gain margin is at least 10 dB

$$Gc(s) = Kc\beta \frac{Ts+1}{\beta Ts+1} = Kc \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}, (\beta > 1)$$

$$G1(s) = KG(s) = \frac{K}{s(s+1)(0.5s+1)}$$

$$Kc\beta = K$$

The first step in the design is to adjust the gain K to meet the required static velocity error constant

$$Kv = \lim_{s\to 0} sGc(s)G(s) = \lim_{s\to 0} s\frac{Ts+1}{\beta Ts+1}G_1(s) = \lim_{s\to 0} sG_1(s)$$

$$Kv = \lim_{s\to 0} \frac{sK}{s(s+1)(0.5s+1)} = K = 5$$

$$K = 5$$

The addition of a lag compensator modifies the phase curve of the Bode diagram, then we must add 5 to 12 to the specific phase margin to compensate for the modification of the phase

Since the frequency corresponding to a phase margin of 40 is 0.7 rad/sec, the new gain crossover frequency must be chosen near this value. To avoid overly large time constant for the lag compensator, we should choose the corner frequency w=1/T which corresponds to the zero of the lag compensator to be 0.1 rad/sec.

We add about 12 to the given phase margin as an allowance to account for the lag angle introduced by the lag compensator. The required phase margin is 52. the phase angle of the uncompensated open loop transfer function is -128 at about w=0.5rad/sec. We choose the new gain crossover frequency to be 0.5rad/sec. To bring the magnitude curve down to 0 dB at this new ghain crossover frequency, the lag compensator must give the necessary attenuation which in this case is -20 dB.

$$20\log\frac{1}{\beta} = -20$$

$$\beta = 10$$

$$w = \beta T$$

$$\frac{1}{\beta T} = 0.01$$

$$Gc(s) = Kc(10)\frac{10s+1}{100s+1} = Kc\frac{s+\frac{1}{10}}{s+\frac{1}{100}}$$

$$Kc = \frac{K}{\beta} = \frac{5}{10} = 0.5$$

$$Gc(s)G(s) = \frac{5(10s+1)}{s(100s+1)(s+1)(0.5s+1)}$$

LEAD COMPENSATION

1. Assume the following lead compensator:

The open-loop transfer function of the compensated system is

$$Gc(s) = Kca\frac{Ts+1}{aTs+1} = Kc\frac{s+\frac{1}{T}}{s+\frac{1}{aT}}, (0 < a < 1)$$

$$Kca = K$$

$$Gc(s) = K\frac{Ts+1}{aTs+1}$$

$$Gc(s)G(s) = K\frac{Ts+1}{aTs+1}G(s) = \frac{Ts+1}{aTs+1}KG(s) = \frac{Ts+1}{aTs+1}G_1(s)$$

$$G_1(s) = KG(s)$$

Determine gain K to satisfy the requirement on the given static error constant.

2. Using the gain K thus determined, draw a Bode diagram G(jw), the gain adjusted but uncompensated system. Evaluate the phase margin

3. Determine the necessary phase lead angle to be added to the system. Add an additional 5 to 12 to the phase lead angle required, because the addition of the lead compensator shifts the gain crossover frequency to the right and decreases the phase margin

4. Determine the attenuation factor a. Determine the frequency where the magnitude of the uncompansated system G(jw) is equal to -20log(1/sqrt(a)). Select this frequency as the new gain crossover frequency. This frequency corresponds to wm=1/sqrt(aT), and maximum phase shift occurs at this frequency.

5. Determine the corner frequencies of the lead comnpansator as:

Zero of lead compensator: w=1/T
Pole of lead compensator: w=1/aT

6. Using the value of K and a, calculate Kc=K/a

7. Check the gain margin to be sure it is satisfactory. If not repeat the design process by modifying the pole zero location of the compansator until a satisfactort result is obtained.

The open loop transfer function is

$$G(s) = \frac{4}{s(s+2)}$$

It is desired to design a compensator for the system so that the static velocity error constant Kv is 20 and the phase margin is at least 50 and the gain margin is at least 10 dB

$$Gc(s) = Kca\frac{Ts+1}{aTs+1} = Kc\frac{s+\frac{1}{T}}{s+\frac{1}{aT}}$$

The compansated system will have the open loop transfer function Gc(s)G(s)

where K=Kca

$$G_1(s) = KG(s) = \frac{4K}{s(s+2)}$$

The first step in the design is to adjust the gain K to meet the steady state performance specification or to provide the required static velocity error constant. Since this constant is given as 20

$$Kv = \lim_{s\to 0} sGc(s)G(s) = \lim_{s\to 0} s\frac{Ts+1}{aTs+1}G_1(s) = \lim_{s\to 0}\frac{s4K}{s(s+2)} = 2K = 20$$

$$K = 10$$

$$G_1(jw) = \frac{40}{jw(jw+2)} = \frac{20}{jw(0.5jw+1)}$$

LAG-LEAD COMPENSATION

$$Gc(s) = Kc\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)$$

$$\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}} = \frac{1}{\gamma}\left(\frac{T_1 s+1}{\frac{T_1}{\gamma}s+1}\right), (\gamma > 1)$$

$$\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}} = \beta\left(\frac{T_2 s+1}{\beta T_2 s+1}\right), (\beta > 1)$$

The frequency w1 is the frequency at which the phase angle is zero. It is given by

$$w_1 = \frac{1}{\sqrt{T_1 T_2}}$$