Experiment – 9 Extended Surface Experiment

Aim of this Experiment

The Extended Surface Heat Transfer experiment allows investigation of one-dimensional conduction from a fin.

Experimental Set – up

The Extended Surface Heat Transfer experiment allows investigation of one-dimensional conduction from a fin. A small diameter metal rod is heated at one end and the remaining exposed length is allowed to cool by natural convection and radiation. This results in a diminishing temperature distribution along the bar that is measured by regularly spaced thermocouples.

The accessory comprises a 10mm diameter brass rod (1) of approximately 350mm effective length mounted horizontally with a support (6) at the heated end and a mounting steady (10) at the opposite end. Inside an insulated housing (3) is a 240V electric heater E1(5) in direct contact with the brass rod. The heater has a nominal power rating of approximately 30 Watts at 240V AC. The power supplied to the heated cylinder is provided by the Heat Transfer Service Unit H111 through the power lead (4). The Heat Transfer Service Unit H111 also allows the operator to vary the power input to the heater by control of the voltage supply to the heater element.

For safety purposes a thermostat (2) limits the maximum temperature of the heater to approximately 150°C.
Eight thermocouples (11) are located at 50mm intervals along the rod to record the surfacetemperature. These connect to the Heat Transfer Service Unit H111 through the miniature plugs (12). The thermocouples are attached to the rod in order to minimise errors from conductioneffects.

An additional thermocouple (9) is mounted on the unit to record the ambient air temperature.

In order to protect the thermocouples from damage all lead terminations are mounted firmly into trunking and conduit (8).

The rod is coated with a heat resistant matt black paint in order to provide a constant radiantemissivity close to 1.

**Schematic Representation of Linear Conduction Experiment Unit**
Capabilities Of The Extended Surfaces Unit

1. Measuring the temperature distribution along an extended surface and comparing the result with a theoretical analysis.

2. Calculating the heat transfer from an extended surface resulting from the combined modes of free convection and radiation heat transfer and comparing the result with a theoretical analysis.

3. Determining the constant of proportionality (the thermal conductivity $k$) of the rod material.
Operating Procedure Of Extended Surfaces Unit

1. Ensure that the main switch is in the off position (the three digital displays should not be illuminated). Ensure that the residual current circuit breaker on the rear panel is in the ON position.

2. Turn the voltage controller fully anti-clockwise to set the AC voltage to minimum. Ensure the Extended Surface Heat Transfer accessory has been connected to the HeatTransfer Service Unit.

3. Ensure that the heated cylinder (7) is located inside its housing (10) before turning on power to the unit.

4. Turn on the main switch, digital displays should illuminate. Set the temperature selector to display T1. **Rotate the voltage controller to increase the voltage to that specified in the procedure for each experiment.**

5. After adjusting the heater voltage ensure that T1 (the thermocouple closest to the heater) varies in accordance with the sense of adjustment. i.e if the voltage has been increased the temperature T1 should also increase, if the voltage is reduced the temperature T1 will reduce. Note that if T1 is close to 100°C and the current (Amps) displays zero, it may be that the safety thermostat (2) has activated. Reduce the voltage and wait for the thermostat to reset.

6. Allow the system to reach stability, and take readings and make adjustments as instructed in the individual procedures for each experiment. Note that due to the process of conduction and the small differential temperatures involved for reasons of safety the time taken to achieve stability can be long.

7. When the experimental procedure is completed, it is good practice to turn off the power to the heater by reducing the voltage to zero and monitor T1 until the rod has cooled. Then turn off the main switch. Allow the components to cool completely to ambient before storing them away safely.
Experiment -9.1

Measuring the temperature distribution along an extended surface and comparing the result with a theoretical analysis

Aim of This Experiment

This experiment aims to measure the temperature distribution along an extended surface and compare the practical results with a theoretical analysis.

Procedure

Following the basic OPERATING PROCEDURE and set the voltage controller to give a 120 volt reading.

Select the temperature position T1 using the rotary selector switch and monitor the temperature REGULARLY until T1 reaches approximately 80°C then reduce the heater voltage to approximately 70 volts. This procedure will reduce the time taken for the system to reach a stable operating condition.

It is now necessary to monitor temperature T1 to T8 until all the temperatures are stable.

When T1 through T8 have reached a steady state temperature record the following: t1 to t9, V and I.

If time permits increase the voltage to a 120 volt reading, repeat the monitoring of all temperatures and when stable repeat the above readings.

Once readings have been completed the voltage may be reduced to zero in order to allow the rod to cool. Finally, turn off the main switch. The theory being demonstrated, sample observations are shown in the following pages.

Sample Test Results

<table>
<thead>
<tr>
<th>Sample No</th>
<th>1</th>
<th>2</th>
<th>Distance From T1 (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V volts</td>
<td>69</td>
<td>117</td>
<td>-</td>
</tr>
<tr>
<td>I Amps</td>
<td>0.039</td>
<td>0.073</td>
<td>-</td>
</tr>
<tr>
<td>t1 ºC</td>
<td>43.3</td>
<td>85.2</td>
<td>0</td>
</tr>
<tr>
<td>t2 ºC</td>
<td>36</td>
<td>64.2</td>
<td>0.05</td>
</tr>
<tr>
<td>t3 ºC</td>
<td>30.9</td>
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<td>0.1</td>
</tr>
<tr>
<td>t4 ºC</td>
<td>28.4</td>
<td>41.4</td>
<td>0.15</td>
</tr>
<tr>
<td>t5 ºC</td>
<td>25.5</td>
<td>34.5</td>
<td>0.2</td>
</tr>
<tr>
<td>t6 ºC</td>
<td>24.8</td>
<td>31.6</td>
<td>0.25</td>
</tr>
<tr>
<td>t7 ºC</td>
<td>23.3</td>
<td>28.5</td>
<td>0.3</td>
</tr>
<tr>
<td>t8 ºC</td>
<td>23.6</td>
<td>28.5</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Experiment 9.2

Calculating the heat transfer from an extended surface resulting from the combined modes of free convection and radiation heat transfer and comparing the result with a theoretical analysis

Aim of This Experiment

This experiment aims to calculate measure the heat transfer from an extended surface resulting from the free convection and radiation heat transfer an extended surface and compare the practical results with a theoretical analysis.

Procedure

Following the basic OPERATING PROCEDURE and set the voltage controller to give a 120 volt reading.

Select the temperature position T1 using the rotary selector switch and monitor the temperature REGULARLY until T1 reaches approximately 80°C then reduce the heater voltage to approximately 70 volts. This procedure will reduce the time taken for the system to reach a stable operating condition.

It is now necessary to monitor temperature T1 to T8 until all the temperatures are stable.

When T1 through T8 have reached a steady state temperature record the following: t1 to t9, V and I.

If time permits increase the voltage to a 120 volt reading, repeat the monitoring of all temperatures and when stable repeat the above readings.

Once readings have been completed the voltage may be reduced to zero in order to allow the rod to cool. Finally, turn off the main switch. The theory being demonstrated, sample observations are shown in the following pages.

Sample Test Results

<table>
<thead>
<tr>
<th>Sample No</th>
<th>V volts</th>
<th>I Amps</th>
<th>Distance From T1 (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>69</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td>t1</td>
<td>43.3</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>t2</td>
<td>36</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>t3</td>
<td>30.9</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>t4</td>
<td>28.4</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>t5</td>
<td>25.5</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>t6</td>
<td>24.3</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>t7</td>
<td>23.3</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>t8</td>
<td>23.5</td>
<td>0.35</td>
<td></td>
</tr>
</tbody>
</table>
Experiment 9.3

Determining the constant of proportionality (the thermal conductivity $k$) of the rod material.

Aim of This Experiment

This experiment aims to determine thermal conductivity of the rod material.

Procedure

Following the basic OPERATING PROCEDURE and set the voltage controller to give a 120 volt reading.

Select the temperature position T1 using the rotary selector switch and monitor the temperature REGULARLY until T1 reaches approximately 80°C then reduce the heater voltage to approximately 70 volts. This procedure will reduce the time taken for the system to reach a stable operating condition.

It is now necessary to monitor temperature T1 to T8 until all the temperatures are stable.

When T1 through T8 have reached a steady state temperature record the following: $t_1$ to $t_9$, $V$ and $I$.

If time permits increase the voltage to a 120 volt reading, repeat the monitoring of all temperatures and when stable repeat the above readings.

Once readings have been completed the voltage may be reduced to zero in order to allow the rod to cool. Finally, turn off the main switch. The theory being demonstrated, sample observations are shown in the following pages.

Sample Test Results

<table>
<thead>
<tr>
<th>Sample No</th>
<th>$t_1$ (°C)</th>
<th>$t_2$ (°C)</th>
<th>$t_3$ (°C)</th>
<th>$t_4$ (°C)</th>
<th>$t_5$ (°C)</th>
<th>$t_6$ (°C)</th>
<th>$t_7$ (°C)</th>
<th>$t_8$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V (volts)</td>
<td>65</td>
<td>36</td>
<td>30.9</td>
<td>28.4</td>
<td>25.5</td>
<td>24.8</td>
<td>23.3</td>
<td>23.6</td>
</tr>
<tr>
<td>I (amps)</td>
<td>0.029</td>
<td>0.051</td>
<td>0.056</td>
<td>0.059</td>
<td>0.051</td>
<td>0.052</td>
<td>0.051</td>
<td>0.053</td>
</tr>
<tr>
<td>Distance From T1 (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Theory of Experiments

A pin of length \( L \) diameter \( D \) and cross-sectional area \( A \) (Perimeter \( P \)) and thermal conductivity \( k \) is heated at one end. It has a total surface area \( A_s \) and is in an ambient at temperature \( T_a \).

Due to the heat input from one end, the temperature of the bar is raised above that of the surroundings and heat is convected and radiated away from the surface. As the heat input is from one end only and the bar is thermally conductive, the temperature will vary along the bar from \( T_1 \) at the hot end to \( T_8 \) at the far end. It is assumed that the bar is sufficiently long for there to be negligible heat transfer from the tip. At any distance \( x \) from the heated end the temperature of the material is \( T_x \).

The heat is convected and radiated away from the bar with an overall convection coefficient \( h \).

From the above conditions it is possible to develop the following differential equation to describe conditions along the bar.

\[
\frac{d^2 T_x}{dx^2} - \frac{hP}{kA_s}(T_x - T_a) = 0
\]

Where Perimeter

\[
P = \frac{dA_s}{dx}
\]

By introducing \( \theta = T_x - T_a \)

The equation becomes

\[
\frac{d^2 \theta}{dx^2} - \frac{hP}{kA_s} \theta = 0
\]
If we introduce

\[ m^2 = \frac{hP}{kd} \]

Then the equation can be written

\[ \frac{d^2 \theta}{dx^2} - m^2 \theta = 0 \]

It can be shown that the general solution to this equation may be written as

\[ \theta = C_1 \cosh(mx) + C_2 \sinh(mx) \]

Where \( C_1 \) and \( C_2 \) are constants of integration.

From the diagram the following boundary conditions may be applied.

At \( x = 0 \) (The heat input point), \( T_x = T_1 \)

At \( x = L \) (The end of the rod),

\( \frac{dT}{dx} = 0 \)

(If there is no heat transfer at the tip this implies there is no temperature gradient between the tip and the surroundings).

By applying the boundary conditions it can be shown that

\[ \frac{T_x - T_a}{T_1 - T_a} = \frac{\cosh m(L - x)}{\cosh mL} \]

\[ m = \sqrt{\frac{hP}{kd}} \]

Therefore the equation may be re-written as

\[ \frac{T_x - T_a}{T_1 - T_a} = \frac{\cosh \sqrt{\frac{hP}{kd}} (L - x)}{\cosh \sqrt{\frac{hP}{kd}} L} \]

The heat transferred away by the fin is equal to the heat conducted into it by the heater. Using Fourier’s law

\[ q_x = -k_d \frac{dT}{dx} \bigg|_{x=0} \]

Using further differential analysis and substitution the following expression may be derived.
The heat will be transferred from the rod by a combination of radiation and convection. Hence the overall heat transfer coefficient $h$ for the rod comprises two factors such that

$$h = h_r + h_c$$

Where $h_r$ is the radiant heat transfer coefficient and $h_c$ is the convective heat transfer coefficient.

Note that the convective coefficient may be due to natural convection or forced convection.

The mathematical development of the above theory is greatly simplified and more detailed derivations may be found in typical textbooks on heat transfer.
# Appendix – I Symbols and Units

## SYMBOLS AND UNITS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Cross sectional area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Heat Transfer area (Surface Area)</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$D$</td>
<td>Diameter of Extended Surface</td>
<td>m</td>
</tr>
<tr>
<td>$F$</td>
<td>Shape Factor</td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>Overall convection heat transfer coefficient</td>
<td>W/m$^2$K</td>
</tr>
<tr>
<td>$I$</td>
<td>Heater Current</td>
<td>A</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity</td>
<td>W/mK</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of extended surface</td>
<td>m</td>
</tr>
<tr>
<td>$Q$</td>
<td>Heat loss to natural convection</td>
<td>W</td>
</tr>
<tr>
<td>$t$</td>
<td>Temperature</td>
<td>°C</td>
</tr>
<tr>
<td>$T$</td>
<td>Absolute temperature</td>
<td>K</td>
</tr>
<tr>
<td>$V$</td>
<td>Heater Voltage</td>
<td>V</td>
</tr>
<tr>
<td>$x$</td>
<td>Distance</td>
<td>m</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan Boltzmann constant for radiation</td>
<td></td>
</tr>
<tr>
<td>$\xi$</td>
<td>Emissivity of cylinder</td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>$(T_a - T_s)$</td>
<td></td>
</tr>
</tbody>
</table>

### Subscripts:
- 9: Ambient condition
- 1-8: Rod Surface Temperature
- a: Ambient Condition
- s: Surface Condition
- Mean: Mean Condition ($\frac{T_1 + T_2 + \ldots + T_n}{n}$)
Appendix – II Some Useful Data