ENE 206 – Fluid Mechanics

WEEK 2

- ## Introduction

➢ The variation of pressure in a static fluid

➢ The pressure forces on submerged surfaces

For a fluid at rest, there is no shear stress and only pressure is considered as a source of pressure force acting on a free body besides body forces due to its weight.

- ## Fundamental equations for fluid at rest

The main objective here is to derive an equation for the variation in a fluid.

➢ Considering the body forces and surface forces due to pressure forces acting on an infinitesimal fluid element which is at rest in Cartesian coordinates as shown in Figure 2.1, the total force per unit volume $\left(d\vec{F}/dV\right)$ acting on this fluid element is the sum of the surface forces and the body forces per unit volume of the fluid element and can be expressed as $d\vec{F}/dV = \rho\vec{g} - \vec{\nabla}p$ where ρ is the density, p and $\vec{g}$ are the pressure and the gravitational acceleration, respectively.

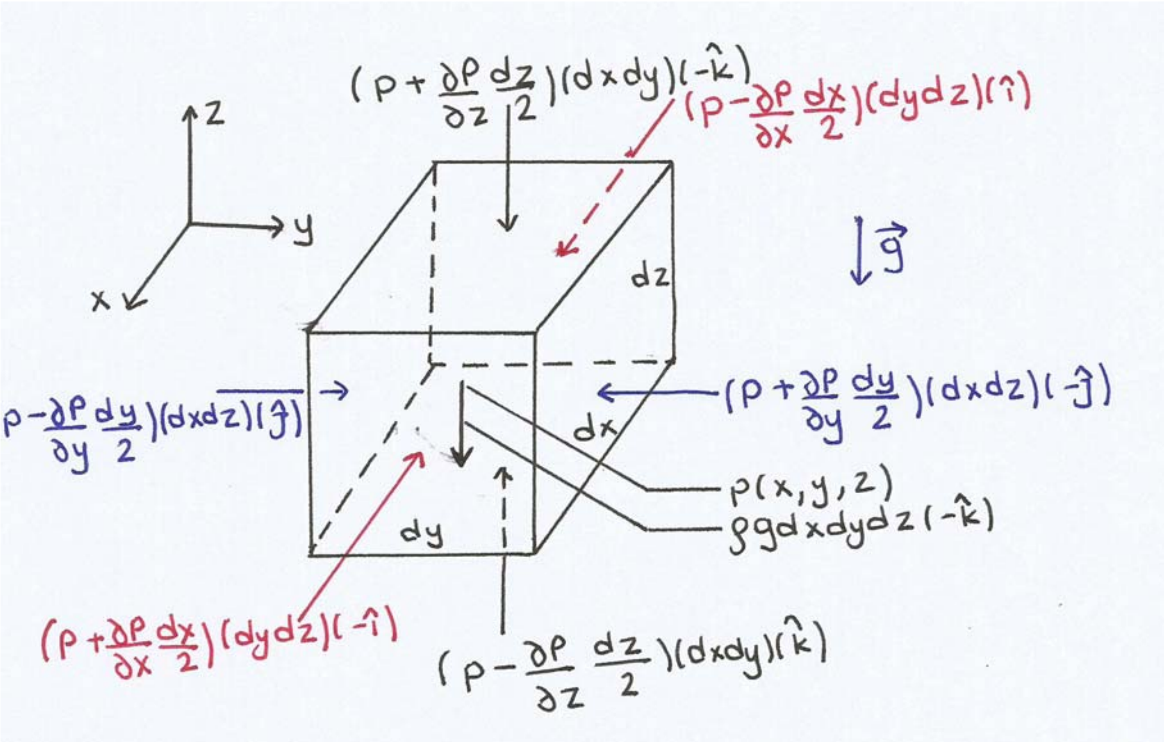

Figure 2.1. Pressure and body forces acting on an infinitesimal fluid element which is at rest

- Since the fluid is at rest, Newton's second law of motion implies that $d\vec{F}/dV = \rho\vec{a} = 0$ where $\vec{a}$ is the fluid acceleration. Thus

  $$d\vec{F}/dV = \rho\vec{g} - \vec{\nabla} p = 0 \quad \text{or} \quad \vec{\nabla} p - \rho\vec{g} = 0 \tag{2.1}$$

  In Eq. 2.1, the first and second terms represent the pressure and body force per unit volume at a point in fluid domain, respectively. This equation can be also expressed in terms of three scalar equations in the-*x*, *y* and *z* directions, respectively in Cartesian coordinates as below:

  $$\frac{\partial p}{\partial x} + \rho g_x = 0 \tag{2.2a}$$

  $$\frac{\partial p}{\partial y} + \rho g_y = 0 \tag{2.2b}$$

  $$\frac{\partial p}{\partial z} + \rho g_z = 0 \tag{2.2c}$$

  If the gravity vector is applied only in the *z*-axis vertically, these equations are further simplified as below:

  $$\frac{\partial p}{\partial x} = 0 \tag{2.3a}$$

  $$\frac{\partial p}{\partial y} = 0 \tag{2.3b}$$

  $$\frac{\partial p}{\partial z} = -\rho g \tag{2.3c}$$

  Eqns. 2.3a - c imply that the pressure is not function of x and y and is only function of *z*. Therefore, at this state instead of partial derivative, total derivative can be used to simply express the variation of pressure in the *z*-direction as below:

  $$\frac{dp}{dz} = -\rho g \tag{2.4}$$

  Eqn. 2.4 is valid for a fluid at rest, when the gravity is only body force term acting in the *z*-direction vertically upward.

- **The pressure variation in a fluid at rest**

- The pressure variation in an incompressible static fluid: The pressure variation in a fluid at rest can be analyzed depending upon the variation of density and gravitational acceleration.
  In the case of an incompressible static fluid, *E* the density is constant. Eqn. 2.4 then becomes
  $\frac{dp}{dz} = -\rho g =$ Constant which enables $p = -\rho g z + C$ where *C* is the constant of integration

- The pressure variation in a compressible static fluid: Pressure variation in this case can also be evaluated by integrating Eqn. 2.4. Before doing this, the density is required to be formulated as a function of one of the other variables in the Eqn. as $\rho=\rho(p,z)$. There are different conditions to be taken into account here as defined below:

  a) Pressure variation in a compressible static fluid: The pressure and density are related to modulus of elasticity, E_v as:

  $$E_v = \rho\frac{dp}{d\rho} \qquad (2.5)$$

  If the modulus of elasticity constant, the density is only dependent on the pressure, and the fluid is considered to be barotropic. Integration of Eqn 2.5 becomes:

  $p = E_v \ln\rho + C$ where C is the constant of integration.

  b) Pressure variation in a compressible static gas: In aeronautics and meteorology, the variation in density, pressure and temperature must be taken into account. For a perfect gas, Eqn. of state gives a relation: $p=\rho RT$ where R is the gas constant and T is the temperature. As long as T is introduced, a new relation is necessary.

  b.1) *Isothermal atmosphere:* The pressure variation is given as below*:*

  $$p=p_0 \exp\left[\frac{-g(T-T_0)}{RT_0}\right] \qquad (2.6)$$

  where p_0, T_0 are reference static pressure and temperature, respectively and are defined at a reference elevation, z_0.

  b.2) *Polytropic atmosphere:* The following below relation should be satisfied first:

  $$\frac{p}{\rho^n}=\frac{p_0}{\rho_0^n}= \text{constant} \qquad (2.7)$$

  n is constant and its value depends on the process. Eqn. 2.4 and 2.7 are combined to obtain:

  $$p=p_0\left[1-\frac{(n-1)}{n}\frac{\rho_0 g}{p_0}(z-z_0)\right]^{\frac{n}{n-1}} \qquad (2.8)$$

  b.3) *Atmosphere with linearly decreasing temperature:* The pressure variation in this case can be derived with the following below approximation:

  $$p=p_0\left[\frac{T_0+mz}{T_0+mz_0}\right]^{\frac{g}{Rm}} \qquad (2.9)$$

  In the above eqn., m is the temperature lapse rate and is used to determine temperature variation with elevation as $T = T_0 + mz$, $m < 0$

- ## References

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