ENE 206 – Fluid Mechanics

WEEK 4

- ## Introduction

➢ Geometry of fluid motion is discussed without giving any consideration to the forces which generate motion:

i) Methods describing fluid motion

ii) Relation between these models

iii) Visualization of fluid flow field

iv) Basic concepts of fluid motion will be described.

- ## Methods for describing the fluid motion

➢ There are two methods two describe the fluid motion

i) Material (Lagrangian) description

ii) Spatial (Eulerian) description

➢ **Lagrangian description:** Identified fluid particles are followed in the course of time (Figure 3.1) and the variation in their properties are recorded. This is widely used in dynamics.

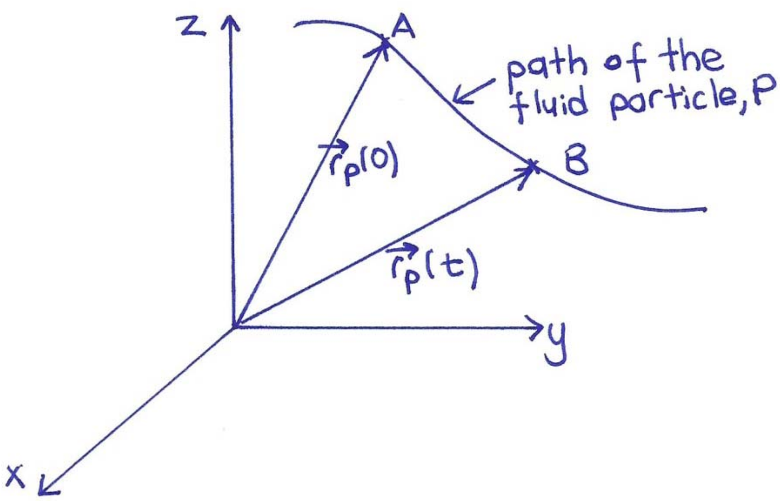

Figure 3.1. Path of a fluid particle

In the Lagrangian description position vector of a fluid particle moving in the fluid domain is described in Cartesian coordinates as follows:

$$\vec{r}_P\,(t) = x_p(t)\,\hat{i} + y_p(t)\,\hat{j} + z_P(t)\hat{k} \tag{3.1}$$

Then the basic properties in Lagrangian description are identified as:

$$\vec{V}_P(t) = \vec{V}_P\left[x_P(t), y_p(t), z_P(t), t\right] \quad (3.2a)$$

$$\rho_P(t) = \rho_P\left[x_P(t), y_p(t), z_P(t), t\right] \quad (3.2b)$$

$$p_P(t) = p_P\left[x_P(t), y_p(t), z_P(t), t\right] \quad (3.2c)$$

Then t is the only independent variable here. x, y, z are dependent variables. The collection of identified fluid particles is known as a material volume or a system which is free to change its position and its shape.

The boundaries are known as **System boundaries** and the fluid particles can not cross these boundaries.

- **Spatial description**: In this description, the attention is focused on a fixed point in space and the variation of properties is considered as the fluid particles passing through this point

Mathematical formulation of the fluid flow

- There are two basic formulations for the mathematical description of fluid motion

  i) Differential formulation

  ii) Integral formulation

- **Differential formulation**:

  i) Every detailed formulation of the flow field is required.

  ii) In Lagrangian description, information about all fluid particles is necessary

  iii) In Eulerian description, information about all points is required.

  The basic laws are applied to a fluid particle in case of Lagrangian description while applied to a fluid element in Eulerian Description.

- **Integral formulation:**

  i) A detailed information is not necessary

  ii) The basic laws are applied to a closed system in the case of Lagrangian description or to a control volume in the case of Eulerian description

Relationship between Material and Spatial description

- In the differential formulation:

  Consider the extensive property (mass dependent), *N*

$$N = N\left[x_P(t), y_p(t), z_P(t), t\right] \quad (3.3)$$

N is the material description of any property. When its total differential is obtained as:

$$\frac{DN}{dt} = \frac{\partial N}{\partial t} + \left(\vec{V}.\nabla\right)N \quad (3.4)$$

The first term on the let hand side of the above equation is referred to as the **total derivative**, the first term on the right hand side is referred to as the **local derivative**, and the last term on the right hand side is the **convective derivative**.

- **Integral formulation:** Properties are classified as extensive which is mass dependent and intensive which is mass independent. If N specifies any arbitrary extensive property, the corresponding intensive property (extensive property per mass) will be specified by η. Thus the Reynolds transport theorem, which is related to the integral formulation of a fluid flow can be expressed as:

$$\frac{DN}{dt} = \frac{D}{dt}\int_{V_s} \rho\eta dV = \frac{\partial}{\partial t}\int_{V} \rho\eta dV + \int_{A} \rho\eta\left(\vec{V}.\hat{n}\right)dA \qquad (3.5)$$

The left hand side of the above equation represents the rate of change of the extensive property N of the system, the first term on the right hand side is the rate of change of the same extensive property with the control volume and the second term on the right hand side is the new rate of efflux of the mentioned property through the control surface.

Classification of fluid flow

- **Steady vs unsteady flow:**

when the fluid properties are independent of time, then the flow is considered to be steady. If any of the fluid property changes with time, then the fluid flow is considered to be unsteady.

$\frac{\partial \vec{V}}{\partial t} = 0$: steady flow

$\frac{\partial \vec{V}}{\partial t} \neq 0$: unsteady flow

- **Dimensionality:**

For the three-dimensional unsteady flow, fluid properties vary in all three directions with time. Such as

$\rho = \rho(x, y, z, t)$

$p = p(x, y, z, t)$

For the two-dimensional unsteady flow: the fluid properties vary in two perpendicular direction, say *x*, *y* with time

$\rho = \rho(x, y, t)$

$p = p(x, y, t)$

For one-dimensional unsteady flow, the fluid properties vary only in one direction say, *x* with time

$\rho = \rho(x, t)$

$p = p(x, t)$

- **Uniformity:**

  The flow property any cross section in the flow domain is represented by its average value and the flow is assumed to be uniform for simplicity over this cross section. Therefore, the flow becomes function of one space variable over the specified cross section.

Visualization of fluid flow

The flow field representation can be achieved with three types of imaginary lines in the flow domain:

i) Pathlines

ii) Streamlines

iii) Streaklines

- **Pathlines**: It is the path or the trajectory flowed by a fluid particle in a flow field. It is used in Lagrangian description.

- **Streamlines:** It is used in Eulerian description. There are lines which are tangent to the velocity vector at all points in the flow field.

- **Streaklines**: Attention is focused on all particles passing through the same point in the flow field.

Acceleration of fluid flow

The acceleration of a fluid particle, $\vec{a}$ can be obtained by differentiating its velocity with respect to time as:

$$\vec{a} = \frac{D\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \left(\vec{V}.\nabla\right)\vec{V} \qquad (3.6)$$

Flow rates

The flow rate is the quantity of fluid flowing per unit time across any cross section in the flow domain.

- **Mass flow rate:**

$$\dot{m} = \int_A \rho\left(\vec{V}.\hat{n}\right)dA \qquad (3.7)$$

- **The volumetric flow rate:**

$$Q = \frac{\dot{m}}{\rho} = \int_A \left(\vec{V}.\hat{n}\right)dA \qquad (3.8)$$

- **The average velocity:**

$$\vec{V} = \frac{Q}{A} = \frac{1}{A}\int_A \left(\vec{V}.\hat{n}\right) dA \qquad (3.9)$$

- ## References

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