ENE 206 – Fluid Mechanics

WEEK 6

- **The first law of thermodynamics**

➢ It is stated as the rate of change of total energy in a system is equal to the sum of the rate of heat transfer to the system and the rate of work done on the system. It can be expressed as follows:

$$\dot{Q}_s + \dot{W}_s = \frac{DE_s}{dt} \qquad (4.13)$$

where $\dot{Q}_s$ is the rate of heat transfer, $\dot{W}_s$ is the work done on the system, and E_s is the total energy of the system at a given state.

➢ **Signs:**

i) the rate of heat transfer is positive when the heat is added to the system from the surroundings, negative when the heat is transferred to the surroundings from the system

ii) the rate of work is positive when work is done on the system and it is negative when work is done on the surroundings by the system.

➢ **Energy form:**

iii) the total energy of the system is composed of the mechanical and thermal energies.

iv) the total mechanical energy is also composed of kinetic $(KE)_s$, potential $(PE)_s$ and internal U_s energy of the system. Hence

$$E_s = (KE)_s + (PE)_s + U_s \qquad (4.14)$$

The total energy of the system can be also expressed as follows.

$$E_s = \int_{m_s} e\,dm = \int_{V_s} \rho e\,dV \qquad (4.15)$$

where *e* is referred to as the total energy per unit mass.

Eqn. 4.13 then becomes:

$$\dot{Q}_s + W_s = \frac{DE_s}{dt} = \frac{D}{dt}\int_{V_s} \rho e\,dV \qquad (4.16)$$

Assuming that total energy per unit mass, *e* is intensive property, and the total energy of the system is extensive property, *E*, then Reynolds' transport theorem can be written for $N = E_s$, and $\eta = e$ as:

$$\dot{Q}_s + \dot{W}_s = \frac{DE_s}{dt} = \frac{D}{dt}\int_{V_s} \rho e\,dV = \frac{\partial}{\partial t}\int_{V} \rho e\,dV + \int_{A} \rho e\left(\vec{V}.\hat{n}\right)dA \qquad (4.17)$$

The total energy per unit mass cam also be expressed as.

$$e = \frac{V^2}{2} + gz + u \tag{4.18}$$

where V is the velocity, z is the elevation, g is the acceleration term and u is the internal energy per unit mass. The first law of thermodynamics then can be written as:

$$\dot{Q}_s + \dot{W}_s = \frac{\partial}{\partial t}\int_V \rho\left(\frac{V^2}{2} + gz + u\right)dV + \int_A \rho\left(\frac{V^2}{2} + gz + u\right)\left(\vec{V}.\hat{n}\right)dA \tag{4.19}$$

Alternatively eqn. 4.19 can be written for the control volume as:

$$\dot{Q} + \dot{W} = \frac{\partial}{\partial t}\int_V \rho\left(\frac{V^2}{2} + gz + u\right)dV + \int_A \rho\left(\frac{V^2}{2} + gz + h\right)\left(\vec{V}.\hat{n}\right)dA \tag{4.20}$$

For a steady flow, eqn 4.20 can be simplified as:

$$\dot{Q} + \dot{W} = \int_A \rho\left(\frac{V^2}{2} + gz + h\right)\left(\vec{V}.\hat{n}\right)dA \tag{4.21}$$

For a steady, one-dimensional flow through a streamtube, the last equation can be also written as follows:

$$_1\dot{Q}_2 + {}_1\ddot{W}_2 = \int_{A_1} \rho_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right)\left(\vec{V}_1\hat{n}_1\right)dA + \int_{A_2}\left(h_2 + \frac{V_2^2}{2} gz_z\right)\left(\vec{V}_2\,\hat{n}_2\right)dA \tag{4.22}$$

- **The second law of thermodynamics**

➢ For a fluid system, the second law of thermodynamics can be stated as follows

$$\frac{DS_s}{dt} \geq \frac{\dot{Q}}{T} \tag{4.23}$$

where S_s represents the entropy of the system and can be given as:

$$S_s = \int_{m_s} s\,dm = \int_{V_s} \rho s\,dV \tag{4.24}$$

In eq. 4.24, s is the entropy per unit mass. Therefore, Eqn. 4.23 becomes

$$\frac{\dot{Q}}{T} \leq \frac{DS_s}{dt} = \frac{D}{dt}\int_{V_s} \rho s\,dV \tag{4.25}$$

In the above equation, inequality implies that the flow process is irreversible, while equality implies that it is reversible process.

The entropy of the system is extensive property while the entropy per mass is an intensive property. Then Reynolds' transport theorem can be written for $N = S_s$, and $\eta = s$ as:

$$\frac{\dot{Q}}{T} \leq \frac{DS_s}{dt} = \frac{D}{dt}\int_{V_s} \rho s\,dV = \frac{\partial}{\Im t}\int_V \rho s\,dV + \int_A \rho s\left(\vec{V}.\hat{n}\right)dA \tag{4.26}$$

For a steady flow, the partial derivatives with respect to time are zero, that is $\partial/\partial t = 0$, and the second law of thermodynamics becomes:

$$\frac{\dot{Q}}{T} \leq \int_{A} \rho s \left(\vec{V}.\hat{n}\right) dA \qquad (4.27)$$

For one-dimensional steady flow through a streamtube, the second law of thermodynamics of fluid flow through the streamtube can be formulated as:

$$\frac{\dot{Q}}{T} \leq \int_{A_1} \rho_1 s_1 \left(\vec{V}_1.\hat{n}_1\right) dA + \int_{A_2} \rho_2 s_2 \left(\vec{V}_2.\hat{n}_2\right) dA \qquad (4.28)$$

For one-dimensional flow, the velocity vectors are perpendicular to the inlet and outlet surface of the streamtubes. Then

$$\frac{\dot{Q}}{T} \leq \int_{A_1} \rho_2 s_2 V_2 \, dA - \int_{A_2} \rho_1 s_1 V_1 dA \qquad (4.29)$$

The properties are also uniform over each cross section, so that

$$\frac{\dot{Q}}{T} \leq \rho_2 s_2 V_2 A_2 - \rho_1 s_1 V_1 A_1 \qquad (4.30)$$

using the continuity equation $\dot{m} = \rho V A$, eqn. 4.30 becomes:

$$\frac{\dot{Q}}{T} \leq \dot{m}\left(s_2 - s_1\right) \qquad (4.31)$$

If there is no heat transfer, eqn. 4.31 becomes

$$s_2 \geq s_1 \qquad (4.32)$$

When the flow process is reversible and adiabatic, i.e. isentropic eqn. 4.32 holds:

$$s_1 = s_2 \qquad (4.33)$$

- ## References

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