# ENE 206 – Fluid Mechanics

## WEEK 8

- ## Introduction

➢ The differential formulation involves an infinitesimal control volume, and the governing flow equations are derived by applying the conservation laws to this control volume. The fundamental principles here:

i) conservation of mass

ii) conservation of momentum

iii) conservation of energy

iv) second law of thermodynamics

These laws are applied to system of fluid particles. The mathematical statements of these fundamental principles yield the integral form of governing flow equations, which are.

i) continuity equation

ii) momentum equation

iii) energy equation

iv) equation for the second law of thermodynamics.

- ## Motion of a fluid element

➢ When a fluid element moves in a flow field, it can undergo following motions:

i) translation

ii) linear deformation

iii) rotation

iv) angular deformation

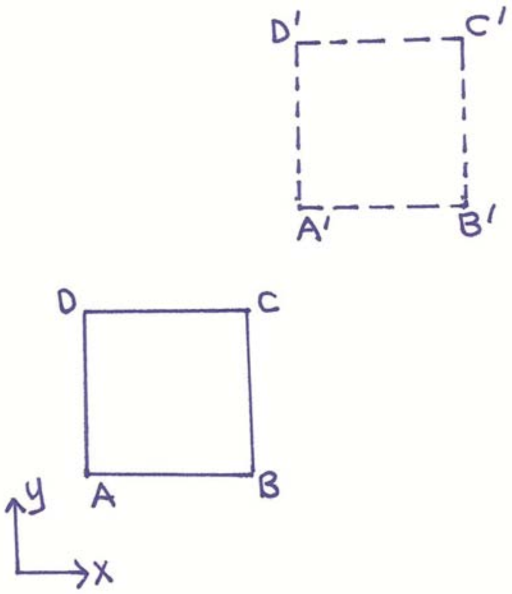


Figure 6.1. Translation of two-dimensional face (ABCD) of the infinitesimal fluid element

- **Translation**: During translation, the position of the fluid element changes, but its shape, size and orientation remain the same. Figure 6.1 shows the translation of a two-dimensional face of a fluid element in the Cartesian coordinates.

- **Linear deformation:** Each side of the infinitesimal fluid element elongated linearly under the action of linear deformation i.e. linear strain. The change in the length of each of the three sides of the infinitesimal fluid element changes its volume as shown in Figure 6.2.

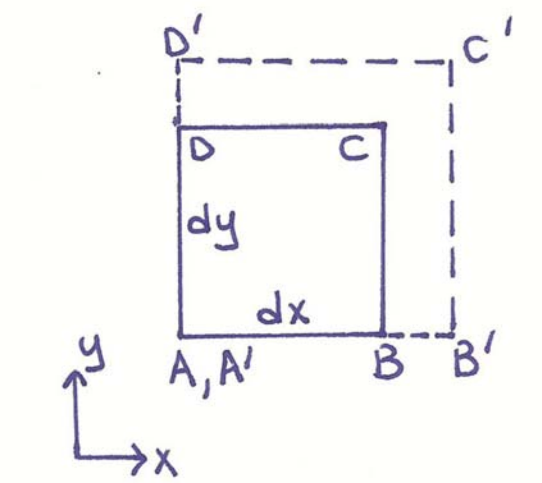


Figure 6.2. The linear deformation of two-dimensional face (ABCD) of the infinitesimal fluid element

The rate of the change of the volume per unit volume is given as:

$$\frac{1}{V_i}\left(\frac{V_f - V_i}{dt}\right) = \frac{1}{V_i}\frac{dV}{dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla.\vec{V} \qquad (6.1)$$

where $V_i$ and $V_f$ are the initial and final volumes of the infinitesimal fluid element, *u, v* and *w* are the velocity components in the x-direction, y-direction and the z-direction respectively. The eqn. 6.1. is related to the compressibility of the fluid, which is known as dilation. For incompressible fluids, the dilation is zero.

During the linear deformation, the size of the fluid element changes but its position, shape and the orientation remain same.

- **Rotation:** The rate of rotation of the infinitesimal fluid element about the *z*-axis, $\omega_z$ is the average of the angular velocities of two mutually perpendicular lines (Figure 6.3).

The orientation of the face ABCD before and after the rotation is shown in Figure 6.3. The angular velocity vector, $\omega$ can be evaluated as:

$$\vec{\omega} = \vec{\omega}_x + \vec{\omega}_y + \vec{\omega}_z = \frac{1}{2}\left\{\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\hat{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\hat{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k}\right\} \qquad (6.2)$$

using the vector notation, angular velocity vector becomes:

$$\omega = \frac{1}{2}\left(\nabla x \vec{V}\right) \tag{6.3}$$

where $\left(\nabla x \vec{V}\right)$ is known as $curl\,\vec{V}$ .

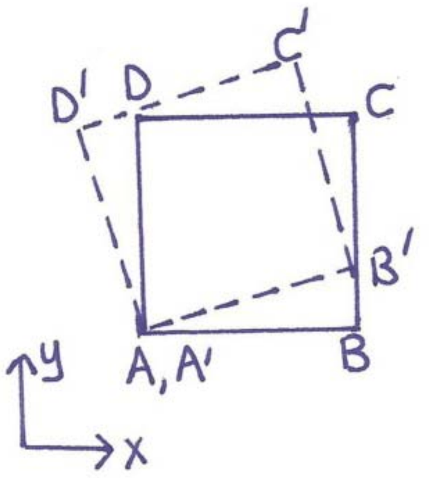


Figure 6.3. The angular rotation of two-dimensional face (ABCD) of the infinitesimal fluid element

- **Angular deformation:** Angular deformation of the two-dimensional face of the infinitesimal fluid element is shown in Figure 6.4. Sides AD and AB are deformed due to the angular deformation and the rate of shear strain can be expressed as:

$$\frac{\partial \alpha_{xy}}{\partial t} = \frac{d\alpha_{AB}}{dt} + \frac{d\alpha_{AD}}{dt} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{6.4}$$

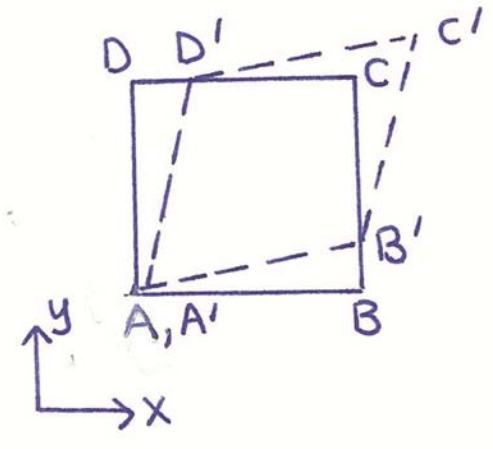


Figure 6.4. Angular deformation of two-dimensional face (ABCD) of the infinitesimal fluid element

- **References**

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