# ENE 206 – Fluid Mechanics

## WEEK 10

- ## Introduction

➢ Similitude: The performance of the prototype can be simulated in a model using similitude. There are three basic laws of similitude and all of them must be satisfied for obtaining complete similarity between fluid flow phenomena in a prototype and in a model: These are:

i) geometric similarity

ii) kinematic similarity and

iii) dynamic similarity.

➢ **Geometric similarity:**

The prototype and the model must be identical in shape but different in size. ratios of the corresponding linear dimensions in the prototype and model must be the same.

The length scale is defined as:

$$L_r = \frac{L_p}{L_m} \tag{7.1}$$

➢ **Kinematic similarity:**

The flow fields in the prototype and the model must have geometrically similar sets of streamlines. For the kinematic similarity, geometric similarity should be also satisfied. The velocities and the acceleration:

i) are parallel at all corresponding sets of points and

ii) have the same ratio of magnitudes between all corresponding sets of points.

Then, the velocity scale, $V_r$ and acceleration scale, $a_r$ are given as follows:

$$V_r = \frac{\vec{V}_p}{\vec{V}_m} \tag{7.2}$$

$$a_r = \frac{a_p}{\vec{a}_m} \tag{7.3}$$

➢ **Dynamic similarity:**

Force distribution between the two flow fields (prototype and model) is such that the identical types of forces

i) are parallel

ii) have the same ratios of magnitude.

1) inertia, $\vec{F}_i$

2) Gravity, $\vec{F}_g$

3) Viscosity, $\vec{F}_{\mu}$

4) Pressure, $\vec{F}_{p}$

5) Compressibility $\vec{F}_{e}$

6) Surface tension, $\vec{F}_{\sigma}$

$$\vec{F}_{gm} + \vec{F}_{\mu m} + \vec{F}_{pm} + \vec{F}_{\sigma m} + \vec{F}_{em} = \vec{F}_{im} \tag{7.4}$$

$$\vec{F}_{gp} + \vec{F}_{\mu p} + \vec{F}_{pp} + \vec{F}_{\sigma p} + \vec{F}_{ep} = \vec{F}_{ip} \tag{7.5}$$

$$\frac{\vec{F}_{gp}}{\vec{F}_{gm}} = \frac{\vec{F}_{\mu p}}{\vec{F}_{\mu m}} = \frac{\vec{F}_{pp}}{\vec{F}_{pm}} = \frac{\vec{F}_{\sigma p}}{\vec{F}_{\sigma m}} = \frac{\vec{F}_{ep}}{\vec{F}_{em}} = \frac{\vec{F}_{ip}}{\vec{F}_{im}} \tag{7.6}$$

This means five simultaneous equations as follows:

$$\frac{\vec{F}_{gp}}{\vec{F}_{gm}} = \frac{\vec{F}_{ip}}{\vec{F}_{im}} \tag{7.7a}$$

$$\frac{\vec{F}_{\mu p}}{\vec{F}_{\mu m}} = \frac{\vec{F}_{ip}}{\vec{F}_{im}} \tag{7.7b}$$

$$\frac{\vec{F}_{pp}}{\vec{F}_{pm}} = \frac{\vec{F}_{ip}}{\vec{F}_{im}} \tag{7.7c}$$

$$\frac{\vec{F}_{\sigma p}}{\vec{F}_{\sigma m}} = \frac{\vec{F}_{ip}}{\vec{F}_{im}} \tag{7.7d}$$

$$\frac{\vec{F}_{ep}}{\vec{F}_{em}} = \frac{\vec{F}_{ip}}{\vec{F}_{im}} \tag{7.7e}$$

With reference to the history of fluid mechanics, it is useful to define the following set of non-dimensional numbers of the dynamic similarity as:

$$Re = \frac{F_i}{F_\mu} = \frac{\rho V L}{\mu} \tag{7.8a}$$

$$Fr = \sqrt{\frac{F_i}{F_g}} = \frac{V}{\sqrt{gL}} \tag{7.8b}$$

$$Eu = \frac{F_p}{F_i} = \frac{\Delta p}{\rho V^2} \tag{7.8c}$$

$$Ma = \sqrt{\frac{F_i}{F_e}} = \sqrt{\frac{\rho V^2}{E_v}} \tag{7.8d}$$

$$We = \sqrt{\frac{F_i}{F_\sigma}} = \sqrt{\frac{\rho V^2 L}{\sigma}} \tag{7.8d}$$

where *Re*, *Fr*, *Eu*, *Ma*, *We* denote Reynolds, Froude, Euler, Mach and Weber numbers. Combining equations 7.7 and 7.8:

$Re_p = Re_m$ (7.9a)

$Fr_p = Fr_m$ (7.9b)

$Eu_p = Eu_m$ (7.9c)

$Ma_p = Ma_m$ (7.9d)

$We_p = We_m$ (7.9e)

- ## Dimensional analysis:

- Fourier's principle of dimensional homogeneity. By using the method of dimensional analysis, the physical phenomenon can be formulated as a relation between a set of non-dimensional groups of variables such that the number of non-dimensional groups is less than the number of variables.

$g_1 = f(g_2, g_3, \ldots\ldots, g_n)$

Basic dimensions are:

i) *L* (length)

ii) *M* (mass)

iii) *T* (time)

iv) $\Theta$ (temperature)

The dimensional analysis is based on the Buckingham-Pi theorem. This theorem proves that in a physical problem including:

n variable and

m basic dimensions

Number of non-dimensional terms: n – m

- ## References

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