WEEK 6: SOLAR ENERGY

INTRODUCTION

During the day the sun has different positions. For low concentration systems (and low temperatures) tracking can be avoided (or limited to a few positions per year) if non-imaging optics are used. For higher concentrations, however, if the mirrors or lenses do not move, then the focus of the mirrors or lenses changes (but also in these cases non-imaging optics provides the widest acceptance angles for a given concentration). Therefore, it seems unavoidable that there needs to be a tracking system that follows the position of the sun (for solar photovoltaic a solar tracker is only optional).

The tracking system increases the cost and complexity. With this in mind, different designs can be distinguished in how they concentrate the light and track the position of the sun.

THEORY

Thermal Radiation

Thermal radiation is a form of energy emission and transmission that depends entirely on the temperature characteristics of the emissive surface. Thermal radiation is in fact an electromagnetic wave that travels at the speed of light (C = 300,000 km/s in a vacuum). This speed is related to the wavelength (λ) and frequency (v of the radiation as given by the equation:

$$C = \lambda v \text{ (eq.1)}$$

When a beam of thermal radiation is incident on the surface of a body, part of it is reflected away from the surface, part is absorbed by the body, and part is transmitted through the body. The various properties associated with this phenomenon are the fraction of radiation reflected, called *reflectivity* (ρ); the fraction of radiation absorbed, called *absorptivity* (α); and the fraction of radiation transmitted, called *transmissivity* (τ). The three quantities are related by the following equation:

$$\rho + \alpha + \tau = 1 \text{ (eq.2)}$$

The following equation is used to express the dependence of these properties on the wavelength:

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1 \quad \textbf{(eq. 3)}$$

where ρ_λ = Spectral reflectivity; α_λ = Spectral absorptivity; τ_λ = Spectral transmissivity

If a body absorbs all the impinging thermal radiation such that $\tau = 0$, $\rho = 0$, and $\alpha = 1$, regardless of the spectral character or directional preference of the incident radiation, it is called a *blackbody*.

A blackbody is not only a perfect absorber, it is also characterized by an upper limit to the emission of thermal radiation. The energy emitted by a blackbody is a function of its temperature and is not evenly distributed over all wavelengths. The rate of energy emission per unit area at a particular wavelength is termed the *monochromatic emissive power*. Max Planck was the first to derive a functional relation for the monochromatic emissive power of a blackbody in terms of temperature and wavelength. This was done by using the quantum theory, and the resulting equation, called *Planck's equation for blackbody radiation*, is given by

$$E_{b\lambda} = \frac{C_1}{\lambda^5 \left(e^{C_2/\lambda T} - 1 \right)} \quad \textbf{(eq. 4)}$$

where

$E_{b\lambda}$ = monochromatic emissive power of a blackbody (W/m^2-μm).

T = temperature of the body (K).

λ = wavelength (μm).

C_1 = constant = 3.74 x 10^8 W-μm^4/m^2.

C_2 = constant = 1.44 x 10^4 μm-K.

By differentiating Eq. 4 and equating to 0, the wavelength corresponding to the maximum of the distribution can be obtained and is equal to $\lambda_{max}T$ = 2897.8 μm-K. This is known as *Wien's displacement law*. Figure 1 shows the spectral radiation distribution for blackbody radiation at three temperature sources. The curves have been obtained by using the Planck's equation.
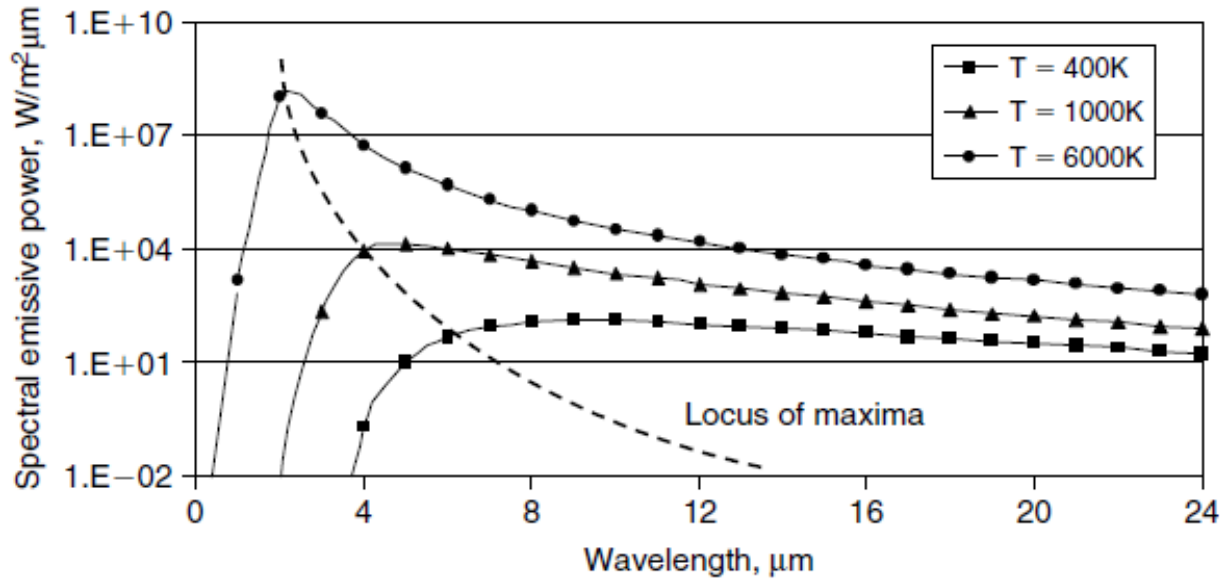
Figure 1. Spectral distribution of blackbody radiation

The total emissive power, E_b, and the monochromatic emissive power, $E_{b\lambda}$, of a blackbody are related by

$$E_b = \int_0^\infty E_{b\lambda}\, d\lambda$$

(eq. 5)

Substituting Eq. (4) into Eq. (5) and performing the integration results in the Stefan-Boltzmann law:

$$E_b = \sigma T^4$$ **(eq. 6)**

where α = the Stefan-Boltzmann constant = 5.6697×10^{-8} W/m²-K⁴.

In many cases, it is necessary to know the amount of radiation emitted by a blackbody in a specific wavelength band $\lambda_1 \rightarrow \lambda_2$. We get;

$$E_b(\lambda_1 T \rightarrow \lambda_2 T) = \int_{\lambda_1 T}^{\lambda_2 T} \frac{E_{b\lambda}}{T}\, d\lambda T$$

(eq. 7)

which results in $E_b(0 \rightarrow \lambda_1 T) - E_b(0 \rightarrow \lambda_2 T)$. Table 2.4 presents a tabulation of $E_b(0 \rightarrow \lambda T)$ as a fraction of the total emissive power, $E_b = \sigma T^4$, for various values of λT.

A blackbody is also a perfect diffuse emitter, so its intensity of radiation, I_b, is a constant in all directions, given by

$$E_b = \pi I_b$$ **(eq. 8)**

Real surfaces emit less energy than corresponding blackbodies. The ratio of the total emissive power, E, of a real surface to the total emissive power, E_b, of a blackbody, both at the same temperature, is called the *emissivity* (ε) of a real surface; that is;

$$\varepsilon = \frac{E}{E_b}$$ **(eq. 9)**

Table 1. Fraction of Blackbody Radiation as a Function of λT

λT (µm-K)	$E_b(0 \to \lambda T)/\sigma T^4$	λT (µm-K)	$E_b(0 \to \lambda T)/\sigma T^4$	λT (µm-K)	$E_b(0 \to \lambda T)/\sigma T^4$
555.6	1.70E–08	4000.0	0.48085	7444.4	0.83166
666.7	7.56E–07	4111.1	0.50066	7555.6	0.83698
777.8	1.06E–05	4222.2	0.51974	7666.7	0.84209
888.9	7.38E–05	4333.3	0.53809	7777.8	0.84699
1000.0	3.21E–04	4444.4	0.55573	7888.9	0.85171
1111.1	0.00101	4555.6	0.57267	8000.0	0.85624
1222.2	0.00252	4666.7	0.58891	8111.1	0.86059
1333.3	0.00531	4777.8	0.60449	8222.2	0.86477
1444.4	0.00983	4888.9	0.61941	8333.3	0.86880
1555.6	0.01643	5000.0	0.63371	8888.9	0.88677
1666.7	0.02537	5111.1	0.64740	9444.4	0.90168
1777.8	0.03677	5222.2	0.66051	10000.0	0.91414
1888.9	0.05059	5333.3	0.67305	10555.6	0.92462
2000.0	0.06672	5444.4	0.68506	11111.1	0.93349
2111.1	0.08496	5555.6	0.69655	11666.7	0.94104
2222.2	0.10503	5666.7	0.70754	12222.2	0.94751
2333.3	0.12665	5777.8	0.71806	12777.8	0.95307
2444.4	0.14953	5888.9	0.72813	13333.3	0.95788
2555.5	0.17337	6000.0	0.73777	13888.9	0.96207
2666.7	0.19789	6111.1	0.74700	14444.4	0.96572
2777.8	0.22285	6222.1	0.75583	15000.0	0.96892
2888.9	0.24803	6333.3	0.76429	15555.6	0.97174
3000.0	0.27322	6444.4	0.77238	16111.1	0.97423
3111.1	0.29825	6555.6	0.78014	16666.7	0.97644
3222.2	0.32300	6666.7	0.78757	22222.2	0.98915
3333.3	0.34734	6777.8	0.79469	22777.8	0.99414
3444.4	0.37118	6888.9	0.80152	33333.3	0.99649
3555.6	0.39445	7000.0	0.80806	33888.9	0.99773
3666.7	0.41708	7111.1	0.81433	44444.4	0.99845
3777.8	0.43905	7222.2	0.82035	50000.0	0.99889
3888.9	0.46031	7333.3	0.82612	55555.6	0.99918

To express the dependence on wavelength, the monochromatic or spectral emissivity, ε_λ, is defined as the ratio of the monochromatic emissive power, E_λ, of a real surface

to the monochromatic emissive power, $E_{b\lambda}$, of a blackbody, both at the same wavelength and temperature:

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}$$ (eq. 10)

Kirchoff's law of radiation states that, for any surface in thermal equilibrium, monochromatic emissivity is equal to monochromatic absorptivity:

$$\varepsilon_\lambda(T) = \alpha_\lambda(T)$$ (eq. 11)

Equation (11) can be generalized as

$$\varepsilon(T) = \alpha(T)$$ (eq. 12)

Similar to Eq. (2.37) for a real surface, the radiant energy leaving the surface includes its original emission and any reflected rays. The rate of total radiant energy leaving a surface per unit surface area is called the *radiosity* (J), given by

$$J = \varepsilon E_b + \rho H$$ (eq. 13)

where

Eb = blackbody emissive power per unit surface area (W/m2).

H = irradiation incident on the surface per unit surface area (W/m2).

ε = emissivity of the surface.

ρ = reflectivity of the surface.

A real surface is both a diffuse emitter and a diffuse reflector and hence, it has diffuse radiosity; i.e., the intensity of radiation from this surface (I) is constant in all directions. Therefore, the following equation is used for a real surface:

$$J = \pi \times I$$ (eq. 14)

Transparent Plates

When a beam of radiation strikes the surface of a transparent plate at angle θ_1, called the *incidence angle*, as shown in Figure 2, part of the incident radiation is reflected and the remainder is refracted, or bent, to angle θ_2, called the *refraction angle*, as it passes through the interface. Angle θ_1 is also equal to the angle at which the beam is specularly reflected from the surface. Angles θ_1 and θ_2 are not equal when the density of the plane is different from that of the medium through which the radiation travels. Additionally, refraction causes the transmitted beam to be bent toward the perpendicular to the surface of higher density. The two angles are related by the Snell's law:

$$n = \frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} \text{ (eq. 15)}$$

where n_1 and n_2 are the refraction indices and n is the ratio of refraction index for the two media forming the interface. The refraction index is the determining factor for the reflection losses at the interface. A typical value of the refraction index is 1.000 for air, 1.526 for glass, and 1.33 for water.
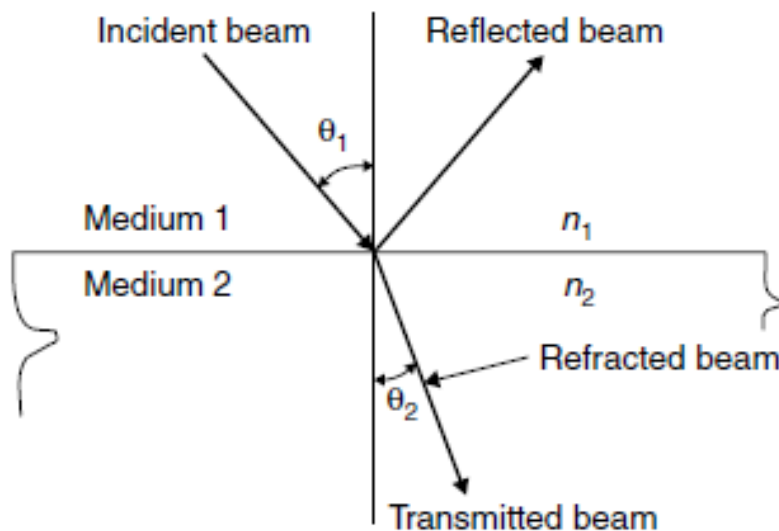

Figure 2. Incident and refraction angles for a beam passing from a medium with refraction index n_1 to a medium with refraction index n_2.

Expressions for perpendicular and parallel components of radiation for smooth surfaces were derived by Fresnel as

$$r_\perp = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} \quad \textbf{(eq. 16)}$$

$$r_\parallel = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} \quad \textbf{(eq. 17)}$$

Equation (16) represents the perpendicular component of unpolarized radiation and Eq. (17) represents the parallel one. It should be noted that *parallel* and *perpendicular* refer to the plane defined by the incident beam and the surface normal.

Properties are evaluated by calculating the average of these two components as

$$r = \frac{1}{2}(r_\perp + r_\parallel) \quad \textbf{(eq. 18)}$$

Similarly, the transmittance, τ_r (subscript r indicates that only reflection losses are considered), can be calculated from the average transmittance of the two components as follows:

$$\tau_r = \frac{1}{2}\left(\frac{1 - r_\parallel}{1 + r_\parallel} + \frac{1 - r_\perp}{1 + r_\perp}\right) \quad \textbf{(eq. 19)}$$

For a glazing system of N covers of the same material, it can be proven that

$$\tau_r = \frac{1}{2}\left(\frac{1 - r_\parallel}{1 + (2N - 1)r_\parallel} + \frac{1 - r_\perp}{1 + (2N - 1)r_\perp}\right) \quad \textbf{(eq. 20)}$$

The transmittance, τ_a (subscript α indicates that only absorption losses are considered), can be calculated from

$$\tau_a = e^{\left(-\frac{KL}{\cos\theta_2}\right)} \quad \textbf{(eq. 21)}$$

where K is the extinction coefficient, which can vary from 4 m^{-1} (low-quality glass) to 32 m^{-1} (high-quality glass), and L is the thickness of the glass cover.

The transmittance, reflectance, and absorptance of a single cover (by considering both reflection and absorption losses) are given by the following expressions. These

expressions are for the perpendicular components of polarization, although the same relations can be used for the parallel components:

$$\tau_\perp = \frac{\tau_\alpha(1-r_\perp)^2}{1-(r_\perp\tau_\alpha)^2} = \tau_\alpha\frac{1-r_\perp}{1+r_\perp}\left(\frac{1-r_\perp^2}{1-(r_\perp\tau_\alpha)^2}\right) \quad \text{(eq. 22)}$$

$$\rho_\perp = r_\perp + \frac{(1-r_\perp)^2\tau_\alpha^2 r_\perp}{1-(r_\perp\tau_\alpha)^2} = r_\perp(1+\tau_\alpha\tau_\perp) \quad \text{(eq. 23)}$$

$$\alpha_\perp = (1-\tau_\alpha)\left(\frac{1-r_\perp}{1-r_\perp\tau_\alpha}\right) \quad \text{(eq. 24)}$$

Since, for practical collector covers, τ_α is seldom less than 0.9 and r is on the order of 0.1, the transmittance of a single cover becomes

$$\tau \cong \tau_\alpha\tau_r \quad \text{(eq. 25)}$$

The absorptance of a cover can be approximated by neglecting the last term of Eq. (24):

$$\alpha \cong 1 - \tau_\alpha \quad \text{(eq. 26)}$$

and the reflectance of a single cover could be found (keeping in mind that $\rho = 1 - \alpha - \tau$) as

$$\rho \cong \tau_\alpha(1-\tau_r) = \tau_\alpha - \tau \quad \text{(eq. 27)}$$

For a two-cover system of not necessarily same materials, the following equation can be obtained (subscript 1 refers to the outer cover and 2 to the inner one):

$$\tau = \frac{1}{2}\left[\left(\frac{\tau_1\tau_2}{1-\rho_1\rho_2}\right)_\perp + \left(\frac{\tau_1\tau_2}{1-\rho_1\rho_2}\right)_\parallel\right] = \frac{1}{2}(\tau_\perp + \tau_\parallel) \quad \text{(eq. 28)}$$

$$\rho = \frac{1}{2}\left[\left(\rho_1 + \frac{\tau\rho_2\tau_1}{\tau_2}\right)_\perp + \left(\rho_1 + \frac{\tau\rho_2\tau_1}{\tau_2}\right)_\parallel\right] = \frac{1}{2}(\rho_\perp + \rho_\parallel) \quad \text{(eq. 29)}$$

Thermal Analyses of Flat Plate Collectors

A flat surface absorbs beam (G_{Bt}), diffuse (G_{Dt}), and ground-reflected (G_{Gt}) solar radiation; that is,

$$G_t = G_{Bt} + G_{Dt} + G_{Gt} \quad \text{(eq. 1)}$$

the beam radiation on a tilted surface is

$$G_{Bt} = G_{Bn} \cos(\theta) \quad \text{(eq. 2)}$$

and on a horizontal surface,

$$G_B = G_{Bn} \cos(\Phi) \quad \text{(eq. 3)}$$

where

G_{Bt} = beam radiation on a tilted surface (W/m^2).

G_B = beam radiation on a horizontal surface (W/m^2).

It follows that;

$$R_B = \frac{G_{Bt}}{G_B} = \frac{\cos(\theta)}{\cos(\Phi)} \quad \text{(eq. 4)}$$

where *RB* is called the *beam radiation tilt factor*.

So the beam radiation component for any surface is

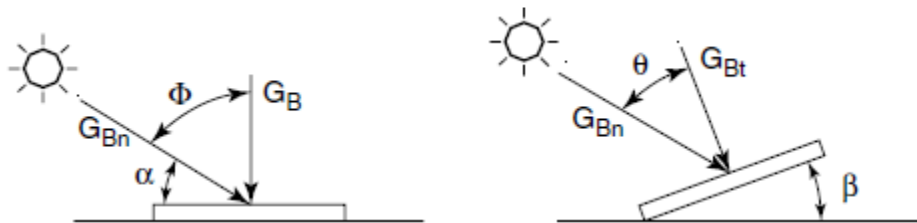$$G_{Bt} = G_B R_B \quad \text{(eq. 5)}$$

Figure 1. Beam radiation on horizontal and tilted surfaces.

Many models give the solar radiation on a tilted surface. The first one is the isotropic sky model developed originally by Hottel and Woertz (1942) and refined by Liu and Jordan (1960). According to this model, radiation is calculated as follows.

Diffuse radiation on a horizontal surface,

$$G_D = 2 \int_0^{\pi/2} G_R \cos(\Phi) \, d\Phi = 2G_R$$

(eq. 6)

where GR = diffuse sky radiance (W/m²-rad).

Diffuse radiation on a tilted surface,

$$G_{Dt} = \int_0^{\pi/2-\beta} G_R \cos(\Phi) \, d\Phi + \int_0^{\pi/2} G_R \cos(\Phi) \, d\Phi$$

(eq. 7)

where β is the surface tilt angle as shown in figure 1.

From Eq.6, the second term of Eq. 7 becomes $G_R = G_D/2$.

Therefore, Eq. 7 becomes

$$G_{Dt} = \frac{G_D}{2} \int_0^{\pi/2-\beta} \cos(\Phi) \, d\Phi + \frac{G_D}{2} = \frac{G_D}{2}\left[\sin\left(\frac{\pi}{2} - \beta\right)\right] + \frac{G_D}{2} = G_D\left[\frac{1 + \cos(\beta)}{2}\right]$$

(eq. 8)

Similarly, the ground-reflected radiation is obtained by $\rho_G(G_B = G_D)$, where ρ_G is ground albedo.

Therefore, G_{Gt} is obtained as follows.

Ground-reflected radiation,

$$\rho_G(G_B + G_D) = 2 \int_0^{\pi/2} G_r \cos(\Phi) \, d\Phi = 2G_r$$

(eq. 9)

where Gr is the isotropic ground-reflected radiance (W/m2-rad).

Ground-reflected radiation on tilted surfaces,

$$G_{Gt} = \int_{\pi/2-\beta}^{\pi/2} G_r \cos(\Phi) \, d\Phi$$

(eq. 10)

Combining Eq. 9 and 10 as before,

$$G_{Gt} = \rho_G(G_B + G_D)\left[\frac{1 - \cos(\beta)}{2}\right]$$

(eq. 11)

Therefore, inserting Eqs. 8 and 11 into Eq. 1, we get

$$G_t = R_B G_B + G_D \left[\frac{1 + \cos(\beta)}{2} \right] + (G_B + G_D)\rho_G \left[\frac{1 - \cos(\beta)}{2} \right]$$
(eq. 12)

The total radiation on a horizontal surface, G, is the sum of horizontal beam and diffuse radiation; that is,

$$G = G_B + G_D$$
(eq. 13)

Therefore, Eq. 12 can also be written as

$$R = \frac{G_t}{G} = \frac{G_B}{G} R_B + \frac{G_D}{G} \left[\frac{1 + \cos(\beta)}{2} \right] + \rho_G \left[\frac{1 - \cos(\beta)}{2} \right]$$
(eq. 14)

where R is called the *total radiation tilt factor*.

Using the isotropic model;

$$G_t = R_B G_B + G_D \left[\frac{1 + \cos(\beta)}{2} \right] + (G_B + G_D)\rho_G \left[\frac{1 - \cos(\beta)}{2} \right]$$

Eq. 12 can be modified to give the absorbed radiation, S, by multiplying each term with the appropriate transmittance-absorptance product as follows:

$$S = I_B R_B (\tau\alpha)_B + I_D (\tau\alpha)_D \left[\frac{1 + \cos(\beta)}{2} \right]$$
$$+ \rho_G (I_B + I_D)(\tau\alpha)_G \left[\frac{1 - \cos(\beta)}{2} \right]$$
(eq. 15)

where the terms [1 + cos(β)]/2 and [1 – cos(β)]/2 are the view factors from the collector to the sky and from the collector to the ground, respectively.

The combination of cover with the absorber plate is shown in Figure 2, together with a ray tracing of the radiation. As can be seen, of the incident energy falling on the collector, τα is absorbed by the absorber plate and (1 - α)τ is reflected back to the glass cover. The reflection from the absorber plate is assumed to be diffuse, so the fraction (1 - α)τ that strikes the glass cover is diffuse radiation and (1 - α)τρ$_D$ is reflected back to the absorber plate. The multiple reflection of diffuse radiation continues so that the fraction of the incident solar energy ultimately absorbed is

$$(\tau\alpha) = \tau\alpha\sum_{n=1}^{\infty}\left[(1-\alpha)\rho_D\right]^n = \frac{\tau\alpha}{1-(1-\alpha)\rho_D}$$

(eq. 16)

Typical values of (τα) are 0.7–0.75 for window glass and 0.9–0.85 for low-iron glass. A reasonable approximation of Eq. 16 for most practical solar collectors is
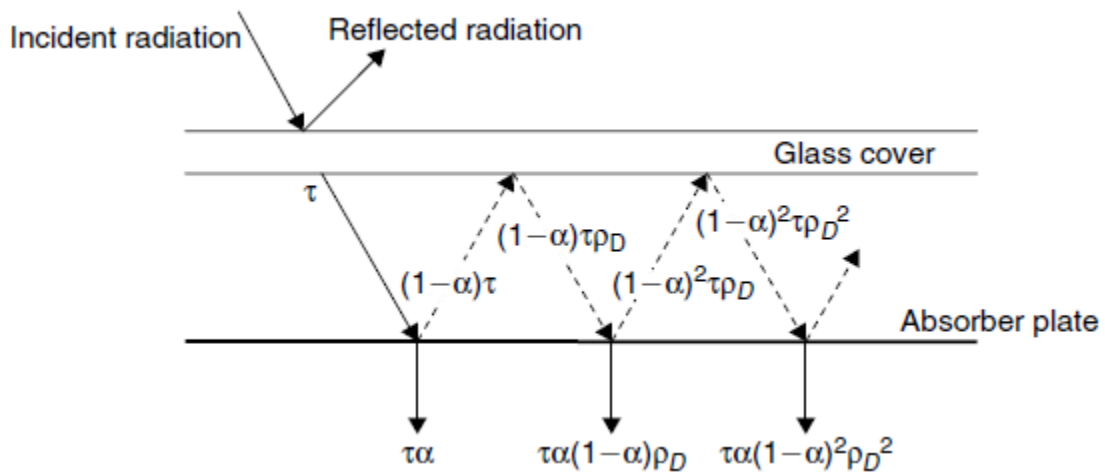
$$(\tau\alpha) \cong 1.01\tau\alpha$$

(eq. 17)

Figure 2. Radiation transfer between the glass cover and absorber plate.

The reflectance of the glass cover for diffuse radiation incident from the absorber plate, ρ_D, can be estimated as the difference between τ_α and τ at an angle of 60°. For single covers, the following values can be used for ρ_D:

For $KL = 0.0125$, $\rho_D = 0.15$.
For $KL = 0.0370$, $\rho_D = 0.12$.
For $KL = 0.0524$, $\rho_D = 0.11$.

For a given collector tilt angle, β, the following empirical relations, derived by Brandemuehl and Beckman (1980), can be used to find the effective incidence angle for diffuse radiation from sky, $\theta_{e,D}$, and ground-reflected radiation, $\theta_{e,G}$:

$$\theta_{e,D} = 59.68 - 0.1388\beta + 0.001497\beta^2$$

$$\theta_{e,G} = 90 - 0.5788\beta + 0.002693\beta^2$$

(eq. 18a-b)

where β = collector slope angle in degrees.

The angle dependent absorptance from 0 to 80° can be obtained from (Beckman et al., 1977):

$$\frac{a}{a_n} = 1 + 2.0345 \times 10^{-3}\theta_e - 1.99 \times 10^{-4}\theta_e^2$$
$$+ 5.324 \times 10^{-6}\theta_e^3 - 4.799 \times 10^{-8}\theta_e^4 \qquad \textbf{(eq.19)}$$

where

θ_e = effective incidence angle (degrees).

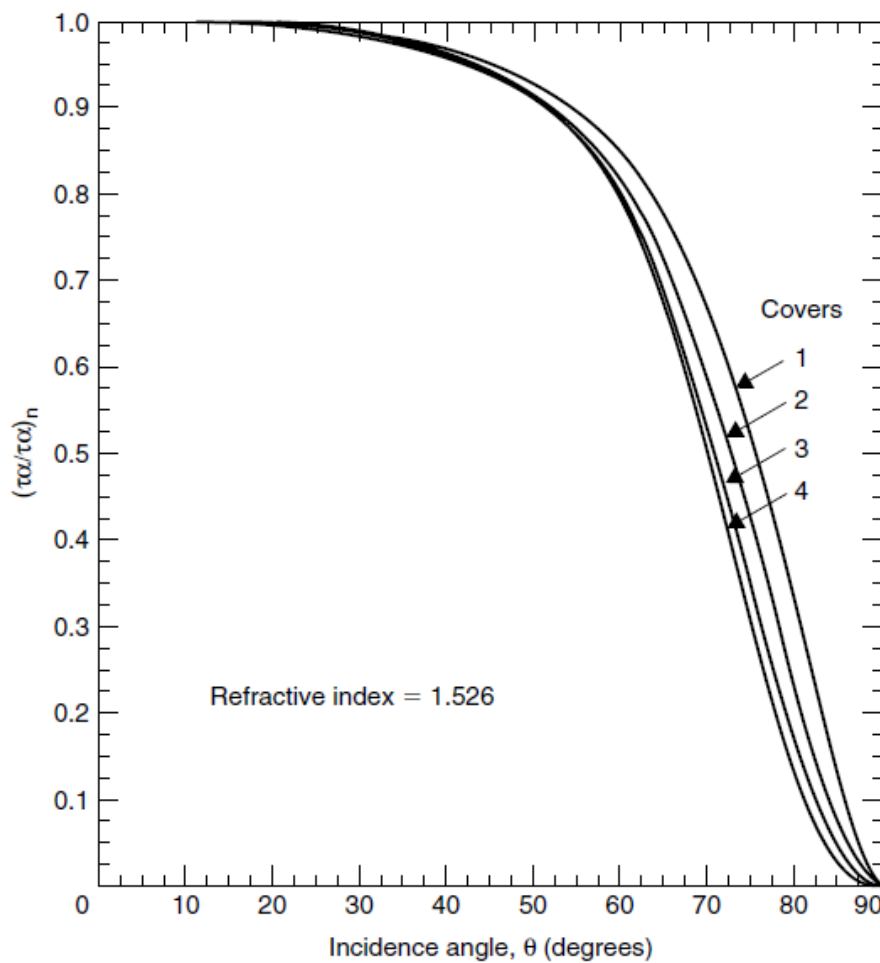θ_n = absorptance at normal incident angle, which can be found from the properties of the absorber.

Figure 3. Typical (τα)/(τα)n curves for one to four glass covers.

Subsequently, Eq. 16 can be used to find $(\tau\alpha)_D$ and $(\tau\alpha)_G$. The incidence angle, θ of the beam radiation required to estimate R_B can be used to find $(\tau\alpha)_B$.

Alternatively, (τα)n can be found from the properties of the cover and absorber materials, and Figure 3 can be used at the appropriate angle of incidence for each radiation component to find the three transmittance-absorptance products.

When measurements of incident solar radiation (I_t) are available, instead of Eq. 15, the following relation can be used:

$$S = (\tau\alpha)_{av}I_t \quad \textbf{(eq. 20)}$$

where $(\tau\alpha)_{av}$ can be obtained from

$$(\tau\alpha)_{av} \cong 0.96(\tau\alpha)_B \quad \textbf{(eq. 21)}$$

PROBLEM SETS

Problem 1: A glass with transmissivity of 0.92 is used in a certain application for wavelengths 0.3 and 3.0 μm. The glass is opaque to all other wavelengths. Assuming that the sun is a blackbody at 5760 K and neglecting atmospheric attenuation, determine the percent of incident solar energy transmitted through the glass. If the interior of the application is assumed to be a blackbody at 373 K, determine the percent of radiation emitted from the interior and transmitted out through the glass.

Solution 1:

For the incoming solar radiation at 5760 K, we have

$$\lambda_1 T = 0.3 \times 5760 = 1728\,\mu m\text{-}K$$
$$\lambda_2 T = 3 \times 5760 = 17280\,\mu m\text{-}K$$

From Table 1 by interpolation, we get

$$\frac{E_b(0 \rightarrow \lambda_1 T)}{\sigma T^4} = 0.0317 = 3.17\%$$

$$\frac{E_b(0 \rightarrow \lambda_2 T)}{\sigma T^4} = 0.9778 = 97.78\%$$

Therefore, the percent of solar radiation incident on the glass in the wavelength range 0.3–3 μm is

$$\frac{E_b(\lambda_1 T \rightarrow \lambda_2 T)}{\sigma T^4} = 97.78 - 3.17 = 94.61\%$$

In addition, the percentage of radiation transmitted through the glass is 0.92 x 94.61 = 87.04%.

For the outgoing infrared radiation at 373 K, we have

$\lambda_1 T = 0.3 \times 373 = 111.9 \mu\text{m-K}$

$\lambda_2 T = 3 \times 373 = 1119.0 \mu\text{m-K}$

From Table 1, we get

$$\frac{E_b(0 \to \lambda_1 T)}{\sigma T^4} = 0.0 = 0\%$$

$$\frac{E_b(0 \to \lambda_2 T)}{\sigma T^4} = 0.00101 = 0.1\%$$

The percent of outgoing infrared radiation incident on the glass in the wavelength 0.3–3 µm is 0.1%, and the percent of this radiation transmitted out through the glass is only 0.92 x 0.1 = 0.092%. This example, in fact, demonstrates the principle of the greenhouse effect; i.e., once the solar energy is absorbed by the interior objects, it is effectively trapped.

Problem 2: For a clear winter day, I_B = 1.42 MJ/m² and I_D = 0.39 MJ/m². Ground reflectance is 0.5, incidence angle is 23°, and R_B = 2.21. Calculate the absorbed solar radiation by a collector having a glass with KL = 0.037, the absorptance of the plate at normal incidence, α_n = 0.91, and the refraction index of glass is 1.526. The collector slope is 60°. (Given: for the beam radiation 23°, τ_α = 0.962, τ_r = 0.916, $\tau = \tau_\alpha \times \tau_r$; for the diffuse radiation at 57°, τ_α = 0.957, τ_r = 0.858, $\tau = \tau_\alpha \times \tau_r$ and for the ground reflected radiation 23°, τ_α = 0.955, τ_r = 0.792, $\tau = \tau_\alpha \times \tau_r$)

Solution 2:

Using Eq. 19 for the beam radiation at θ = 23°,

$$\begin{aligned}\frac{a}{a_n} &= 1 + 2.0345 \times 10^{-3}\theta_e - 1.99 \times 10^{-4}\theta_e^2 \\ &\quad + 5.324 \times 10^{-6}\theta_e^3 - 4.799 \times 10^{-8}\theta_e^4 \\ &= 1 + 2.0345 \times 10^{-3} \times 23 - 1.99 \times 10^{-4} \times 23^2 \\ &\quad + 5.324 \times 10^{-6} \times 23^3 - 4.799 \times 10^{-8} \times 23^4 \\ &= 0.993\end{aligned}$$

$$\tau \cong \tau_\alpha \tau_r = 0.962 \times 0.916 = 0.881$$

From eq. 17

$$(\tau\alpha)_B = 1.01\tau \, (\alpha/\alpha_n)\alpha_n = 1.01 \times 0.881 \times 0.91 \times 0.993 = 0.804 \approx 0.80$$

From Eq. 18a, the effective incidence angle for diffuse radiation is

$$\theta_{e,D} = 59.68 - 0.1388\beta + 0.001497\beta^2$$
$$= 59.68 - 0.1388 \times 60 + 0.001497 \times 60^2 = 57°$$

Using Eq. 19 for the beam radiation at $\theta = 57°$, $\alpha/\alpha_n = 0.949$

$$\tau = 0.957 \times 0.858 = 0.821$$

From eq. 17,

$$(\tau\alpha)_D = 1.01\tau(\alpha/\alpha_n)\alpha_n = 1.01 \times 0.821 \times 0.949 \times 0.91 = 0.716 \approx 0.72$$

From Eq. (3.4b), the effective incidence angle for ground reflected radiation is

$$\theta_{e,G} = 90 - 0.5788\beta + 0.002693\beta^2$$
$$= 90 - 0.5788 \times 60 + 0.002693 \times 60^2 = 65°$$

Using Eq. 19 for the ground reflected radiation at $\theta = 65°$, $\alpha/\alpha_n = 0.897$

$$\tau = 0.792 \times 0.955 = 0.756$$

From eq. 17

$$(\tau\alpha)_G = 1.01\tau(\alpha/\alpha_n)\alpha_n = 1.01 \times 0.756 \times 0.897 \times 0.91 = 0.623 \approx 0.62$$

In different way from eq. 17

From the figure 3, for the beam radiation at $\theta = 23°$;

$$(\tau\alpha)/(\tau\alpha)_n = 0.98 \text{ and } (\tau\alpha)_B = 0.812 \times 0.98 = 0.796 \approx 0.80$$

From the figure 3, for diffuse radiation at $\theta = 57°$;

$$(\tau\alpha)/(\tau\alpha)_n = 0.89 \text{ and } (\tau\alpha)_D = 0.812 \times 0.89 = 0.722 \approx 0.72$$

From the figure 3, for ground-reflected radiation at $\theta = 65°$;

$$(\tau\alpha)/(\tau\alpha)_n = 0.76 \text{ and } (\tau\alpha)_G = 0.812 \times 0.76 = 0.617 \approx 0.62$$

All these values are very similar to the previously found values, but the effor required is much less.

Finally, the absorbed solar radiation is obtained from Eq. 15:

$$S = I_B R_B (\tau\alpha)_B + I_D (\tau\alpha)_D \left[\frac{1 + \cos(\beta)}{2}\right]$$

$$+ \rho_G (I_B + I_D)(\tau\alpha)_G \left[\frac{1 - \cos(\beta)}{2}\right]$$

$$= 1.42 \times 2.21 \times 0.80 + 0.39 \times 0.72 \left[\frac{1 + \cos(60)}{2}\right]$$

$$+ 0.5 \times 1.81 \times 0.62 \left[\frac{1 - \cos(60)}{2}\right] = 2.72 \text{ MJ/m}^2$$

Photovoltaic (PV) Cells

PVs or *solar cells* convert sunlight directly into electricity. When photons strike certain semiconductor materials, such as silicon, they dislodge electrons, which causes a potential difference to form between the specially treated front surface of the solar cells and the back surface. In order to increase the voltage, individual cells are combined in a panel form. The most advanced photon utilization technology is the solar cell to which the PV effects of semiconductors are applied.

Solar cells are the standard-bearer of the new energy technologies because of their great potential. Their successful development is dependent on cost reduction of the power-generating systems that include SCs.

Photovoltaic cells consist of a junction between two thin layers (positive, p, and negative, n) of dissimilar semiconducting materials. When a valance electron of an atom absorbs a photon of light, the energy of the electron is increased by the amount of energy of the photon. If the energy of the photon is equal to or more than the band gap of the semiconductor, the electron with the excess energy will jump into the conduction band where it can move freely. Figure 1. Shows the PV device schematically. These solar cell contains a junction of a p-type and an n-type semiconductor (a p-n junction).
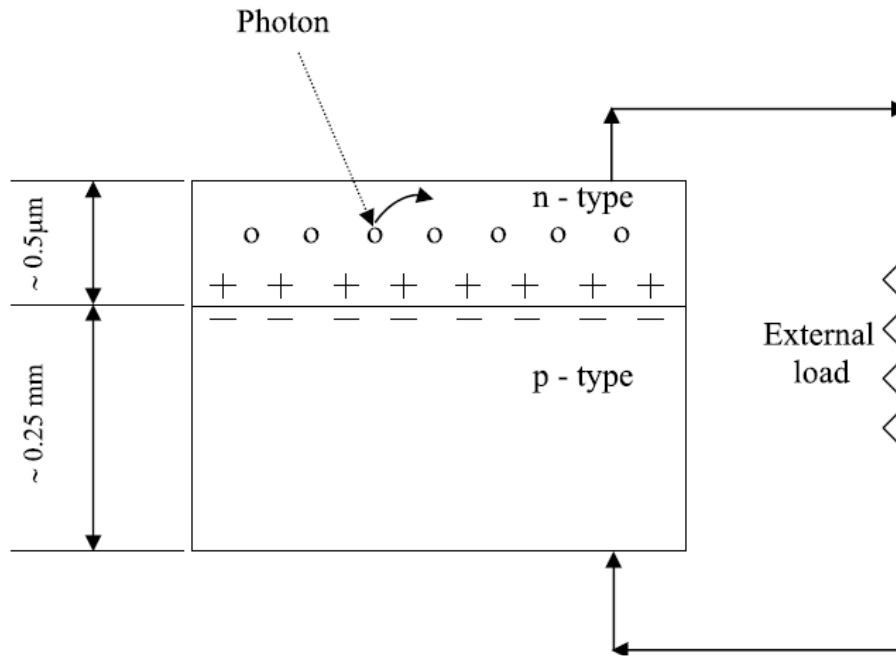
Figure 1. Simple PV cell and resistive load

The thickness of the n-type layer in a typical crystalline silicon cell is about 0.5 µm, whereas that of the p-type layer is about 0.25 mm. Thermal radiation is in fact an electromagnetic wave that travels at the speed of light (C = 300,000 km/s in a vacuum). This speed is related to the wavelength (λ) and frequency (v) of the radiation as given by the equation:

$$C = \lambda v \quad \textbf{(Eq. 1)}$$

The energy contained in a photon, Ep, is given by

$$E_p = h v \quad \textbf{(Eq. 2)}$$

where

h = Planck's constant, = 6.625 x 10⁻³⁴ J-s.

v = frequency (s⁻¹).

Combining Eq. 1 with Eq. 2, we get

$$E_p = \frac{hC}{\lambda} \quad \textbf{(Eq. 3)}$$

Silicon has a band gab of 1.11 eV (1 eV = 1.6 x 10⁻¹⁹ J); therefore, by using Eq. 3, it can be found that photons with wavelength of 1.12 μm or less are useful in creating electron-hole pairs and thus electricity. The number of photons, n_p, incident on a cell can be estimated from the intensity of light, I_p:

$$n_p = \frac{I_p}{E_p}$$ (Eq. 4)

When solar energy (photons) hits the solar cell, electrons are knocked loose from the atoms in the semiconductor material, creating electron-hole pairs. If electrical conductors are attached to the positive and negative sides, forming an electrical circuit, the electrons are captured in the form of electric current, called *photocurrent*, I_{ph}. During darkness the solar cell is not active and works as a diode, i.e., a p-n junction that does not produce any current or voltage. If, however, it is connected to an external, large voltage supply, it generates a current, called the *diode* or *dark current*, I_D. A solar cell is usually represented by an electrical equivalent one-diode model, shown in Figure 2.
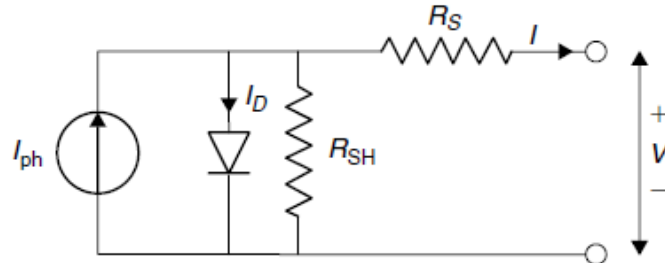

Figure 2. Single solar cell model.

As shown in Figure 2, the model contains a current source, I_{ph}, one diode, and a series resistance *RS*, which represents the resistance inside each cell. The diode has also an internal shunt resistance, as shown in Figure 2. The net current is the difference between the photocurrent, I_{ph}, and the normal diode current, I_D, given by

$$I = I_{ph} - I_D = I_{ph} - I_o \left\{ \exp\left[\frac{e(V + IR_S)}{kT_C} \right] - 1 \right\} - \frac{V + IR_S}{R_{SH}}$$ (Eq. 5)

It should be noted that the shunt resistance is usually much bigger than a load resistance, whereas the series resistance is much smaller than a load resistance, so that less power is dissipated internally within the cell. Therefore, by ignoring these two resistances, the net current is the difference between the photocurrent, I_{ph}, and the normal diode current, I_D, given by

$$I = I_{ph} - I_D = I_{ph} - I_o \left[\exp\left(\frac{eV}{kT_C} \right) - 1 \right]$$

(Eq. 6)

where

k = Boltzmann's gas constant, = 1.381 x 10^{-23} J/K.

T_C = absolute temperature of the cell (K).

e = electronic charge, = 1.602 x 10^{-19} J/V.

V = voltage imposed across the cell (V).

I_o = dark saturation current, which depends strongly on temperature (A).

Figure 3 shows the I-V characteristic curve of a solar sell for a certain irradiance (G_t) at a fixed cell temperature, T_C. The current from a PV cell depends on the external voltage applied and the amount of sunlight on the cell. When the cell is short-circuited, the current is at maximum (short-circuit current, I_{sc}), and the voltage across the cell is 0. When the PV cell circuit is open, with the leads not making a circuit, the voltage is at its maximum (open-circuit voltage, V_{oc}), and the current is 0. In either case, at open circuit or short circuit, the power (current times voltage) is 0. Between open circuit and short circuit, the power output is greater than 0.
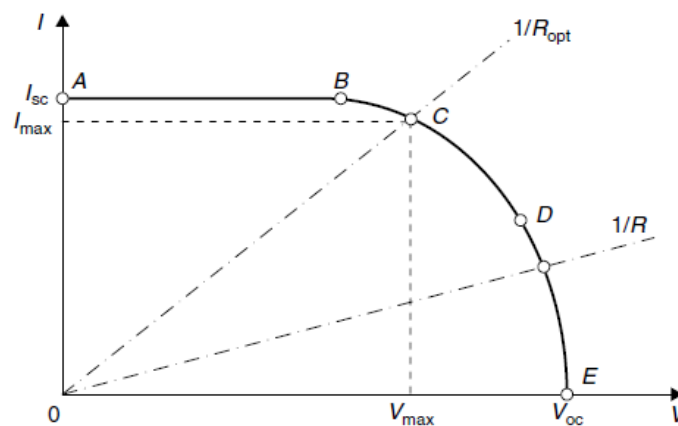

Figure 3. Representative current-voltage curve for photovoltaic cells.

The load characteristic is a straight line with a slope $1/V = 1/R$. If the load resistance is small, the cell operates in the region *AB* of the curve, where the cell behaves as a constant current source, almost equal to the short-circuit current. On the other hand, if the load resistance is large, the cell operates on the region *DE* of the curve, where the cell behaves more as a constant voltage source, almost equal to the open circuit voltage. The power can be calculated by the product of the current and voltage. If this exercise is performed and plotted on the same axes, then Figure 4 can be obtained.
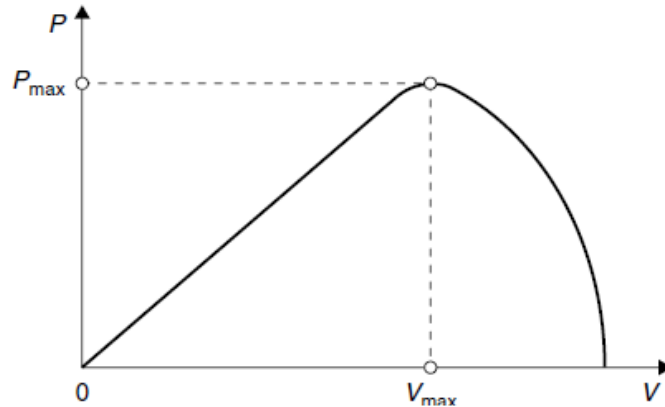

Figure 4. Representative power-voltage curve for photovoltaic cells.

The maximum power passes from a maximum power point (point *C* on Figure 3), at which point the load resistance is optimum, R_{opt}, and the power dissipated in the resistive load is maximum and given by

$$P_{max} = I_{max} V_{max} \quad \text{(Eq. 7)}$$

Point *C* on Figure 3 is also called the *maximum power point*, which is the operating point P_{max}, I_{max}, V_{max} at which the output power is maximized. Given P_{max}, an additional parameter, called the *fill factor*, FF, can be calculated such that

$$P_{max} = I_{sc} V_{oc} FF \quad \text{(Eq. 8)}$$

Or

$$FF = \frac{P_{max}}{I_{sc} V_{oc}} = \frac{I_{max} V_{max}}{I_{sc} V_{oc}} \quad \text{(Eq. 9)}$$

The fill factor is a measure of the real *I-V* characteristic. For good cells, its value is greater than 0.7. The fill factor decreases as the cell temperature increases.

Thus, by illuminating and loading a PV cell so that the voltage equals the PV cell's *V*max, the output power is maximized. The cell can be loaded using resistive loads, electronic loads, or batteries. Typical parameters of a single-crystal solar cell are current density I_{sc} = 32 mA/cm^2, V_{oc} = 0.58 V, V_{max} = 0.47 V, FF = 0.72, and P_{max} = 2273 mW.

Other fundamental parameters that can be obtained from Figure 3 are the short-circuit current and the open circuit voltage. The short-circuit current, I_{sc}, is the higher value of the current generated by the cell and is obtained under short-circuit conditions, i.e., *V* = 0, and is equal to I_{ph}. The open circuit voltage corresponds to the voltage drop across the diode when it is traversed by the photocurrent, I_{ph}, which is equal to I_D, when the generated current is *I* = 0. This is the voltage of the cell during nighttime and can be obtained from Eq. 6:

$$\exp\left(\frac{eV_{oc}}{kT_C}\right) - 1 = \frac{I_{sc}}{I_o} \quad \textbf{(Eq. 10)}$$

which can be solved for V_{oc}:

$$V_{oc} = \frac{kT_C}{e}\ln\left(\frac{I_{sc}}{I_o} + 1\right) = V_t\ln\left(\frac{I_{sc}}{I_o} + 1\right) \quad \textbf{(Eq. 11)}$$

where V_t = thermal voltage (V) given by

$$V_t = \frac{kT_C}{e} \quad \textbf{(Eq. 12)}$$

The output power, *P*, from a photovoltaic cell is given by

$$P = IV \quad \textbf{(Eq. 13)}$$

The output power depends also on the load resistance, *R*; and by considering that *V* = *IR*, it gives

$$P = I^2R \quad \textbf{(Eq. 14)}$$

Substituting Eq. 6 into Eq. 13 gives

$$P = \left\{ I_{sc} - I_o \left[\exp\left(\frac{eV}{kT_C} \right) - 1 \right] \right\} V$$
(Eq. 15)

Equation 15 can be differentiated with respect to V. By setting the derivative equal to 0, the external voltage, V_{max}, that gives the maximum cell output power can be obtained:

$$\exp\left(\frac{eV_{max}}{kT_C} \right)\left(1 + \frac{eV_{max}}{kT_C} \right) = 1 + \frac{I_{sc}}{I_o}$$
(Eq. 16)

This is an explicit equation of the voltage V_{max}, which maximizes the power in terms of the short-circuit current ($I_{sc} = I_{ph}$), the dark saturation current (I_o), and the absolute cell temperature, T_C. If the values of these three parameters are known, then V_{max} can be obtained from Eq. 16 by trial and error.

The load current, I_{max}, which maximizes the output power, can be found by substituting Eq. 16 into Eq. 6:

$$I_{max} = I_{sc} - I_o \left[\exp\left(\frac{eV}{kT_C} \right) - 1 \right] = I_{sc} - I_o \left[\frac{1 + \dfrac{I_{sc}}{I_o}}{1 + \dfrac{eV_{max}}{kT_C}} - 1 \right]$$
(Eq. 17)

which gives

$$I_{max} = \frac{eV_{max}}{kT_C + eV_{max}} (I_{sc} + I_o)$$
(Eq. 18)

By using Eq. 7,

$$P_{max} = \frac{eV_{max}^2}{kT_C + eV_{max}} (I_{sc} + I_o)$$
(Eq. 19)

Efficiency is another measure of PV cells that is sometimes reported. *Efficiency* is defined as the maximum electrical power output divided by the incident light power. Another parameter of interest is the maximum efficiency, which is the ratio between the maximum power and the incident light power, given by

$$\eta_{max} = \frac{P_{max}}{P_{in}} = \frac{I_{max}V_{max}}{AG_t}$$ **(Eq. 20)**

where A = cell area (m²).

PROBLEM SETS

Problem 1: If the dark saturation current of a solar cell is 1.7 x 10⁻⁸ A/m², the cell temperature is 27°C, and the short-circuit current density is 250 A/m², calculate the open circuit voltage, V_{oc}; voltage at maximum power, V_{max}; current density at maximum power, I_{max}; maximum power, P_{max}; and maximum efficiency, η_{max}. What cell area is required to get an output of 20 W when the available solar radiation is 820 W/m²?

Solution 1:

First the value of e/kT_C is evaluated, which is used in many relations:

$$\frac{e}{kT_C} = \frac{1.602 \times 10^{-19}}{1.381 \times 10^{-23} \times 300} = 38.67\,\text{V}^{-1}$$

Using Eq. 11,

$$V_{oc} = \frac{kT_C}{e} \ln\left(\frac{I_{sc}}{I_o} + 1\right) = \frac{1}{38.67}\ln\left(\frac{250}{1.7 \times 10^{-8}} + 1\right) = 0.605\,\text{V}$$

Voltage at maximum power can be found from Eq. 16 by trial and error:

$$\exp\left(\frac{eV_{max}}{kT_C}\right)\left(1 + \frac{eV_{max}}{kT_C}\right) = 1 + \frac{I_{sc}}{I_o}$$

or

$$\exp(38.67V_{max})(1 + 38.67V_{max}) = 1 + \frac{250}{1.7 \times 10^{-8}}$$

which gives V_{max} = 0.47 V.

The current density at maximum power point can be estimated from Eq. 18:

$$I_{max} = \frac{eV_{max}}{kT_C + eV_{max}}(I_{sc} + I_o)$$

$$= \frac{1.602 \times 10^{-19} x 0.47}{1.381 \times 10^{-23} \times 300 + 1.602 x 10^{-19} \times 0.47}(250 + 1.7 \times 10^{-8})$$

$$= 237\,A/m^2$$

Maximum power, P_{max}, is obtained from Eq. 7:

$$P_{max} = I_{max}V_{max} = 237 \times 0.47 = 111.4\,W/m^2$$

Maximum efficiency, η_{max}, is obtained from Eq. 20:

$$\eta_{max} = \frac{P_{max}}{P_{in}} = \frac{111.4}{820} = 13.58\%$$

Finally, the cell area required to get an output of 20 W is

$$A = \frac{P_{req}}{P_{max}} = \frac{20}{111.4} = 0.18\,m^2$$

References:

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