WEEK 7: GEOTHERMAL ENERGY

INTRODUCTION

Use of a geothermal resource ultimately depends on the resource temperature. Thus, there have been a number of classification methods aimed at categorizing geothermal resources by temperature. Here, we will use the following gross temperature categories:

(a) high-temperature uses: $T_{resource}$ > 150 °C

(b) medium-temperature uses: 90 °C<$T_{resource}$ < 150 °C

(c) low-temperature uses: 30 °C< $T_{resource}$ < 90 °C

(d) ambient temperatures (heat pump uses): 0 °C<$T_{resource}$

Note that there is no distinct break between categories. The geothermal power industry typically uses only the top three temperature categories (a, b, and c), based on cut-off temperatures of economical electric power generation, which has historically not been economical for resources with temperatures below about 150 C. However, binary organic Rankine cycle power plants, under favorable circumstances, have demonstrated that it is possible to generate electricity economically above 90 °C. A fourth category (d) is added here to distinguish the geothermal heat pump applications.

Figure 1 shows some of the many past and/or current uses of geothermal energy worldwide. As shown in this figure, there are many other 'high-temperature' resource use possibilities aside from electric power generation. Many of the medium-temperature uses are termed 'direct uses' because there is no energy conversion process, and the resource temperature matches or exceeds that required by the load. However, ambient groundwater can also be used for direct cooling applications.

Currently operating geothermal power plants make use of so-called hydrothermal resources. Other types of resource exist, but their utilization is still considered a developing and emerging use. The main types of geothermal power plant are summarized in Table 1: (i) binary, (ii) flash steam, and (iii) dry steam.

Table 1. Summary of geothermal Power Plant Types

Power plant type	Geothermal resource	Working fluid	Occurrence
Binary cycle	Single-phase fluid (compressed liquid)	Engineered fluid (low-boiling-point refrigerant or hydrocarbon)	Most common type (generally considered feasible at resource temperatures up to ~175 °C)
Flash-type	Two-phase fluid	Geothermal fluid	Moderately common
Dry steam	Single-phase fluid (superheated vapor)	Geothermal fluid	Very rare (only a few plants worldwide)

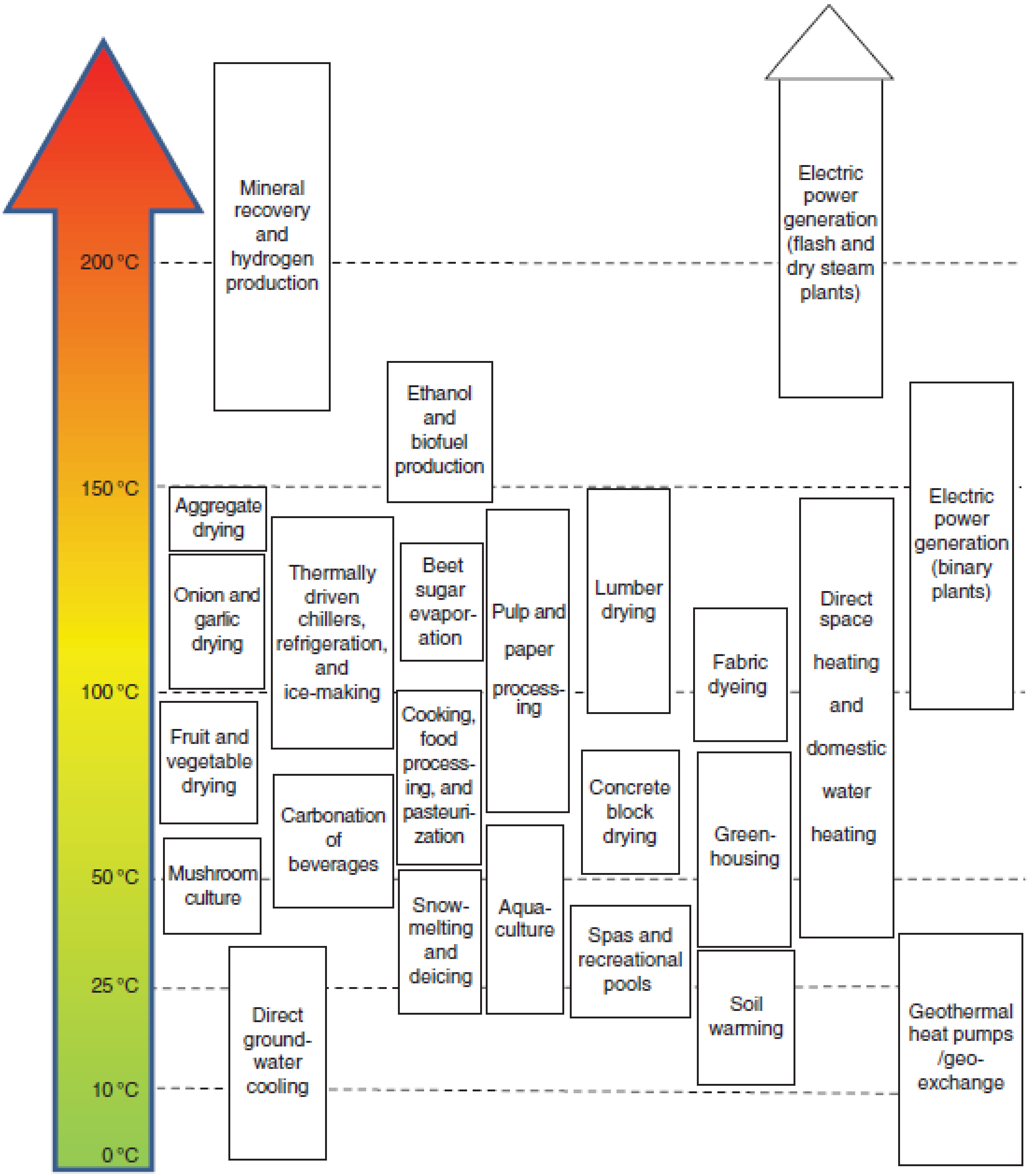

Figure 1. Worldwide past and present utilization of geothermal energy based on resource temperature

THEORY

Equations of Groundwater Flow (Darcy's Law)

Darcy conducted a number of experiments with the column tilted at various angles from the vertical, by introducing water into the column, which was allowed to flow through the column until the inflow rate ($\dot{Q}$) equilibrated with the outflow rate. Darcy noted that $\dot{Q}$ was directly proportional to the difference in equilibrated water levels in the manometers (i.e., the change in hydraulic head, or Δh) and the cross-sectional area (A) of the column tube, and that $\dot{Q}$ was indirectly proportional to the length of the column (or ΔL). Thus,

$$\dot{Q} \propto A \frac{\Delta h}{\Delta L}$$ **(Eq. 1)**

Darcy converted the proportionality to an equality with the introduction of a proportionality constant (K), known as the hydraulic conductivity, and thus the law became

$$\dot{Q} = KA \frac{\Delta h}{\Delta L}$$ **(Eq. 2)**

Darcy concluded that this proportionality (K) must be a function of the soil. Subsequent researchers concluded that the hydraulic conductivity must be a function not only of the soil or rock but also of the fluid. Thus, hydraulic conductivity is further defined as

$$K = \frac{k\rho g}{\mu}$$ **(Eq. 3)**

where k is the permeability of the medium (with dimensions of [L^2]) and is a function of the grain size and interconnected pore spaces, ρ and μ are the density and viscosity, respectively, of the fluid, and g is the acceleration due to gravity.

It should be emphasized that Darcy's law is an empirical law; it rests only on experimental evidence. Attempts have been made, however, to derive Darcy's law more fundamentally, from Navier–Stokes equations, which are widely known in fluid mechanics. Several analogies in other forms of energy transfers due to a driving gradient. Some of these analogies are summarized in Table 2.

Table 2. Summary of Analogous One-Dimensional Energy Rate Equations

Energy Form	Energy Flux (common SI units)	Driving Gradient	Proportionality Constant (common units)	Governing Law
Hydraulic (groundwater)	Groundwater flux, $\dot{Q}/m^2$ ($m{\cdot}s^{-1}{\cdot}m^{-2}$)	Hydraulic gradient ($\Delta H/\Delta L$)	Hydraulic Conductivity, K (m/s)	Darcy's law $\dot{q}'=K\frac{\Delta H}{\Delta L}$
Thermal	Conduction heat flux, q/m^2 ($W{\cdot}m^{-2}$)	Temperature gradient ($\Delta T/\Delta x$)	Thermal conductivity, k ($W{\cdot}m^{-1}{\cdot}K^{-1}$)	Fourier's law of heat conduction $\dot{q}'=k\frac{\Delta T}{\Delta x}$
Thermal	Convection heat flux, q/m^2 ($W{\cdot}m^{-2}$)	Temperature gradient (ΔT)	Convection heat transfer coefficient, h_c ($W{\cdot}m^{-2}{\cdot}K^{-1}$)	Newton's law of cooling $\dot{q}'=h_c\Delta T$
Thermal	Radiation heat flux, q/m^2 ($W{\cdot}m^{-2}$)	Temperature gradient (ΔT)	Radiation heat transfer coefficient (linearized), h_r ($W{\cdot}m^{-2}{\cdot}K^{-1}$)	Stefan–Boltzmann's law $\dot{q}'=h_r\Delta T$
Electrical	Current, I (A)	Voltage drop (ΔV)	Electrical resistance, R (ohm)	Ohm's law $I=\frac{1}{R}V$
Chemical/ mass	Chemical flux, J ($mg{\cdot}s^{-1}{\cdot}m^{-2}$)	Concentration gradient ($\Delta C/\Delta x$)	Diffusion coefficient, D ($m^2{\cdot}s^{-1}$)	Fick's law $J=D\frac{\Delta C}{\Delta x}$
Elastic	Force, F (N)	Distance (Δx)	Spring constant, k ($N{\cdot}m^{-1}$)	Hooke's law $F=k\Delta x$

Borehole Heat Exchanger (BHE)

The term borehole heat exchanger (BHE) refers to a closed-loop pipe assembly installed in a vertical borehole with radius r_b over some active depth H for purposes of heat exchange with the Earth, as shown in Figure 2. The top of the active BHE is buried at some depth (D) from the ground surface. These boreholes are designed to extract (or reject) a certain amount of thermal energy ($\dot{q}'$) per unit depth (H) by pumping a fluid, with an average temperature (T_f), through the heat exchanger. Thus, $\dot{q}'$ is the heat extraction or rejection rate divided by the active borehole depth. Heat transfer occurs from the fluid to the ground, the undisturbed 'far-field' temperature of which remains at T_g. T_b represents the average temperature at the borehole wall.

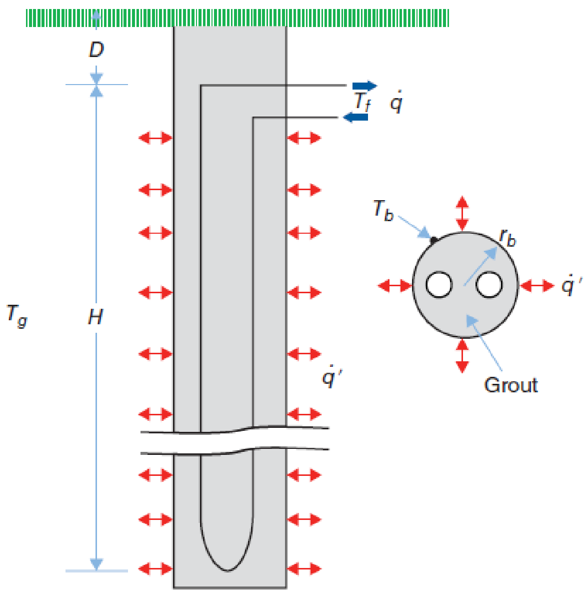

Figure 2. Schematic diagram of a typical geothermal borehole heat exchanger (BHE) comprised of a single U-tube grouted in a vertical borehole

Thermal and Mathematical Considerations for BHEs

General BHE Thermal Considerations

Eskilson (1987) and Hellström (1991) provide a detailed thermal analysis of heat extraction boreholes and describe important parameters in their performance. Eskilson (1987) also identifies negligible parameters and effects on borehole heat exchanger performance. These are:

- Deviations from average thermal conductivity owing to stratified ground,
- Temperature variations at the ground surface,
- The effect of groundwater flow is negligible if the following criterion is met:

$$\frac{H\rho_w c_{p,w}\dot{q}_w}{2k} < 1 \quad \textbf{(Eq. 4)}$$

where H is the borehole depth, ρ_w is the groundwater density, $c_{p,w}$ is the groundwater heat capacity, q_w is the Darcy velocity of the groundwater, and k is the ground thermal conductivity.

- Transient thermal effects in the borehole grout and the heat carrier fluid above timescales of t_b:

$$t_b = \frac{5 r_b^2}{\alpha} \quad \textbf{(Eq. 5)}$$

where r_b is the borehole radius and α is the soil/rock thermal diffusivity. Thus, below this timescale, borehole transient thermal effects may be significant.

Mathematical Models of Heat Transfer around BHEs

The temperature distribution in the ground may be described by the partial differential heat diffusion equation expressed in cylindrical coordinates:

$$\alpha \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right] + G = \frac{\partial T}{\partial t} \quad \textbf{(Eq. 6)}$$

where T is the temperature, α is the thermal diffusivity, r is the radial coordinate, z is the vertical coordinate, t is the time, and G is a general source/sink term.

There are actually two types of line source models used in BHE design: (1) the infinite line source model and (2) the finite line source model.

In infinite line source models, a simplifying assumption is made that $\partial^2 T/\partial z^2 = 0$, and therefore only radial heat transfer in the ground is considered. Such a model is valid under certain restrictions of time and borehole depth, but after long periods of time, and/or with short boreholes, the heat transfer around a BHE takes on a significant vertical component and $\partial^2 T/\partial z^2$ must be accounted for; hence, the need for a finite line source model.

Infinite Line Source Analytical Models of Heat Transfer in the Ground

Infinite line source solutions are general solutions to Equation 6 when $\partial^2T/\partial z^2 = 0$. The initial conditions are prescribed as

$$T(r,t) = T(r,0) = T_g \quad \textbf{(Eq. 7)}$$

and the boundary conditions are given by

$$T(r,t) = T(\infty,t) = T_g \quad \textbf{(Eq. 8)}$$

$$\lim_{r\to 0}\left(r\frac{\partial T}{\partial r}\right) = \frac{\dot{q}'}{2\pi k} \quad \textbf{(Eq. 9)}$$

where T is temperature, T_g is the undisturbed ground temperature, $\dot{q}'$ is the heat transfer rate per length of line source (W.m^{-1} or Btu/h/ft), and k is the thermal conductivity of the medium (W.m^{-1}.K^{-1} or Btu/h/ft/°F).

Ingersoll and Plass (1948) and Ingersoll et al. (1954) provide an adaptation of Kelvin's line source model for practical use with BHEs

$$\Delta T_r = \frac{\dot{q}'}{4\pi k}\int_u^\infty \frac{e^{-u}}{u}\mathrm{d}u \quad \textbf{(Eq. 10)}$$

where ΔT_r is the temperature change (°C or °F) at radius r, and u is defined as

$$u = \frac{r^2}{4\alpha t} \quad \textbf{(Eq. 11)}$$

where r is the radius from the line source, α is the thermal diffusivity of the medium (m^2s^{-1} or ft^2/h), and t is the time duration (s or h) of the $\dot{q}'$ heat input ($W.m^{-1}$ or Btu/h/ft).

Equation 10 may be written more simply as

$$\Delta T_r = \frac{\dot{q}'}{4\pi k} W(u)$$ **(Eq. 12)**

where W(u) well function. For small values of u, Equation 12 may be approximated as

$$\Delta T_r = \frac{\dot{q}'}{4\pi k}\left(\ln\left(\frac{4\alpha t}{r^2}\right) - \gamma\right)$$ **(Eq. 13)**

where γ is Euler's constant (0.5772). Equations 12 and 13 have a lower limit of time at which the heat pulse has not reached the radius r of interest, and an upper limit of time to ensure radial heat flow from the line source. Eskilson (1987) defines these time limits as

$$5r^2/_{\alpha} < t < {}^{t_s}/_{10}$$ **(Eq. 14)**

where t_s is a characteristic timescale (also known as steady-state time) equivalent to $H^2/(9\alpha)$, where H is the borehole depth (m or ft) and α is the thermal diffusivity ($m^2 . s^{-1}$ or ft^2/h).
Note that Equation 12 can also be expressed as

$$\Delta T_r = \dot{q}' R_g'$$ **(Eq. 15)**

where R_g' is the ground thermal resistance per unit length of bore ($m.K.W^{-1}$). Thus, W(u)/(4πk) is the ground thermal resistance per unit length of bore.

Finite Line Source Analytical Models of Heat Transfer in the Ground

Finite line source solutions are general solutions of Equation 6 when $\partial^2 T/\partial z^2 \neq 0$. The initial conditions for finite line source solutions are prescribed as

$$T(r,z,t) = T(r,z,0) = T_g \quad \textbf{(Eq. 16)}$$

Note that this condition prescribes a geothermal gradient of zero, and assumes a temperature of T_g at a depth D (referring to the notation in Figure 2).
The boundary conditions are similar to those of the infinite line source, but are applied over the finite length of the BHE.

Claesson and Eskilson (1987) provide a simple and computationally efficient solution to Equation 6 as

$$\Delta T_r = \frac{\dot{q}'}{2\pi k} g \quad \textbf{(Eq. 17)}$$

where g is a dimensionless temperature response factor, referred to as the g-function, and is defined as

$$g\left(\frac{t}{t_s}, \frac{r}{H}\right) = \begin{cases} \ln\left(\frac{H}{2r}\right) + \frac{1}{2}\ln\left(\frac{t}{t_s}\right) & , \frac{5r^2}{\alpha} < t < t_s \\ \ln\left(\frac{H}{2r}\right) & , t > t_s \end{cases} \quad \textbf{(Eq. 18a-b)}$$

When Equation 17 is expressed in terms of a thermal resistance $\dot{q}'$ =1/R'.ΔT, then it can readily be seen that g/(2πk) can be described as the ground thermal resistance per unit length (R').

Determining the BHE Fluid Temperature

We are frequently interested in the change in fluid temperature of heat carrier fluid in the borehole heat exchanger. In order to calculate the change in temperature at a given time in the BHE fluid, we first need to calculate the temperature at the borehole wall by substituting the value of r_b in place of r in the above equations, and calculating the ground thermal resistance. Next, we need to account for the thermal resistance of the borehole elements (such as the pipe configuration within the borehole, the borehole grout, and the fluid thermal properties) using the so-called borehole thermal resistance (R'_b):

$$\Delta T_f = \dot{q}' R'_g + \dot{q}' R'_b \quad \textbf{(Eq. 19)}$$

where the subscript f refers to the heat carrier fluid within the BHE, and R'_b is the steady-state borehole thermal resistance per unit length (m.K.W^{-1}).
The average BHE fluid temperature is then simply calculated by adding the change in the fluid temperature ΔT_f to the undisturbed ground temperature (T_g):

$$T_{f,avg} = \Delta T_f + T_g \quad \textbf{(Eq. 20)}$$

It is emphasized here that the fluid temperature calculated by the above approach is the average temperature of the fluid circulating in the BHE. This is actually a simplifying approximation to the three-dimensional nature of the temperature distribution within the BHE fluid itself. As the BHE is a heat exchanger, the fluid temperature entering the BHE will obviously be different from the fluid temperature exiting the BHE.

PROBLEM SETS

Problem 1: Consider a BHE undergoing continuous heat extraction from the Earth at a rate of 5000 W. The borehole depth (H) is 100 m and the borehole diameter is 125 mm. The initial, undisturbed ground temperature (T_g) was 15 °C. The borehole is completely installed in shale with a thermal conductivity (k) of 2.0 W.m^{-1}.K^{-1}, a density (ρ) of 2500 kg.m^{-3}, and a heat capacity (c_p) of 1000 J.kg^{-1}.K^{-1}. The borehole thermal resistance has been estimated to be 0.1000 m.K.W^{-1}. Determine the following:
(a) the time (in hours) after which the transient thermal effects in the borehole grout and the heat carrier fluid are negligible,
(b) the time at which heat extraction reaches steady state,

(c) the time (in hours) after which the heat transfer around the BHE is no longer purely radial,

(d) the average temperature of the BHE circulating fluid after 50 h, and

(e) the average temperature of the BHE circulating fluid at steady-state conditions.

Solution 1:

(a) The time after borehole transients can be ignored is given by

$$t_0 = \frac{5r_b^2}{\alpha} = \frac{5(0.125\,\mathrm{m}/2)^2}{\dfrac{2.0\,\mathrm{W\cdot m^{-1}\cdot K^{-1}}}{2500\,\mathrm{kg\cdot m^{-3}} \times 1000\,\mathrm{J\cdot kg^{-1}\cdot K^{-1}}}} = \frac{5(0.0625\,\mathrm{m})^2}{8.0\mathrm{E\text{-}07}\,\mathrm{m^2\cdot s^{-1}}} = 2.44\mathrm{E{+}04}\,\mathrm{s}$$

note $\alpha = k/(\rho c_p)$.

$$\therefore t_0 = 6.78\,\mathrm{h}$$

(b) The time at which the heat extraction reaches steady state is given by

$$t_s = \frac{H^2}{9\alpha} = \frac{(100\,\mathrm{m})^2}{9 \times 8.0\mathrm{E\text{-}07}\,\mathrm{m^2\cdot s^{-1}}} = 1.39\mathrm{E{+}09}\,\mathrm{s}$$

$$\therefore t_s = 3.86\mathrm{E}+05\,\mathrm{h} = 44\,\mathrm{years}$$

(c) The time after which the heat transfer around the BHE is no longer purely radial can be estimated by $t_s/10$ (see Equation 14).

$$\therefore t_s = 3.86\mathrm{E}+04\,\mathrm{h} = 4.4\,\mathrm{years}$$

(d) The average temperature of the BHE circulating fluid after 50 h of continuous operation is given by

$$T_f = \dot{q}'R_g' + \dot{q}'R_b' + T_g$$

As $t_0 < 50\,\mathrm{h} < t_s$, then

$$R_g' = \frac{1}{2\pi k}g = \left(\frac{1}{2\pi k}\right)\left(\ln\left(\frac{H}{2r_b}\right) + \frac{1}{2}\,\ln\left(\frac{t}{t_s}\right)\right)$$

$$R_g'' = \left(\frac{1}{2\pi \times 2.0\,\mathrm{W\cdot m^{-1}\cdot K^{-1}}}\right)\left(\ln\left(\frac{100\,\mathrm{m}}{0.125\,\mathrm{m}}\right) + \frac{1}{2}\,\ln\left(\frac{50\,\mathrm{h}}{3.86\mathrm{E{+}05}\,\mathrm{h}}\right)\right) = 0.1758\,\mathrm{m\cdot K\cdot W^{-1}}$$

$$\therefore T_f = \frac{-5000\,\mathrm{W}}{100\,\mathrm{m}} \times (0.1758 + 0.1000)\,\mathrm{m\cdot K\cdot W^{-1}} + 15\,^{\circ}\mathrm{C} = 1.21\,^{\circ}\mathrm{C}$$

$$\therefore T_f = 1.2\,^{\circ}\mathrm{C}$$

(e) To calculate the average temperature of the BHE circulating fluid at steady state, the approach is the same as in part (c), except that the g-function differs, so R_g' is now

$$R_g' = \frac{1}{2\pi k} g = \left(\frac{1}{2\pi k}\right)\left(\ln\left(\frac{H}{2r_b}\right)\right) = 0.5319\,\mathrm{m \cdot K \cdot W^{-1}}$$

$$\therefore T_f = -16.6\,^{\circ}\mathrm{C}$$

References:

Andrew D. Chiasson, Geothermal Heat Pump and Heat Engine Systems – Theory and Practice, First Edition, Wiley-ASME Press Series, 2016.