WEEK 9: WIND ENERGY AND TURBINE

PROBLEM SETS

Problem 1: Using the data on the Vesta V90 – 3.0 MW (turbine diametre = 90 m) in Figure 1, find the turbine's efficiency for **A)** just above the cut-in speed (5m/s), **B)** the nominal speed (15 m/s) **C)** the cut-out speed (25 m/s) and compare.

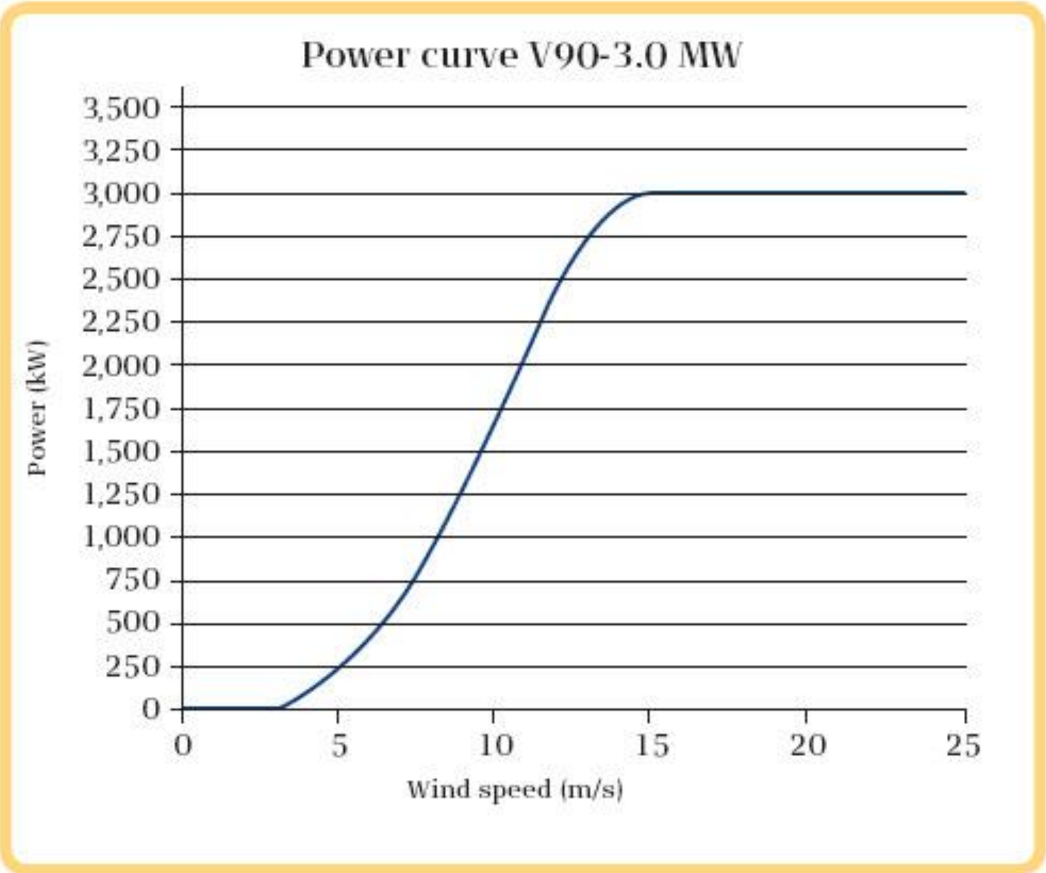

Figure 1. The optimal wind speeds needed to extract varying amounts of power using a Vesta V90 – 3.0 MV wind turbine.

Solution 1A):

Approach: Find the mass of the air going through the turbine each second, use that mass to find the kinetic energy and power of the air, and from that, calculate the efficiency.

What we know:

Wind speed = 5 m/s

Diameter of the turbine = 90 m

Actual power output = 250 Kw

Finding the mass:

The volume of the air passing through the turbine per second

$$= (\text{Area rotor covers})(v\ )$$
$$= \pi\, r^2\, v$$
$$= \pi\ (45\text{m})^2(5\ \text{m/s})$$
$$= 31{,}809\ \text{m}^3/\text{s}$$

The mass of air per second

$$= \rho V$$
$$= (1.2\ \text{kg/m}^3)(31{,}809\ \text{m}^3/\text{s})$$
$$= 38{,}170\ \text{kg/s}$$

Finding the power:

The Kinetic energy of this air per second

$$= \frac{1}{2}\text{mv}^2$$
$$= \frac{1}{2}(38{,}170\ \text{kg/s})(5\ \text{m/s})^2$$
$$= 477{,}125\ \text{W}$$

Finding the efficiency:

Efficiency

$$= \frac{\text{actual output}}{\text{predicted output}}$$
$$= \frac{250\ \text{kW}}{477\ \text{kW}}$$
$$= 52.4\ \%$$

Solution 1B):

This is solved in exactly the same way as part A.

Approach: Find the mass of the air going through the turbine each second, use that mass to find the kinetic energy and power of the air, and from that calculate the efficiency.

What we know:

Wind speed = 15 m/s

Diameter of the turbine = 90 m

Actual power output = 3,000 Kw

Finding the mass:

The volume of the air passing through the turbine per second

$$= (\text{Area rotor covers})(v)$$
$$= \pi r^2 v$$
$$= \pi (45\text{m})^2 (15 \text{ m/s})$$
$$= 95{,}426 \text{ m}^3/\text{s}$$

The mass of air per second

$$= \rho V$$
$$= (1.2 \text{ kg/m}^3)(95{,}426 \text{ m}^3/\text{s})$$
$$= 114{,}511 \text{ kg/s}$$

Finding the power:

The Kinetic energy of this air per second

$$= \frac{1}{2}\text{mv}^2$$
$$= \frac{1}{2}(114{,}511 \text{ kg/s})(15 \text{ m/s})^2$$
$$= 12{,}882 \text{ kW}$$

Finding the efficiency:

Efficiency

$$= \frac{\text{actual output}}{\text{predicted output}}$$
$$= \frac{3{,}000 \text{ kW}}{12{,}882 \text{ kW}}$$
$$= 23.3\ \%$$

Solution 1C):

Approach: Find the mass of the air going through the turbine each second, use that mass to find the kinetic energy and power of the air, and from that calculate the efficiency.

This is solved in exactly the same way as part A.

What we know:

Wind speed = 25 m/s

Diameter of the turbine = 90 m

Actual power output = 3,000 kW

Finding the mass:

The volume of the air passing through the turbine per second

$$= (\text{Area rotor covers})(v\ )$$
$$= \pi\, r^2\, v$$
$$= \pi\ (45\text{m})^2(25\ \text{m/s})$$
$$= 159{,}043\ \text{m}^3/\text{s}$$

The mass of air per second

$$= \rho V$$
$$= (1.2\ \text{kg/m}^3)(159{,}043\ \text{m}^3/\text{s})$$
$$= 190{,}852\ \text{kg/s}$$

Finding the power:

The Kinetic energy of this air per second

$$= \frac{1}{2}\text{mv}^2$$
$$= \frac{1}{2}(190{,}852\ \text{kg/s})(25\ \text{m/s})^2$$
$$= 59{,}641\ \text{kW}$$

The efficiency:

Efficiency $= \dfrac{\text{actual output}}{\text{predicted output}}$

$$= \frac{3{,}000\ \text{kW}}{59{,}641\ \text{kW}}$$

$$= 5.0\ \%$$

Problem 2: Calculate the yearly greenhouse gas emissions from 1 GWe (gigawatt-electric) power stations powered by wind turbines. Wind turbines do not emit CO_2 as they harness the wind but emissions do occur during the manufacturing of the turbine. For Turkey, the CO_2 emission per GDP is approximately 300 Tonnes / $ 1 million of GDP. For wind turbines, it costs $2 million for a 1 MWe plant, which has a capacity factor of 0.3, and the yearly maintenance is approximately 2% of the initial cost. Assume the average lifetime of a wind turbine is 20 years.

Solution 2:

A wind turbine farm

What we know:

It costs $ 2 million for a 1 MWe farm

Capacity factor is 0.3

Maintenance costs 2% of the initial cost per year

For Turkey, the CO_2 emission per GDP is 300 Tonnes / $ 1 million of GDP.

Finding the yearly cost:

We want an output of 1 GWe.

GWe input needed $= \frac{1}{0.3}$

$= 3.3 \text{ GWe}$

Finding the total cost:

Initial costs $= (3.3 \text{ GWe})(\$2 \text{ million/MWe})(1000 \text{ MWe/GWe})$

$= \$\,6600 \text{ million}$

Yearly maintenance costs $= (\$\,6600 \text{ million})(2\%)$

$= \$\,132 \text{ million/year}$

Total costs over lifetime of farm $= \$\,6600 \text{ million} + (20 \text{ years})(\$\,132 \text{ million/year})$

$= \$\,9240 \text{ million}$

$= \$\,9200 \text{ million}$

Finding the mass of CO_2 produced:

CO_2 produced $= (\$\,9240 \text{ million})\left(\frac{300\text{T}}{\$\,1 \text{ million}}\right)$

$= 2{,}772 \text{ x } 10^3 \text{ Tonnes}$

$= 138{,}600 \text{ Tonnes/year}$

Problem 3: Design a set of turbines of a reasonable size that could produce 1GW of electrical power under common wind conditions at a site you can choose in Ankara (for now you can neglect other factors such as land formations and wind consistency). You need to specify the number of turbines and land area needed.

Solution 3:

Approach: Assuming that Ankara has a mean wind speed of 12 m/s. Let us use the Vesta V90 – 3.0 MW wind turbine (Fig. 1) operating at approximately 30% efficiency. We will find the power output for one turbine, and then determine how many turbines are needed. We will then find the land area so that no wind turbine is within 5 rotor diameters from another.

What we know:

Mean wind speed: 12 m/s

Rotor diametre: 90 m

Finding the mass:

The volume of the air passing through the turbine per second

$$
\begin{aligned}
&= (\text{Area rotor covers})(v) \\
&= \pi r^2 v \\
&= \pi\,(45\text{m})^2(12\text{ m/s}) \\
&= 76{,}341\text{ m}^3/\text{s}
\end{aligned}
$$

The mass of air per second

$$
\begin{aligned}
&= \rho V \\
&= (1.2\text{ kg/m}^3)(76{,}341\text{ m}^3/\text{s}) \\
&= 91{,}609\text{ kg/s}
\end{aligned}
$$

Finding the power:

The Kinetic energy of this air per second

$$
\begin{aligned}
&= \frac{1}{2}\text{mv}^2 \\
&= \frac{1}{2}(91{,}609\text{ kg/s})(12\text{ m/s})^2 \\
&= 6{,}596\text{ kW}
\end{aligned}
$$

The efficiency:

Efficiency $= \dfrac{\text{actual output}}{\text{predicted output}}$

$$= \frac{\text{actual output}}{6{,}596\text{ kW}} = 0.3$$

$$\text{Actual output} = (0.3)(6{,}596\text{ kW}) = 1{,}979\text{ kW}$$

Finding the number of turbines required:

Number of turbines needed $= \frac{1 \text{ x } 10^6 \text{ kW}}{1{,}979 \text{ kW}}$

$= 505 \text{ turbines}$

Finding the land area needed:

The area, *A,* needed is found as *A* = (5 *n d*) ^{2}.

For 4 turbines (*n*=1)

For 9 turbines (*n*=2)

:

For N turbines ($n = \sqrt{\#\text{ of turbines}} - 1$)

So, for 505 turbines...

Area $= \left((5)(d)\sqrt{\#\ of\ turbines} - 1\right)^2$

$$= \left((5)(90)\sqrt{505} - 1\right)^2$$

$$= 1.021 \text{ x } 10^8 \text{ m}^2$$

Problem 4: Assuming that Ankara is a potential site for a turbine farm and wind speeds are constant during each season at 8.57 m/s in winter, 5.75 m/s in spring, 3.71 m/s in summer, and 6.64 m/s in fall. Find the mean speed cubed ($\langle v \rangle^3$) and the mean cubed speed ($< v^3 >$).

Solution 4:

To find the mean speed cubed, first calculate the mean speed, then cube it.

$$= \left(\frac{8.57 + 5.75 + 3.71 + 6.64}{4}\right)^3 = 234.60 \text{ m}^3/\text{s}^3 = 235 \text{ m}^3/\text{s}^3$$

To find the mean cubed speed, take the mean of the cubed velocities.

$$= \frac{8.57^3 + 5.75^3 + 3.71^3 + 6.64^3}{4} = 290.84 \text{ m}^3/\text{s}^3 = 291 \text{ m}^3/\text{s}^3$$

The mean of the cubed speeds is larger than the mean speed cubed. In power calculations, we would rather use the mean cubed speed as it more accurately reflects the average power, which is able to be harnessed from the wind.

References:

Andrews J, and Jelley N. Energy Science – Principles, Technologies and Impacts. New York, NY: Oxford University Press, 2007, chapt. 1, p 11.

http://c21.phas.ubc.ca/problemsets/wind-turbines-problem-set-solutions

http://www.windatlas.ca/en/maps.php

http://collaboration.cmc.ec.gc.ca/science/rpn/modcom/eole/CanadianAtlas0.html

http://www.bchydro.com/etc/medialib/internet/documents/info/pdf/rou_wind_garrad_hassan_report.Par.0001.File.rou_wind_garrad_hassan_report.pdf

http://www.vestas.com/en/wind-power-solutions/wind-turbines/3.0-mw.aspx.